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# Polarization of Diffraction Radiation of a Bunch of Charged Particles on a Metal Sphere

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Received September 12, 2023; revised November 25, 2023; accepted November 25, 2023

**Abstract**—Diffraction radiation is widely used for the nondestructive diagnostics of charged particle beams. In the series of previous works, a method was developed for describing the diffraction radiation of a nonrelativistic particle on a perfectly conducting sphere, based on the method of images well-known from electrostatics. This method allows one to derive the analytic formulas for two main radiation characteristics, i.e., spectral angular density and polarization. The characteristic features of these values allow the possibility of developing, on their basis, new methods for monitoring the parameters of the trajectory of a moving particle in relation to the sphere center. In this work, formulas are obtained that describe the polarization of the coherent diffraction radiation on a metal sphere from a pancake-bunch of charged particles. It is shown that the polarization of the radiation in this case makes it possible to estimate the positions of bunch edges relative to the center of the sphere. This can be used for the nondestructive measurement of characteristic bunch dimensions.

**Keywords:** diffraction radiation, conducting sphere, method of images, polarization, particles monitoring, beam diagnostics

**DOI:** 10.1134/S1027451024700502

## INTRODUCTION

Diffraction radiation occurs when a charged particle moves uniformly near the inhomogeneity of the electromagnetic properties of the medium; in the simplest case, near a conducting half-plane in vacuum [1, 2]. This phenomenon can be used for the nondestructive diagnostics of charged particle beams (e.g., monographs [3, 4]). In [5, 6], an approach was proposed to describe the diffraction radiation of a nonrelativistic charged particle on a perfectly conducting sphere, based on the image method known from electrostatics (e.g., [7, 8]). In [9, 10], this approach was used to find the polarization of radiation generated by an individual particle flying past the sphere. This paper examines the polarization characteristics of coherent diffraction radiation generated on a sphere by a short bunch (pancake-bunch) of nonrelativistic charged particles and demonstrates how measuring the polarization of the radiation makes it possible to diagnose the position of the edges of the bunch.

Note that the diagnostics of bunches of nonrelativistic particles using the long-wave transition and diffraction radiation generated by them on various targets is quite widely discussed in modern literature (e.g., [11, 12]).

## METHOD

The imaging method consists of reproducing the field created by a charged particle in the presence of conducting bodies by introducing, along with a real point charge $e_0$, one or more fictitious charges ("images" of a real charge). In the case of rectilinear and uniform motion of a real charge (at velocity $v_0$) near the conducting sphere, its image will describe a circle (Fig. 1); that is, the charge will move with acceleration, which leads to the generation of radiation.

The amplitude of the diverging wave of the vector potential of the radiation field is proportional to the quantity (e.g., [13–15]):

$$\mathbf{I} = \int_{-\infty}^{\infty} e(t)\,\mathbf{v}(t)\exp[i(\omega t - \mathbf{k}\mathbf{r}(t))]\,dt, \tag{1}$$

where $\omega$ and $\mathbf{k}$ are the frequency and wave vector of radiation, $|\mathbf{k}| = \omega/c$ and $e(t)$, $\mathbf{r}(t)$, and $\mathbf{v}(t)$ is the magnitude, position, and velocity of the "image" of the charge, respectively. The spectral-angular density of radiation with a certain polarization will be described by the following formula:

$$\left(\frac{dE}{d\omega d\Omega}\right)_\alpha = \frac{\omega^2}{4\pi^2 c^3}|\mathbf{e}_\alpha \cdot \mathbf{I}|^2, \tag{2}$$

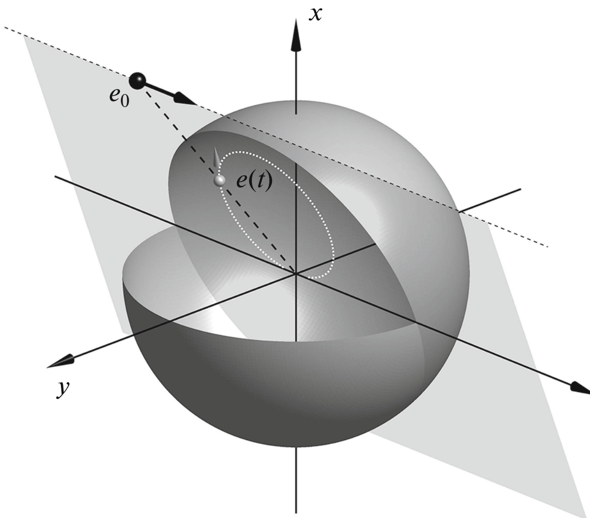


**Fig. 1.** Circular path (dashed line) of "image" $e(t)$ of charge $e_0$ when a charged particle moves near a conducting sphere.

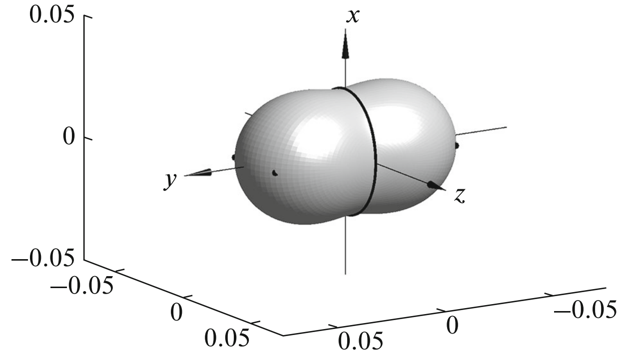


**Fig. 2.** Directional pattern of diffraction radiation on a sphere at $b = R + 0$, $v_0 = 0.1c$, and $R\omega/v_0 = 2.34$: the solid line refers to the directions of radiation with linear polarization (100%); dots refer to circular polarization (100%).

where $\mathbf{e}_\alpha$ $(\alpha = 1,2)$ are two polarization vectors orthogonal to the wave vector $\mathbf{k}$ and to each other:

$$\mathbf{e}_1 = \frac{\mathbf{k}\times\mathbf{v}_0}{|\mathbf{k}\times\mathbf{v}_0|} = \mathbf{e}_x \sin\varphi - \mathbf{e}_y \cos\varphi,$$
$$\mathbf{e}_2 = \frac{\mathbf{k}\times\mathbf{e}_1}{k} = \mathbf{e}_x \cos\theta\cos\varphi \qquad (3)$$
$$-\ \mathbf{e}_y \cos\theta\cos\varphi - \mathbf{e}_z \sin\theta,$$

where $\mathbf{v}_0$ is the velocity vector of the incident particle.

The maximum radiation intensity of a nonrelativistic particle occurs in the region of long wavelengths:

$$\lambda \gg 2\pi R^2/b, \quad \omega \ll cb/R^2, \qquad (4)$$

where $R$ is radius of the sphere and $b = \sqrt{x^2 + y^2}$ is the impact parameter of the incident particle. In this case, the vector components $\mathbf{I}$ will be equal to [5, 6]:

$$I_x^{(1)} = \frac{4}{3} e_0 R^3 \frac{\omega^2}{v_0^2} i \frac{x}{b} K_1\left(\frac{\omega}{v_0} b\right),$$
$$I_y^{(1)} = \frac{4}{3} e_0 R^3 \frac{\omega^2}{v_0^2} i \frac{y}{b} K_1\left(\frac{\omega}{v_0} b\right), \qquad (5)$$
$$I_z^{(1)} = -\frac{4}{3} e_0 R^3 \frac{\omega^2}{v_0^2}\left[K_0\left(\frac{\omega}{v_0} b\right) + \frac{v_0}{2\omega b} K_1\left(\frac{\omega}{v_0} b\right)\right],$$

where $e_0$ is the charge of the incident particle and $K_0(x)$ and $K_1(x)$ are modified Bessel functions of the second kind (MacDonald functions). Dimensionless quantity $\left(4\pi^2 c/e_0^2\right)\sum_\alpha \left(dE/d\omega d\Omega\right)_\alpha$, characterizing the spectral-angular radiation density summed over polarization vectors, is presented in Fig. 2 in the form of a radiation pattern for the case when the trajectories of the incident particle and its images lie in the plane $(x, z)$. The maximum spectral radiation density occurs at combinations of parameter values $\omega b/v_0 \approx 2.34$ [5, 9, 10].

The polarization tensor [16] taking into account (2) will be equal to

$$\rho_{\alpha\beta} = \frac{(\mathbf{e}_\alpha \cdot \mathbf{I})(\mathbf{e}_\beta \cdot \mathbf{I})^*}{(\mathbf{e}_1 \cdot \mathbf{I})(\mathbf{e}_1 \cdot \mathbf{I})^* + (\mathbf{e}_2 \cdot \mathbf{I})(\mathbf{e}_2 \cdot \mathbf{I})^*}. \qquad (6)$$

Expressing its components through the Stokes parameters [16] allows one to extract information about the degree of radiation polarization:

$$\rho_{\alpha\beta} = \frac{1}{2}\begin{pmatrix} 1+\xi_3 & \xi_1 - i\xi_2 \\ \xi_1 - i\xi_2 & 1-\xi_3 \end{pmatrix}, \qquad (7)$$

where $\xi_1 = l\sin 2\phi$, $\xi_2 = A$, $\xi_3 = l\cos 2\phi$, $l$ is the degree of maximum linear polarization, $\phi$ is an angle between the direction of maximum linear polarization and vector $\mathbf{e}_1$, and $A$ is the degree of circular polarization.

In the case of a single incident particle, substitution of (5) into (6) shows that $\xi_1^2 + \xi_2^2 + \xi_3^2 = 1$; that is, the radiation is completely polarized, in the general case elliptically, and linearly in the plane containing the center of the sphere and the trajectory of the incident particle (the plane in Fig. 1). On the other hand, for radiation directions perpendicular to this plane and close to them (within very wide limits), the radiation polarization will be close to circular (right for $y < 0$ and left for $y > 0$, depending on the sign of $\xi_2$). This creates a fundamental possibility for monitoring the trajectory of a flying particle, which is discussed in detail in [9, 10, 17].

## RESULTS AND DISCUSSION

Let us now consider the coherent radiation of a bunch of charged particles, the longitudinal size of which is negligible compared to the transverse one (pancake-bunch), with the charge density distribution

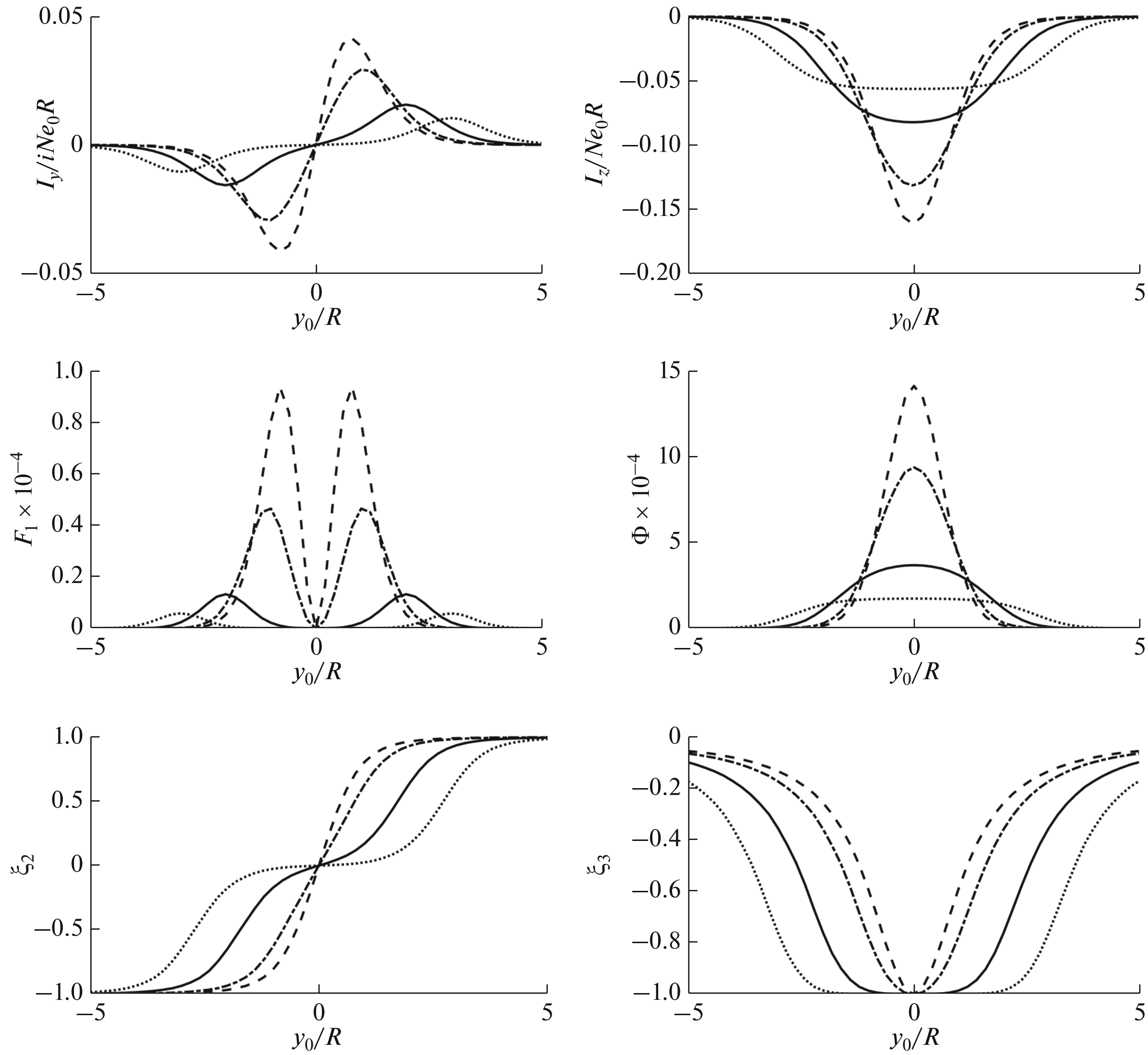


**Fig. 3.** Characteristics of diffraction radiation of the rectangular homogeneous bunch of particles on the sphere in direction $\theta = \pi/2$, $\varphi = 0$ with dimensions $2a_x = R$, $2a_y = 6R$ (dotted line), $4R$ (solid line), $2R$ (dash-dotted line), and $R$ (dashed line). Impact parameter of the bunch center in the direction of the axis $x$ equals $x_0 = 1.6R$, impact parameter of the bunch center in the direction of the axis $y$ shown on the $x$-axis, and other parameters coincide with the parameters in Fig. 2.

function $Nn(x, y)$, where $N$ is the total number of particles in the bunch and $n(x, y)$ is function normalized to unity:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n(x, y)\,dxdy = 1.$$

In the case when the size of the bunch is much smaller than the radiation wavelength, components $\mathbf{I}$, describing the coherent radiation of the bunch, can be found by integrating (5) with the distribution function:

$$I_j = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_j^{(1)}(x, y)\,n(x, y)\,dxdy.$$

In particular, for components $I_y$ after integration by parts we get:

$$I_y = N\frac{4}{3}e_0R^3\frac{\omega^2}{v_0^2}i\frac{v_0}{\omega} \times \int K_0\left(\frac{\omega}{v_0}\sqrt{x^2 + y^2}\right)\frac{\partial n(x, y)}{\partial y}dxdy. \quad (8)$$

Substitution of component $\mathbf{I}$ in (2) leads to formulas for the spectral-angular density of coherent radiation from a bunch with a certain polarization:

$$\left(\frac{dE}{d\omega d\Omega}\right)_\alpha = \frac{\omega^2 N^2}{4\pi^2 c^3}\left|\mathbf{e}_\alpha \cdot \mathbf{I}\right|^2 \equiv \frac{e_0^2 N^2}{4\pi^2 c}F_\alpha, \quad (9)$$

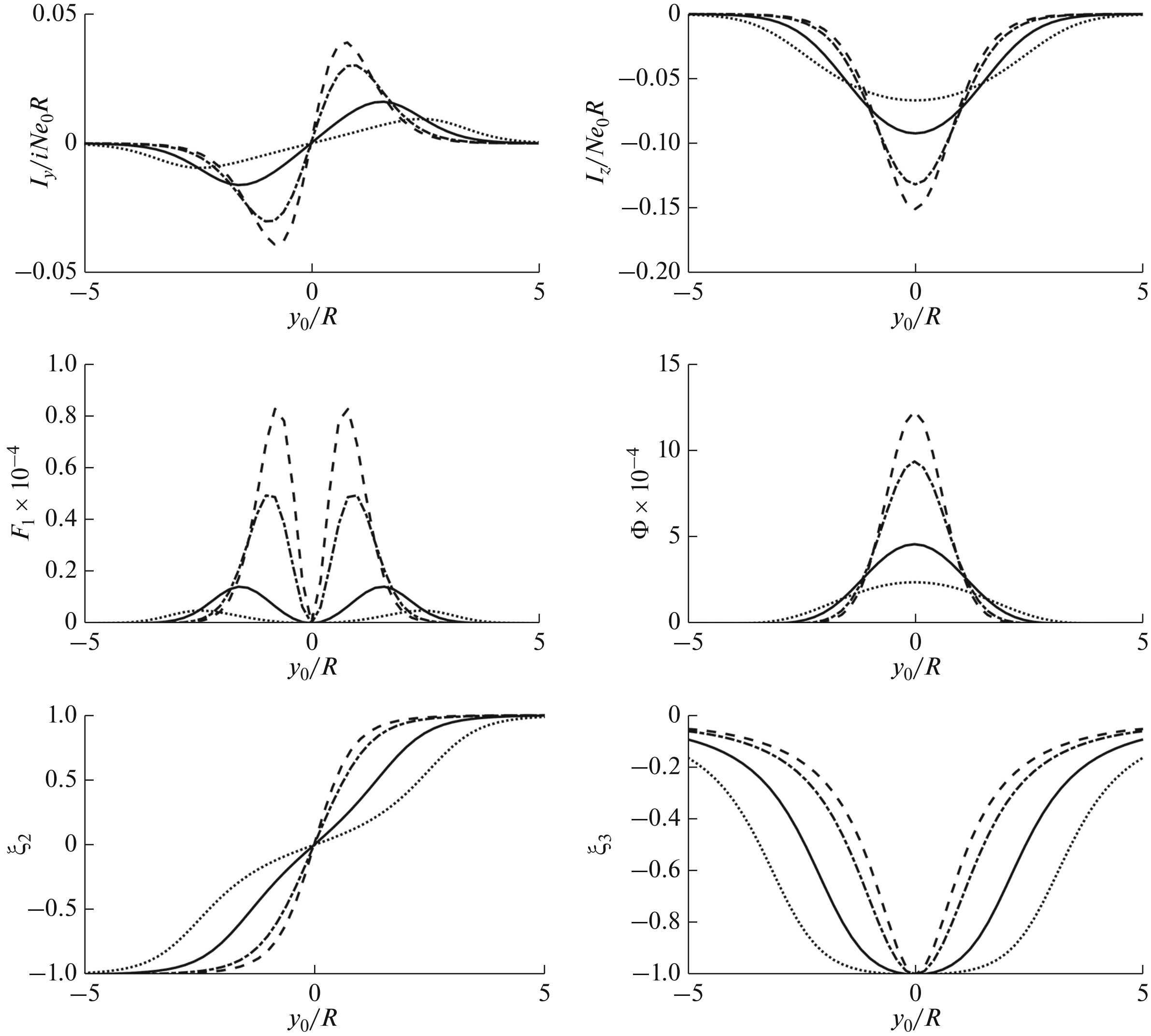


**Fig. 4.** The same as in Fig. 3 but for the elliptical homogeneous bunch of particles, the lengths of the axes of which are equal to the sides of the rectangular bunch.

and summed over polarization vectors:

$$\frac{dE}{d\omega d\Omega} = \frac{\omega^2 N^2}{4\pi^2 c^3} \sum_{\alpha=1}^{2} \left|\mathbf{e}_\alpha \cdot \mathbf{I}\right|^2 \equiv \frac{e_0^2 N^2}{4\pi^2 c} \Phi, \tag{10}$$

where for the convenience of presentation of graphs the dimensionless function $F_\alpha$ and $\Phi$ are highlighted

From formula (8) it follows that radiation polarized in the direction of the axis $y$ will be maximum when the maximum change in the density of a bunch of particles along this axis passes over the top point of the sphere (left or right border of the bunch). This circumstance makes it possible to detect the edges of the bunch.

In the case when the radiation detector is located above the top point of the sphere ($\theta = \pi/2$, $\varphi = 0$), the unit polarization vectors will be equal to $\mathbf{e}_1 = -\mathbf{e}_y$, $\mathbf{e}_2 = -\mathbf{e}_z$ and the characteristics of radiation arising in this direction for a homogeneous bunch of rectangular and elliptical shapes are presented in Figs. 3 and 4, respectively. It can be seen that the maximum of radiation polarized in the direction of the axis $y$ will be observed when the right or left edges of the bunch pass over the top point of the sphere, as follows from (8). Thus, by moving the sphere and the detector and recording the intensity of radiation polarized in the direction of the axis $y$, it is possible to nondestructively find the positions of the left and right edges of the bunch, and, therefore, to determine its size in this direction. In addition, the appearance of a significant fraction of circular polarization of the radiation or a significant decrease in the magnitude of the linear polarization indicates the proximity of the edge of the bunch to the top point of the sphere.

## CONCLUSIONS

Coherent diffraction radiation from a short bunch of nonrelativistic charged particles flying past a conducting sphere is considered. It has been established that the formulas describing the polarization characteristics of such radiation include coordinate derivatives of the particle density distribution function in the bunch.

Thus, the polarization of the bunch radiation turns out to be sensitive to the position of its edges relative to the plane containing the velocity of the beam particles, the center of the sphere, and the direction to the radiation detector. This creates opportunities for the nondestructive assessment of the bunch size and the position of its edges.

## FUNDING

This work was supported by ongoing institutional funding. No additional grants to carry out or direct this particular research were obtained.

## CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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