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About the Best Basis for Cognitive Radio Signal Development

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Abstract: The considered the problem of data transmitting and receiving control in the presence of a large number of electromagnetic radiation sources interfering during unpredictable time and in unpredictable frequency intervals. One of the most natural and effective methods of difficulty overcoming for the provision of electromagnetic compatibility is the use of adaptive determination procedures for frequency intervals that are available for information transfer. It is important to create such signal-code constructions that ensure minimal leakage of a channel signal energy beyond this interval which is equivalent to the minimization of this inter-channel interference measure. In this study, based on the formulation and the solution of the corresponding variational problem they showed that the best basis for the signal-code construction development with a minimum level of energy infiltration beyond the specified frequency interval are the eigenfunctions of the integral ratio core which (the core) is called subband one. The conditions are established that allow to achieve zero infiltration of energy, the procedures for the synthesis of signal-code structures and their decoding re developed in the presence of interference.

Key words: Cognitive radio, eigenfunctions of subband cores, time and frequency resources of radio communication systems, variational problems, set frequency subband, energy

INTRODUCTION

The current electromagnetic environment in which radio communication is carried out is characterized by the presence of frequency subbands suitable for information transfer during unpredictable time intervals. Therefore, now a days one of the main trends in the development of radio communication systems is the application of flexible reorganization principles concerning the procedures of radio signal transmitting and receiving, the parameters of which are used for the transmitted information encoding (Chen and Prasad, 2009; Doyle, 2009; Akyildiz *et al*., 2006; Yucek and Arslan, 2009; Zhilyakov, 2012; Shakhnovich, 2006; Andrews, 2005). The need for such a restructuring is determined by the desire to improve the efficiency of data transmission and reception in the presence of a large number of electromagnetic radiation sources that operate at unpredictable time intervals and a broad band of frequencies. In order to increase the efficiency of time-frequency resources of radio communication system use it is advisable to use signal-code constructions that allow to generate channel signals which have the maximum energy concentration in a given frequency subband at a given duration.

Thus, the problem consists in channel signal generation with a predetermined duration, the energy of which is concentrated in the selected subband. This problem is considered in the framework of this study and a new basis of orthonormal functions is proposed for its solution.

MAIN PART (THEORETICAL BASIS OF SIGNAL SUBBAND ANALYSIS AND SYNTHESIS)

Let x (t), t∈ (0, T) is a certain continuous signal (time function) of finite duration and energy whose fourier transformant (spectrum) has the following Eq. 1:

$$X(\omega) = \int_0^T x(t) \exp(-j\omega t)\, dt \tag{1}$$

Where:
$\omega = 2\pi v$, v-frequency
$j = (-1)^{1/2}$

In accordance with (Zhilyakov, 2012), let:

$$P_r(x) = \int_{\omega \in \Omega_r} |X(\omega)|^2 d\omega/2\pi \tag{2}$$

where, the subband of the following type is meant:

$$\Omega_r = [-\Omega_{2r}, -\Omega_{1r}) \cup [\Omega_{1r}, \Omega_{2r}), \Omega_{1r} \geq 0 \tag{3}$$

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It seems natural to call the characteristic Eq. 2 as apart of the energy falling in a frequency sub-band of the Eq. 3 symmetrically located relative to the coordinate origin. Substituting the representation Eq. 1 into the right-hand side of Eq. 2, we obtain an important representation for further research after simple transformations:

$$P_r(x) = \int_0^T\int_0^T A_r(t-\tau)\, x(t)\, x(\tau)\, dtd\tau \tag{4}$$

Determining the part of the energy directly in the region of the originals (time). Here, $A_r(t-\tau)$ is subband core:

$$A_r(t-\tau) = \int_{\omega\in\Omega_k} \exp(-j\omega(t-\tau))d\omega/2\pi \tag{5}$$

which in accordance with the definition Eq. 3 is easy to represent in a more convenient form for the calculations in the original region:

$$A(t) = 2\sin(\Delta_r t/2)\cos(\omega_r t)/\pi t \tag{6}$$

Where:

$$\Delta_r = \Omega_{2r}-\Omega_{1r};\ \omega_r = (\Omega_{2r}+\Omega_{1r})/2 \tag{7}$$

It is clear that the subband core (Eq. 5) is symmetric and it follows directly from the definition (Eq. 2) and the representation Eq. 4 that it is positive definite. Therefore, it can be represented in the Eq. 8 (Smirnov, 1974):

$$A(t-\tau) = \sum_{k=1}^{\infty} \lambda_k g_k(t) g_k(\tau) \tag{8}$$

where, the summands in the sum are determined by the eigenvalues and the functions of the subband core which satisfy the equations of the following Eq. 9:

$$\lambda_k g_k(t) = \int_0^T A_r(t-\tau) g_k(\tau) d\tau \tag{9}$$

and are orthonormal, that is the following equalities are fulfilled:

$$(g_k, g_i) = \int_0^T g_k(t)\, g_i(t) dt = 0,\ i \neq k \tag{10}$$

$$\|g_k\|^2 = (g_k, g_k) = 1 \tag{11}$$

Therefore, the representation (Eq. 4) may have the following form after the substitution of Eq. 8:

$$P_r(x) = \sum_{k=1}^{\infty} \lambda_k \alpha_k^2 \tag{12}$$

Where:

$$\alpha_k = (x, g_k) \tag{13}$$

Own functions and numbers of a subband core have a number of properties useful for signal analysis and synthesis. In particular, due to the nonnegative definiteness of the core its eigenvalues are also nonnegative and further without the loss of generality we assume that they are ordered by decrease:

$$\lambda_k \geq \lambda_{k+1},\ \forall k \geq 1 \tag{14}$$

Using the representation Eq. 8 and the property Eq. 11, we have:

$$A_r(0)\int_0^T dt = \sum_{k=1}^{\infty} \lambda_k$$

Together with the definition Eq. 5, it gives the following equality for the sum of eigenvalues:

$$\sum_{k=1}^{\infty} \lambda_k = T\Delta_r/\pi \tag{15}$$

Let:

$$Q_k(\omega) = \int_0^T q_k(t) \exp(-j\omega t) dt \tag{16}$$

Then, after the substitution of the subband core Eq. 5 into the Eq. 9 it is not difficult to obtain the following relation:

$$\lambda_k g_k(t) = \int_{\omega\in\Omega_r} Q_k(\omega) \exp(j\omega t) d\omega/2\pi \tag{17}$$

Which shows that the eigenfunction corresponding nonzero eigennumber is completely determined by the fourier transformant segment from the given subband. On the basis of the relaltion (Eq. 17), we have the following equation:

$$\lambda_k \int_0^T g_k(t) \exp(-jzt) dt = \int_{\omega\in\Omega_r} Q_k(\omega) \int_0^T \exp(jt(\omega-z)) dt d\omega/2\pi$$

which is converted into the following equation:

$$\lambda_k Q_k(z) = \int_{\omega\in\Omega_r} D(z-\omega) Q_k(\omega) d\omega \tag{18}$$

Where:

$$D(z\text{-}\omega) = \int_0^T \exp(-jt(z\text{-}\omega)dt/2\pi = \exp(-jT(z\text{-}\omega)/2)\sin(T(z\text{-}\omega)/2)/\pi(z\text{-}\omega) \quad (19)$$

Thus, the fourier transformants of the eigenfunctions are also, the eigenfunctions of the core in the following (Eq. 19) at the same eigenvalues. Let's, also note that at nonzero eigenvalues to the relation (Eq. 18) determines the extension of the fourier transformants concerning the eigenfunctions to the entire frequency axis. On the other hand, the relation (Eq. 17) allows us to obtain the following equality:

$$\lambda_k \int_0^T g_k(t)\, g_m\ (t)dt = \int_{\omega\in\Omega_r} Q_k(\omega)\, Q_m(-\omega)d\omega/2\pi$$

Hence, in accordance with Eq. 10 and 11 we have the following:

$$\int_{\omega\in\Omega r} Q_k(\omega)\, Q_m(-\omega)d\omega/2\pi = 0,\ k \neq m \quad (20)$$

$$\lambda_k = \int_{\omega\in\Omega_r} |Q_k(\omega)|^2\ d\omega/2\pi \leq 1 \quad (21)$$

Thus, the Fourier transformants of the eigenfunctions are orthogonal not only on the entire frequency axis (like fourier transformants of orthonormal functions) but also in the selected subband (dual orthogonality property). The inequality in the right-hand side of relation (Eq. 21) is obtained from those considerations that in accordance with the Parseval equation (Khurgin, 1971) for all eigenfunctions in Eq. 11 the integral value is achieved when along the whole frequency axis while during the integration over the subband such an effect can be observed only for some of them.

From the finiteness of the right-hand side of Eq.15, it follows that with the increase of the index value the corresponding eigenvalues must decrease so that this series of positive numbers converges. The extensive computational experiments showed that starting from the index:

$$J = 2[T\Delta_r]+4 \quad (22)$$

The following equations are performed quite accurately:

$$\lambda_{J+k} = 0,\ \forall k \geq 1 \quad (23)$$

The square bracket in Eq. 22 means the taking an entire piece of content. Another very important property of eigenvalues is that some of them, viz:

$$J_1 = J\text{-}8 \quad (24)$$

can be practically equal to one, thus we can assume that:

$$\lambda_k = 1, k = 1, \ldots, J_1 \quad (25)$$

Let's note that for the fulfillment of these equalities it is necessary to obtain the positivity of the right-hand side in Eq. 24 by choosing the product in the square brackets from relation Eq. 22. Otherwise, there will not be a single eigenvalue close to one. Bearing in mind the equality Eq. 21 it is easy to understand that the fulfillment of equalities in the form of Eq. 25 means almost complete concentration of the energy concerning the corresponding eigenfunctions of the subband core in a given subband. It is also clear that any linear combination of these eigenfunctions will have the complete energy concentration in the subband:

$$f(t) = \sum_{k=1}^{J_1} c_k g_k(t) \quad (26)$$

where, c_k, $k = 1, \ldots, J_1$ are real numbers. It is easy to understand that this property of the subband core eigenfunctions makes it possible to synthesize signals with a minimal infiltration of energy beyond an allocated subband. Another widely used technique of the subband analysis is to isolate the components from a certain signal $x(t)$, $0 \leq t \leq T$ of the same duration the fourier transformant of which must satisfy the following requirements in an ideal case:

$$Y(\omega) = X(\omega),\ \forall\omega\in\Omega_r \quad (27)$$

$$Y(\omega) = 0,\ \omega\notin\Omega_r \quad (28)$$

In particular, the property Eq. 27 and 28 is reasonable to provide at a channel signal allocation so that it is determined only by the energy of all signals mixture simultaneously used during the transmission of information.

At the same time it is known (Khurgin, 1971) that it is impossible to fulfill both the requirements, Eq. 27 and 28 simultaneously due to the uncertainty relation. But we can use some measure of deviations from an ideal, the minimization of which will make it possible to obtain an optimal component in the sense of this criterion. The following functional is proposed to use as such measure (criterion):

$$F(y,\beta) = \beta \int_{\omega\in\Omega_r} |X(\omega)-Y(\omega)|^2\, d\omega/2\pi + (1-\beta)\int_{\omega\in\Omega} |Y(\omega)|^2 d\omega/2\pi \quad (29)$$

where the parameter determines the weight of the components concerning the errors of execution for the requirement Eq. 27 (the first integral) or Eq. 28 (the second integral) and is chosen from the condition:

$$0 \le \beta \le 1 \quad (30)$$

Bearing in mind the definition (Eq. 1) the representation Eq. 4 and the Parseval equality (Khurgin, 1971), the functional Eq. 29 can be presented in the following Eq. 31:

$$F(y,\beta) = \beta P_r(x) - 2\beta\int_0^T\int_0^T x(t)\,y(\tau)A_r(t-\tau)dtd\tau + (1-\beta)\|y\|^2 + (2\beta-1)\,P_r(y) \quad (31)$$

Which allows to solve the variational problem of finding its minimum in an analytical form for a fixed value of the weight parameter in the time domain directly:

$$F(y,\beta) = \min \quad (32)$$

where, the search for a minimum takes place in the space of continuous functions with a bounded euclidean norm (energy) and the domain of definitio $t\in[0, T]$. The minimization of the functional Eq. 31 gives the following integral equation for the desired function (signal):

$$(1-\beta)\,y(t)+(2\beta-1)\int_0^T A_r(t-\tau)y(\tau)d\tau = \beta\int_0^T A_r(t-\tau)\,x(\tau)d\tau \quad (33)$$

Using the completeness of the orthonormal basis of eigenfunctions of the subband core (Smirnov, 1974) it is advisable to represent the desired function in the form of their linear combination:

$$y(t) = \sum_{k=1}^{\infty} d_k g_k(t) \quad (34)$$

The substitution of which into Eq. 33 taking into account the representations Eq. 8 and 13 provides the following:

$$\sum_{k=1}^{\infty}((1-\beta+(2\beta-1)\lambda_k)d_k-\beta\lambda_k\alpha_k)\,g_k(t) \equiv 0,\ t\in[0,T] \quad (35)$$

Because of the orthonormality of the subband core eigenfunctions, the fourier series on the left-hand side of Eq. 35 will converge to the zero function if and only if all the coefficients of the series are zero so that the following equalities are fulfilled:

$$d_k = \beta\lambda_k\alpha_k)/(1-\beta+(2\beta-1)\lambda_k),\ \forall k\ge 1 \quad (36)$$

Thus, the representation Eq. 34 for a desired optimal component can have the following Eq. 37:

$$y(t) = \beta\sum_{k=1}^{\infty}\lambda_k\alpha_k g_k(t)/(1-\beta+(2\beta-1)\lambda_k) \quad (37)$$

If the last relation is assumed as follows:

$$\beta = 0{,}5 \quad (38)$$

Then it gives:

$$y(t) = \sum_{k=1}^{\infty}\lambda_k\alpha_k g_k(t) \quad (39)$$

Thus, taking into account the definition Eq. 13 and the representations Eq. 8 we obtain the following:

$$y(t) = \int_0^T A_r(t-\tau)\,x(\tau)\,d\tau \quad (40)$$

If we substitute the representation for a subband core of the Eq. 5, we obtain the following:

$$y(t) = \int_{\omega\in\Omega_r} X(\omega)d\omega/2\pi \quad (41)$$

Thus, the use of the representation Eq. 39 allows us to obtain a signal component that depends only on a segment of the fourier transformant from a given subband. Let's note that due to the property Eq. 23 the series Eq. 37 and 39 will have a finite number of terms.

The synthesis of channel signals for subband information transfer: Let it is necessary to transfer the information vector:

$$\vec{e} = (e_1, \dots, e_M)' \quad (42)$$

where, the stroke denotes transposition and the components are real numbers. A channel signal is synthesized for this as the function of time with a finite determination domain whose parameters depend on a transmitted information vector:

$$x(t) = f(t, \vec{e}),\ t \in [0, T] \quad (43)$$

The main requirement for a channel signal is an unambiguous interpretation of the transmitted numbers on the basis of a certain operator:

$$e_m = H_k(f) \quad (44)$$

Without violating the generality of the conclusions, we assume that the energy expended for the transmission is determined by the following relation:

$$\|f\|^2 = \|\vec{e}\|^2 = \sum_{m=1}^{M} e_m^2 \quad (45)$$

and the frequency subband of the Eq. 3 is allocated for transmission. From the point of view of inter-channel interference minimization, the natural requirement for the synthesized channel signal is the following variational condition:

$$\|f\|^2 - P_r(f) = \min \quad (46)$$

where, the minimum is at a fixed information vector, taking into account the additional condition (Eq. 45). The following assertion is valid. The solution of the variational problem Eq. 46 and 45 has the following form:

$$f(t, \vec{e}) = \sum_{m=1}^{M} e_m g_m(t) \quad (47)$$

In this study, the decryption operator Eq. 44 is determined by the scalar products of the following form:

$$e_m = (f, g_m) \quad (48)$$

In order to prove this statement we use the fact that, in view of the eigenfunctions of the subband core basis completeness, the channel signal can be represented in the form of the following fourier series:

$$f\left(t, \vec{e}\right) = \sum_{k=1}^{\infty} c_k\left(\vec{e}\right) g_k(t) \quad (49)$$

Therefore, the variational problem Eq. 46 and 45 can be reformulated with respect to the coefficients of this series. The substitution of the representation Eq. 49 into Eq. 45 and 46 gives a new representation of the variational conditions:

$$\sum_{k=1}^{\infty} (1-\lambda_k) c_k^2(\vec{e}) = \min \quad (50)$$

$$\sum_{k=1}^{\infty} c_k^2(\vec{e}) = \|\vec{e}\|^2 \quad (51)$$

Let's note that in this case the information vector is also, fixed and the conditional minimum is determined on the set of all real expansion coefficients for the unknown function with respect to the eigenfunctions of the subband core. Then, we will use induction. Let it is known that when the number of components of the information vector equal to M = K-1 the solution of the variational problem has the form Eq. 47. We will show that this form must also be preserved at M = K. Indeed, under the conditions indicated above, the requirements Eq. 50 and the equality Eq. 49 are transformed to the following equation:

$$\begin{gathered} \sum_{k=K}^{\infty} (1-\lambda_k) c_k^2(\vec{e}) = \min \\ \sum_{k=K}^{\infty} c_k^2(\vec{e}) = e_K^2 \end{gathered} \quad (52)$$

Bearing in mind the ordering of the eigenvalues in descending order Eq. 14 and the inequality Eq. 21, we obtain the following inequality for the left-hand side of Eq. 52:

$$\sum_{k=K}^{\infty} (1-\lambda_k)\, c_k^2(\vec{e}) \geq (1-\lambda_K) \sum_{k=K}^{\infty} c_k^2(\vec{e}) = (1-\lambda_K) e_K^2 \quad (53)$$

Obviously, the equality here corresponds to the minimum of the left side and it is achieved when the following conditions are met: $c_K(\vec{e}) = e_K;\ c_{K+i}(\vec{e}) = 0,\ \forall i \geq 1$. Now let's M = 1. Then, taking into account the condition Eq. 14, we obtain the following solution of problem Eq. 50 and 51:

$$f(t, \vec{e}) = e_1 g_1(t) \quad (54)$$

For the remaining values of the information vector dimension, we obtain the representation Eq. 47 by induction which proves the assertion. Thus, the representation Eq. 47 determines the channel signal optimal in the sense of the criterion Eq. 45 and 46 for the transmission of the information vector Eq. 42. In this study, decoding is performed on the basis of scalar products in the following Eq. 48.

Summary: It was shown that the notion of the signal energy fraction in a given frequency subband is the natural basis to control the processes of adaptive shaping and the processing of channel signals in cognitive radio systems. Theoretical bases of subband analysis and synthesis for finite duration signals were developed which made it possible to formulate and solve the variational problem of optimal signal-code structure synthesizing with minimal infiltration of energy beyond the specified frequency interval (the minimum level of interchannel interference). The procedures for signal-code structures decoding were developed on the basis of subband core eigenfunctions.

CONCLUSION

The best basis for the development of cognitive radio signals, the use of which makes it possible to minimize the fraction of energy infiltration beyond a given frequency interval is the set of eigenfunctions for subband cores that determine integral representations for signal energy fractions.

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