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PARTIAL DIFFERENTIAL EQUATIONS

Pseudodifferential Equations and Boundary Value Problems in a Multidimensional Cone

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Abstract—We consider a special boundary value problem in the Sobolev–Slobodetskii space for a model elliptic pseudodifferential equation in a multidimensional cone. Taking into account the special factorization of the elliptic symbol, we write the general solution of the pseudodifferential equation that contains an arbitrary function. To determine it unambiguously, some integral condition is added to the equation, which makes it possible to write the solution in Fourier transforms.

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INTRODUCTION

The theory of pseudodifferential operators and equations [1, 2] is currently a fairly developed branch of modern mathematics. As a rule, the main works concerned operators and equations either on a smooth manifold without a boundary or on a smooth manifold with a smooth boundary. In recent decades, there has been a certain interest in nonsmooth manifolds, both without and with a boundary [3–6]. The main question that aroused interest was the description of the conditions for an operator to be Fredholm and the presentation of a formula for its index.

In [7, 8], a new approach to constructing the theory of pseudodifferential equations in domains with a nonsmooth boundary was proposed; it is based on a special factorization of the elliptic operator symbol. Moreover, a general concept of an elliptic operator on a manifold with a boundary that can have singularities was proposed in [9]. The implementation of this concept requires the study of the unique solvability of model pseudodifferential equations in canonical domains, which are cones in multidimensional spaces. Within the framework of this approach, model elliptic pseudodifferential operators (equations) and their invertibility (unique solvability) in Sobolev–Slobodetskii spaces were studied in [15–18]. Model equation–operator equation in a cone means that the symbol of the pseudodifferential operator does not depend on the spatial variable. The issues of unique solvability of such equations were investigated using a special factorization of the elliptic symbol, the presence of which is one of the main requirements.

Here we will consider one of the cases of solvability of a pseudodifferential equation with an additional integral condition for a convex cone of arbitrary shape.

1. MAIN NOTATION AND DEFINITIONS

Let $C \subset \mathbb{R}^m$ be a convex cone that does not contain an entire line. We write the equation of the cone surface in the form $x_m = \varphi(x'), x' = (x_1, \cdots, x_{m-1})$ assuming that $\varphi(x')$ is a smooth function in $\mathbb{R}^{m-1}$, $\varphi(0) = 0$.

Let A be a model pseudodifferential operator $\mathbb{R}^3$ with symbol $A(\xi)$ independent of the spatial variable x, defined by the formula

$$(Au)(x) = \int\limits_{\mathbb{R}^3} e^{-ix\cdot\xi} A(\xi)\tilde{u}(\xi)\, d\xi,$$

where $\tilde{u}(\xi)$ is the Fourier transformation of the function $u(x), x \in \mathbb{R}^m$,

$$\tilde{u}(\xi) = \int\limits_{\mathbb{R}^m} e^{ix\cdot\xi} u(x)\, dx, \quad x\cdot\xi = \sum_{k=1}^{m} x_k\xi_k.$$

We will consider a class of symbols that satisfy the condition

$$c_1\big(1+|\xi|\big)^{\alpha} \le \big|A(\xi)\big| \le c_2\big(1+|\xi|\big)^{\alpha}, \quad \alpha\in\mathbb{R},$$

with positive constants c_1 and c_2. An operator A with such a symbol is a bounded linear operator $H^s(\mathbb{R}^m)\to H^{s-\alpha}(\mathbb{R}^m)$. Recall [2] that the space consists of (generalized) functions u with a finite norm

$$\|u\|_s = \left(\int\limits_{\mathbb{R}^m} \big|\tilde u(\xi)\big|^2\big(1+|\xi|\big)^{2s}\,d\xi\right)^{1/2}.$$

We will be interested in the solvability of the equation

$$(Au)(x)=0, \quad x\in C. \tag{1}$$

Note that a homogeneous equation was chosen only for simplicity; the presence of the right-hand side in the equation does not add any fundamental difficulties.

The description of the solvability picture of Eq. (1) depends on the special representation of the elliptic symbol $\tilde A(\xi)$. Before giving a precise definition [7, 8] for such a representation, we introduce some concepts of multidimensional complex analysis [10, 11].

The radial tubular domain over a cone C is the subset of the multidimensional complex space $\mathbb{C}^m$ of the following form

$$T(C)=\{z\in\mathbb{C}^m: z=x+iy,\, x\in\mathbb{R}^m,\, y\in C\}.$$

The dual cone $\overset{*}{C}$ of the cone C is a cone of the form

$$\overset{*}{C}{}^{a}_{+}=\{x\in\mathbb{R}^m: x\cdot y>0,\,\forall y\in C\}.$$

Definition. The wave factorization of a symbol relative to the cone C is its representation in the form

$$A(\xi)=A_{\neq}(\xi)A_{=}(\xi),$$

where the factors $A_{\neq}(\xi), A_{=}(\xi)$ must satisfy the following conditions:

1. $A_{\neq}(\xi)$, $A_{=}(\xi)$ are defined for all values of $\xi\in\mathbb{R}^m$ except for possibly points $\partial\left(\overset{*}{C}\cup(-\overset{*}{C})\right)$.
2. $A_{\neq}(\xi)$, $A_{=}(\xi)$ admit analytical continuation into radial tubular domains $T(\overset{*}{C}{}^{a}_{+}), T(-\overset{*}{C}{}^{a}_{+})$, respectively, for which one has the estimates

$$\big|A_{\neq}^{\pm1}(\xi+i\tau)\big|\le c_1\big(1+|\xi|+|\tau|\big)^{\pm æ},$$
$$\big|A_{=}^{\pm1}(\xi-i\tau)\big|\le c_2\big(1+|\xi|+|\tau|\big)^{\pm(\alpha-æ)}, \quad \forall\tau\in\overset{*}{C}.$$

The number $æ\in\mathbb{R}$ is the index of wave factorization.

2. PRESENTATION OF SOLUTION

Here we will make some assumptions regarding the wave factorization index and describe the structure of the general solution of Eq. (1). We assume that the symbol $A(\xi)$ admits wave factorization with respect to C with an index $æ$ such that $æ-s=n+\varepsilon$, $n\in\mathbb{N}$, $|\varepsilon|<1/2$. To make the presentation more complete, we present here some calculations (details can be found in [14, 15]).

The main stages of constructing a solution are standard [7]. The function $v(x)=-(Au)(x)$ is introduced so that $v(x)=0$ for $x\in\mathbb{R}^m\setminus C$, and Eq. (1) is rewritten as a pair equation in the space $\mathbb{R}^m$,

$$(Au)(x)+v(x)=0, \quad x\in\mathbb{R}^m.$$

Then we apply the Fourier transform

$$A(\xi)\tilde u(\xi)+\tilde v(\xi)=0$$

and the wave factorization

$$A(\xi) = A_{\neq}(\xi)A_{=}(\xi),$$

after which the last equation takes the form

$$\tilde{A}_{\neq}(\xi)\tilde{u}(\xi) = -\tilde{A}_{=}^{-1}(\xi)\tilde{v}(\xi). \tag{2}$$

Due to the analytical properties of $A_{\neq}(\xi)$ in the radial tubular domain $T(\overset{*}{C})$ and the properties of the carriers $F^{-1}_{\xi\to x}(\tilde{A}_{\neq}(\xi))$ and $v(x)$ (details can be found in [7]), we conclude that

$$\operatorname{supp} F^{-1}_{\xi\to x}\big(\tilde{A}_{\neq}(\xi)\tilde{u}(\xi)\big) \subset \overline{C},$$
$$\operatorname{supp} F^{-1}_{\xi\to x}\big(\tilde{A}_{=}^{-1}(\xi)\tilde{v}(\xi)\big) \subset \overline{\mathbb{R}^3 \setminus C}.$$

According to relation (2), this means that both the left- and right-hand sides of this relation can only be a (generalized) function concentrated on the boundary of the cone ∂C.

Since the form of the generalized function concentrated on the line [11] and, as a consequence, on the hyperplane [2] is known, we introduce the transformation $T_\varphi : \mathbb{R}^m \to \mathbb{R}^m$ that translates ∂C^a_+ into the hyperplane $x_m = 0$, and is as follows:

$$\begin{cases} t_1 = x_1 \\ \dots\dots\dots\dots \\ t_{m-1} = x_{m-1} \\ t_m = x_m - \varphi(x'). \end{cases}$$

If we apply the operator T_φ to the function $F^{-1}_{\xi\to x}(\tilde{A}_{\neq}(\xi)\tilde{u}(\xi))$, then the generalized function looks as follows:

$$F^{-1}_{\xi\to x}\big(\tilde{A}_{\neq}(\xi)\tilde{u}(\xi)\big) = \sum_{k=0}^{l}\Big(c_k\big(x'\delta^{(k)}(x_m)\big)\Big),$$

or, in Fourier transforms,

$$\sum_{k=0}^{l}\tilde{c}_k(\xi')\xi_m^k;$$

the number of terms must be such that each element in the sum belongs to the space $H^{s-æ}$. Let us verify

$$\int\limits_{\mathbb{R}^m}\big|\tilde{c}_k(\xi')\xi_m^k\big|^2\big(1+|\xi|\big)^{2(s-æ)}\,d\xi = \int\limits_{\mathbb{R}^{m-1}}\big|\tilde{c}_k(\xi')\big|^2\left(\int\limits_{-\infty}^{+\infty}|\xi_m|^{2k}\big(1+|\xi|\big)^{2(s-æ)}\,d\xi_m\right)d\xi'$$
$$\sim \int\limits_{\mathbb{R}^{m-1}}\big|\tilde{c}_k(\xi')\big|^2\big(1+|\xi'|\big)^{2(s-æ+k+1/2)}\,d\xi',$$

since the integral over ξ_m will exist only under the condition $2(s-æ+k) < -1$. Since $2(s-æ+k) = 2(-n-\varepsilon+k)$, then the inequality $-n-\varepsilon+k < -1/2$ can only be satisfied when $k = 0, 1, \dots, n-1$. In this case, $c_k \in H^{s_k}(\mathbb{R}^{m-1})$, $s_k = s - æ + k + 1/2$. Then we obtain

$$FT_\varphi F^{-1}\big(\tilde{A}_{\neq}(\xi)\tilde{u}(\xi)\big) = \sum_{k=0}^{n-1}\tilde{c}_k(\xi')\xi_m^k,$$

or, eventually,

$$\tilde{u}(\xi) = \tilde{A}_{\neq}^{-1}(\xi) V_{\varphi}^{-1}\left(\sum_{k=0}^{n-1} \tilde{c}_k(\xi')\xi_m^k\right),$$

where

$$V_{\varphi} = FT_{\varphi}F^{-1}.$$

The operators T_{φ} and V_{φ} are defined for generalized functions in [18], where a more detailed description of their properties can be found. Let us formulate the result obtained.

Theorem 1 (on general solution). *Let the symbol $A(\xi)$ admit wave factorization with respect to C with an index æ such that $æ - s = n + \varepsilon$, $n \in \mathbb{N}$, $|\varepsilon| < 1/2$. The general solution of Eq. (1) in Fourier transforms is given by the formula*

$$\tilde{u}(\xi) = A_{\neq}^{-1}(\xi) V_{\varphi}^{-1}\left(\sum_{k=0}^{n-1} \tilde{c}_k(\xi')(x_m^k\right), \tag{3}$$

where $c_k(x') \in H^{s_k}(\mathbb{R}^{m-1})$ are arbitrary functions, $s_k = s - æ + k + 1/2$, $k = 0, 1, 2, \ldots, n-1$.

3. DETAILING

To describe the operator in more detail, let us establish a connection between the Fourier transform and the operator T_{φ},

$$\begin{aligned}(FT_{\varphi}u)(\xi) &= \int\limits_{\mathbb{R}^m} e^{ix\cdot\xi} u\big(x_1, \ldots, x_{m-1}, x_m - \varphi(x')\big)\, dx \\ &= \int\limits_{\mathbb{R}^m} e^{iy'\xi'} e^{i(y_m + \varphi(y'))\xi_m} u(y_1, \ldots, y_{m-1}, y_m)\, dy \\ &= \int\limits_{\mathbb{R}^{m-1}} e^{i\varphi(y')\xi_m} e^{iy'\xi'} \hat{u}(y_1, \ldots, y_{m-1}, \xi_m)\, dy',\end{aligned}$$

where $\hat{u}$ denotes the Fourier transform with respect to the last variable and the Jacobian of the transformation T_{φ} is equal to one. Thus, the last formula contains the $(m-1)$-dimensional Fourier transform of two functions $e^{i\varphi(y')\xi_m}$ and $\hat{u}(y_1, \ldots, y_{m-1}, \xi_m)$. As is well known, the result should be the convolution of their Fourier transforms. Let us denote the $(m-1)$-dimensional Fourier transform ($y' \to \xi'$ in the sense of distributions) of the function $e^{i\varphi(y')\xi_m}$ by the symbol $E_{\varphi}(\xi', \xi_m)$ and write the operator V_{φ} using the formula

$$(V_{\varphi}\tilde{u})(\xi) = (E_{\varphi} * \tilde{u})(\xi),$$

where the sign $*$ denotes the convolution over the first $m-1$ variables and the multiplication operator over the last variable ξ_m. Thus, V_{φ} is a specific combination of convolution and multiplier with kernel $E_{\varphi}(\xi', \xi_m)$. In more detail,

$$V_{\varphi} : \tilde{u}(\xi) \to \int\limits_{\mathbb{R}^{m-1}} E_{\varphi}(\xi' - \eta', \xi_m)\tilde{u}(\eta', \xi_m) d\eta'. \tag{4}$$

Unfortunately, the rather general form of operator (4) makes it difficult to use in further research; however, in some specific cases it can be described using one-dimensional singular integral operators [12–14], and in its general form it allows one to write the solution of one boundary value problem with an additional integral condition (see below).

3.1. Examples

All examples given below concern the case of $n = 1$. Consequently, only one arbitrary function $\tilde{c}_0(\xi')$ is involved in the formula.

– In the two-dimensional case, we set $\varphi(x_1)=a|x_1|$, $a>0$. Then [16]

$$(V_\varphi\tilde c_0)(\xi_1,\xi_2)=\frac{\tilde c_0(\xi_1+a\xi_2,\xi_2)+\tilde c_0(\xi_1-\xi_2,\xi_2)}{2} + \frac{i}{2\pi}\text{p.v.}\int\limits_{-\infty}^{+\infty}\frac{\tilde c_0(\eta,\xi_2)\,d\eta}{\xi_1+a\xi_2-\eta}-\frac{i}{2\pi}\text{p.v.}\int\limits_{-\infty}^{+\infty}\frac{\tilde c_0(\eta,\xi_2)\,d\eta}{\xi_1-a\xi_2-\eta}, \tag{5}$$

where p.v. denotes the integral in the sense of principal value [11–13]. For a more convenient recording of the formula (5), we introduce the following notation:

$$(Su)(\xi)=\frac{i}{\pi}\text{p.v.}\int\limits_{-\infty}^{+\infty}\frac{\tilde u(\eta,\xi_2)\,d\eta}{\xi_1-\eta},$$

$$P=\frac{1}{2}(I+S),\quad Q=\frac{1}{2}(I-S),$$

where I is the identity operator. In this notation, formula (3) takes the form

$$\tilde u(\xi)=\tilde A_{\neq}^{-1}(\xi)\big((P\tilde c_0)(\xi_1-a\xi_2)+(Q\tilde c_0)(\xi_1+a\xi_2)\big)$$

– In the three-dimensional case, we set $\varphi(x_1,x_2)=a|x_1|+b|x_2|$, $a,b>0$. Then [15]

$$\begin{aligned}(V_\varphi\tilde c_0)(\xi)=&\frac{1}{4}\tilde c_0(\xi_1-a\xi_3,\xi_2-b\xi_3)-\frac{1}{2}(S_1\tilde c_0)(\xi_1-a\xi_3,\xi_2-b\xi_3)\\&-\frac{1}{2}(S_2\tilde c_0)(\xi_1-a\xi_3,\xi_2-b\xi_3)+(S_1S_2\tilde c_0)(\xi_1-a\xi_3,\xi_2-b\xi_3)\\&+\frac{1}{4}\tilde c_0(\xi_1-a\xi_3,\xi_2+b\xi_3)-\frac{1}{2}(S_1\tilde c_0)(\xi_1-a\xi_3,\xi_2+b\xi_3)\\&+\frac{1}{2}(S_2\tilde c_0)(\xi_1-a\xi_3,\xi_2+b\xi_3)-(S_1S_2\tilde c_0)(\xi_1-a\xi_3,\xi_2+b\xi_3)\\&+\frac{1}{4}\tilde c_0(\xi_1+a\xi_3,\xi_2-b\xi_3)+\frac{1}{2}(S_1\tilde c_0)(\xi_1+a\xi_3,\xi_2-b\xi_3)\\&-\frac{1}{2}(S_2\tilde c_0)(\xi_1+a\xi_3,\xi_2-b\xi_3)-(S_1S_2\tilde c_0)(\xi_1+a\xi_3,\xi_2-b\xi_3)\\&+\frac{1}{4}\tilde c_0(\xi_1+a\xi_3,\xi_2+b\xi_3)+\frac{1}{2}(S_1\tilde c_0)(\xi_1+a\xi_3,\xi_2+b\xi_3)\\&+\frac{1}{2}(S_2\tilde c_0)(\xi_1+a\xi_3,\xi_2+b\xi_3)+(S_1S_2\tilde c_0)(\xi_1+a\xi_3,\xi_2+b\xi_3),\end{aligned}$$

where we have introduced the notation

$$(S_1u)(\xi_1,\xi_2,\xi_3)=\frac{i}{2\pi}\text{p.v.}\int\limits_{-\infty}^{+\infty}\frac{u(\tau,\xi_2,\xi_3)\,d\tau}{\xi_1-\tau},$$

$$(S_2u)(\xi_1,\xi_2,\xi_3)=\frac{i}{2\pi}\text{p.v.}\int\limits_{-\infty}^{+\infty}\frac{u(\xi_1,\eta,\xi_3)\,d\eta}{\xi_2-\eta}.$$

– In the m-dimensional space, we can consider the cone $\varphi(x')=a|x'|$, $a>0$. Then [17]

$$(V_\varphi\tilde c_0)(\xi)=\lim_{\tau\to0+}\frac{1}{(2\pi)^{m-1}}\int\limits_{\mathbb{R}^{m-1}}\frac{iaz_m2^{m-1}\pi^{\frac{m-2}{2}}\Gamma(m/2)\tilde c_0(\eta')\,d\eta'}{\big((\xi_1-\eta_1)^2+(\xi_2-\eta_2)^2+\dots+(\xi_{m-1}-\eta_{m-1})^2-a^2z_m^2\big)^{m/2}},$$

where Γ is the Euler gamma function, $z_m=\xi_m+i\tau$.

3.2. Boundary Value Problems with an Integral Condition

In this section we will return to the general operator, but consider an operator whose symbol admits wave factorization with an index satisfying an inequality. Thus, according to Theorem 1, the general solution contains one arbitrary function. To determine it, we set the integral condition

$$\int u(x', x_m)\, dx_m = f(x'). \tag{6}$$

Theorem 2. *Let the symbol $A(\xi)$ admit wave factorization with respect to C with an index æ such that $1/2 < \text{æ} - s < 3/2$. Then for any function $f \in H^{s+1/2}(R^{m-1})$ problem (1), (6) has a unique solution that can be written in Fourier transforms using the formula*

$$\tilde{u}(\xi) = (2\pi)^{m-1} A_{\neq}(\xi', 0)\tilde{f}(\xi').$$

Proof. In Fourier transforms, condition (6) takes the form

$$\tilde{u}(\xi', 0) = \tilde{f}(\xi'),$$

and then, according to the general solution formula (3), we have the relation

$$A_{\neq}^{-1}(\xi', 0) \int\limits_{\mathbb{R}^{m-1}} E_{\varphi}(\xi' - \eta', 0)\tilde{c}_0(\eta')\, d\eta' = \tilde{f}(\xi') \tag{7}$$

with allowance for formula (4). It remains to clarify what the function $E_{\varphi}(\xi', 0)$ is. By definition of this function

$$F_{x' \to \xi'} e^{i\varphi(x')\xi_m};$$

this implies that this function tends to the Fourier transformation of unity as $\xi_m \to 0$ (recall that we are within the framework of the theory of generalized functions), which will lead to an $(m-1)$-dimensional δ-function with a factor $(2\pi)^{-m+1}$. Equation (7) takes the form

$$(2\pi)^{-m+1} A_{\neq}^{-1}(\xi', 0) \int\limits_{\mathbb{R}^{m-1}} \delta(\xi' - \eta')\tilde{c}_0(\eta')\, d\eta' = \tilde{f}(\xi');$$

based on this, we obtain

$$(2\pi)^{-m+1} A_{\neq}^{-1}(\xi', 0)\tilde{c}_0(\xi') = \tilde{f}(\xi').$$

Finding $\tilde{c}_0$ and substituting it into the general solution formula, we obtain the assertion in Theorem 2.

CONCLUSIONS

In this paper, we limited ourselves to only a convex cone, but it is possible to transfer most of the results to the case where the cone is the complement of a convex cone. In addition, similar methods are used to study situations of multidimensional edges, where the cone is the direct product of an m-dimensional space and a convex cone.

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CONFLICT OF INTEREST

The author of this work declares that he has no conflicts of interest.

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