ISSN 1027-4510, Journal of Surface Investigation: X-ray, Synchrotron and Neutron Techniques, 2023, Vol. 17, No. 2, pp. 368–370. © Pleiades Publishing, Ltd., 2023.
Russian Text © The Author(s), 2023, published in Poverkhnost', 2023, No. 3, pp. 59–62.

On Monitoring Position of a Charged Particle Moving near a Metal Sphere by Means of Diffraction Radiation

V. V. Syshchenko[a], * and A. I. Tarnovsky[a]

[a] *Belgorod National Research University, Belgorod, 308015 Russia*

**e-mail: syshch@yandex.ru*

Received June 20, 2022; revised August 15, 2022; accepted August 15, 2022

Abstract—A uniformly moving charged particle generates transition radiation when moving in an inhomogeneous medium (in particular, when crossing the interface between two media) and diffraction radiation when moving near medium inhomogeneities without crossing their boundaries. Both diffraction and transition radiation can be used to detect particles and monitor beams in accelerators. While methods based on the transition radiation of particles for diagnostics of both relativistic and nonrelativistic beams are widespread, the application of diffraction radiation for these goals remains the subject of research. Diffraction-radiation generation weakly perturbs the motion of a particle beam, which makes it possible to develop nondestructive beam-diagnostics methods. The description of the diffraction radiation of a nonrelativistic charged particle for a conducting sphere was constructed earlier by means of the image method known from electrostatics. The method for finding the parameters of particle flight by the sphere was proposed within the framework of this approach; it used a single point detector recording the intensity and polarization of diffraction radiation. Here we propose a scheme with three detectors that solves the same problem without recording the radiation polarization.

Keywords: diffraction radiation, conducting sphere, image method, monitoring of particles, beam diagnostics, detector, polarization

DOI: 10.1134/S1027451023020167

INTRODUCTION

A uniformly moving particle can generate electromagnetic-wave radiation either in a homogeneous medium if the condition for the appearance of Vavilov–Cherenkov radiation is satisfied or if it interacts with inhomogeneities of the medium. In the latter case, it is accepted to separate the transition radiation that occurs when a moving particle crosses the interface between two media [1–3], and the diffraction radiation corresponding to the case of the motion of the particle near a spatially localized inhomogeneity of the medium (target) without crossing its boundary [4, 5]. Diffraction and transition radiation are widely used for diagnosing and monitoring charged particle beams (for example, [6–8] and the references therein).

In [9–11], an approach was developed to describe the diffraction and the transition radiation generated when a nonrelativistic particle moves near a conducting sphere; it was based on the image method known in electrostatics [2, 3]. In [12, 13], the developed approach was used to calculate the diffraction-radiation polarization. The authors of those papers also proposed a method for determining the azimuth of the trajectory of a flying particle relative to the sphere center; it used a single point detector that recorded the polarization of radiation emitted in a certain selected direction. In this paper, we draw attention to the fundamental possibility of determining the particle position in the plane that is perpendicular to its velocity, this position is unambiguous within one quadrant, and we use three detectors that record only the intensity rather than the radiation polarization.

EXPERIMENTAL

In the image method [12, 13], the effect of the conducting surface on the distribution of the electric field in space is simulated by introducing one or more fictitious charges ("images" of the real charge) along with a real point charge. In particular, the distribution of the electric field of the point charge near a closely located grounded sphere is the same as that of the field of two point charges, namely, the real one located outside the sphere and the fictitious one located inside the sphere (Fig. 1). To satisfy this condition, the real charge e_0 and the fictitious one $e = -e_0 R/r_0$ must be located on a straight line with the center of a sphere with the radius R, where r_0 is the distance from the real charge to the sphere center, and the distance from the fictitious charge to the sphere center is $r = R^2/r_0$.

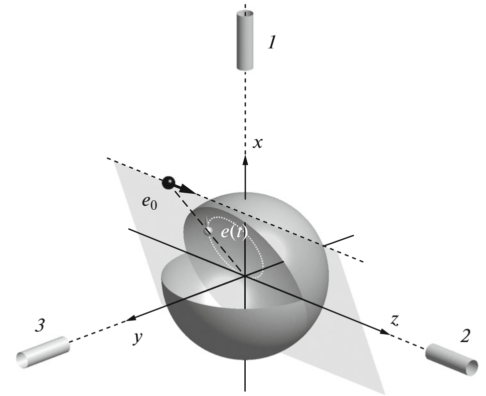

Fig. 1. Positions of the real charge e_0 and its "image" $e(t)$ with respect to a grounded conducting sphere with the radius R and also those of three radiation detectors *1–3*.

We consider the case where the real charge moves rectilinearly and uniformly with velocity $\mathbf{v}_0$ past the sphere center in the case of a two-dimensional impact parameter $\mathbf{b} = (x, y)$. Obviously, in this case, the movement of the fictitious charge is accelerated, which leads to the appearance of radiation.

The amplitude of the diverging wave of the field vector potential is proportional to the quantity [14–16]

$$I = \int_{-\infty}^{\infty} e(t)\mathbf{v}(t)\exp[i(\omega t - \mathbf{kr}(t))]\,dt, \tag{1}$$

where ω and $\mathbf{k}$ are the frequency and the wave vector of the emitted wave; $|\mathbf{k}| = \omega/c$, c is the light velocity in free space; and $e(t)$, $\mathbf{r}(t)$, and $\mathbf{v}(t)$ are the value, the trajectory, and the velocity of the fictitious charge, respectively. The spectral-angular radiation density of an arbitrarily moving charge can be described using the well-known formulas [15, 16]:

$$\left(\frac{dE}{d\omega d\Omega}\right)_a = \frac{\omega^2}{4\pi^2 c^3}|\mathbf{e}_a \mathbf{I}|^2, \tag{2}$$

where $\mathbf{e}_a$, $a = 1, 2$, are the radiation-polarization unit vectors, which are orthogonal to $\mathbf{k}$ and to each other, and the summation over polarizations gives

$$\frac{dE}{d\omega d\Omega} = \frac{1}{4\pi^2 c}|\mathbf{k} \times \mathbf{I}|^2. \tag{3}$$

We consider the system within the limit of small radiation frequencies to which the diffraction-radiation intensity maximum of the nonrelativistic particle corresponds, as was shown in [9, 11]. For example, for a sphere with a radius of 1 cm and for an impact parameter whose value is close to the above-mentioned one, the radiation-intensity maximum of the particle moving with a velocity of $v_0 = 0.1c$, where c is the light velocity, corresponds to a frequency on the order of 1 GHz. In the limit of small radiation frequencies

$$\omega \ll cb/R^2 \tag{4}$$

or, in terms of the wavelength $\lambda = 2\pi c/\omega$,

$$\lambda \gg 2\pi R^2/b\lambda \gg 2\pi R/b,$$

integral (1) can be calculated analytically:

$$I_x = i\frac{4}{3}e_0 R^3 \frac{\omega^2}{v_0^2}\frac{x}{b}K_1\left(\frac{\omega}{v_0}b\right), \tag{5}$$

$$I_y = i\frac{4}{3}e_0 R^3 \frac{\omega^2}{v_0^2}\frac{y}{b}K_1\left(\frac{\omega}{v_0}b\right), \tag{6}$$

$$I_z = -\frac{4}{3}e_0 R^3 \frac{\omega^2}{v_0^2}K_0\left(\frac{\omega}{v_0}b\right) - \frac{2}{3}e_0 R^3 \frac{\omega}{v_0 b}K_1\left(\frac{\omega}{v_0}b\right), \tag{7}$$

where $K_0(x)$ and $K_1(x)$ are the modified Bessel functions of the third kind (Macdonald functions). We see that, thus, within the limit (4), the quantities I_x and I_y turn out to be purely imaginary, and I_z is real.

The calculations [12, 13] showed the fundamental possibility of determining both components of the two-dimensional impact parameter $\mathbf{b} = (x, y)$ of the flying-particle trajectory by recording the radiation intensity and polarization by means of a detector installed at a certain angle $0 < \theta < \pi/2$, $\varphi = 0$. In this paper, we draw attention to a simpler possibility of monitoring this quantity without using polarization-sensitive detectors.

RESULTS AND DISCUSSION

We consider the spectral-angular radiation density (3) summed over the polarization. If the fact that, in the region bounded by expression (4), the components I_x and I_y of vector $\mathbf{I}$ turn out to be imaginary and I_z is real is taken into account, the opening of the modulus of the vector product in (3) yields

$$\begin{aligned}\frac{dE}{d\omega d\Omega} = \frac{\omega^2}{4\pi^2 c^3}\Big\{&\sin^2\theta |I_z|^2 \\ &+ \left(\cos^2\varphi \sin^2\theta + \cos^2\theta\right)|I_y|^2 \\ &+ \left(\sin^2\varphi \sin^2\theta + \cos^2\theta\right)|I_x|^2 \\ &+ 2\sin\varphi\cos\phi\sin^2\theta |I_x||I_y|\Big\}.\end{aligned} \tag{8}$$

It is easy to see that the detector installed in the x-axis direction ($\theta = \pi/2$, $\varphi = 0$, position *1* in Fig. 1) records the radiation intensity that is proportional to

$$\left(\frac{dE}{d\omega d\Omega}\right)_1 = \frac{\omega^2}{4\pi^2 c^3}\left\{|I_z|^2 + |I_y|^2\right\}; \tag{9}$$

the detector located in the y-axis direction ($\theta = \pi/2$, $\varphi = \pi/2$, position *2* in Fig. 1) records the radiation intensity that is proportional to

$$\left(\frac{dE}{d\omega d\Omega}\right)_2 = \frac{\omega^2}{4\pi^2 c^3}\left\{|I_z|^2 + |I_x|^2\right\} \quad (10)$$

and the detector located in the z-axis direction ($\theta = 0$, position *3* in Fig. 1) records the radiation intensity that is proportional to

$$\left(\frac{dE}{d\omega d\Omega}\right)_3 = \frac{\omega^2}{4\pi^2 c^3}\left\{|I_x|^2 + |I_y|^2\right\}. \quad (11)$$

Thus, among the three measured quantities (9)–(11), it is possible to find the absolute values of all three components of the vector **I** up to the sign. If expressions (5) and (6) are taken into account, it can be seen that the measurements of the radiation-diffraction intensity for the sphere in three directions make it possible to determine the coordinates x and y of the flying particle if it is known that they are located within the limit of one quadrant. It is the last restriction that is due to the loss of information about the signs of the components of vector **I** when calculating their absolute values by means of formulas (9)–(11).

CONCLUSIONS

In this paper, we have shown the fundamental possibility of determining the position of a particle in a plane that is perpendicular to its velocity; the determination was unambiguous within one quadrant and used three detectors that recorded only the intensity, but not the radiation polarization. The developed approach is applicable only for nonrelativistic particles. However, such problems are also of considerable interest, as shown, for example, in recent papers [17, 18] dedicated to using transition radiation for diagnosing a beam of nonrelativistic particles.

CONFLICT OF INTEREST

We declare that we have no conflicts of interest.

REFERENCES

1. V. L. Ginzburg and V. N. Tsytovich, *Transition Radiation and Transition Scattering* (Nauka, Moscow, 1984); Adam Hilger, Bristol U.K., 1990).
2. J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962; Mir, Moscow, 1965).
3. L. D. Landau and E. M. Lifshits, *Electrodynamics of Continuous Media* (Nauka, Moscow, 1992) [in Russian].
4. B. M. Bolotovskii and G. V. Voskresenskii, Sov. Phys. Usp. **9**, 73 (1966). https://doi.org/10.1070/PU1966v009n01ABEH002892
5. B. M. Bolotovskii and E. A. Galst'yan, Phys.—Usp. **43**, 755 (2000). https://doi.org/10.1070/PU2000v043n08ABEH000683
6. M. Castellano and V. A. Verzilov, Phys. Rev. Spec. Top.—Accel. Beams **1**, 062801 (1998). https://doi.org/10.1103/PhysRevSTAB.1.06280
7. A. P. Potylitsyn, M. I. Ryazanov, M. N. Strikhanov, and A. A. Tishchenko, *Diffraction Radiation from Relativistic Particles*, Springer Tracts in Modern Physics, Vol. 239 (Springer, Berlin, 2010). https://doi.org/10.1007/978-3-642-12513-3
8. A. P. Potylitsyn, *Electromagnetic Radiation of Electrons in Periodic Structures*, Springer Tracts in Modern Physics, Vol. 243 (Springer, Berlin, 2011). https://doi.org/10.1007/978-3-642-19248-7
9. N. F. Shul'ga, V. V. Syshchenko, and E. A. Larikova, Nucl. Instrum. Methods Phys. Res., Sect. B **402**, 167 (2017). https://doi.org/10.1016/j.nimb.2017.03.013
10. V. V. Syshchenko, E. A. Larikova, and Yu. P. Gladkih, J. Instrum. **12**, C12057 (2017). https://doi.org/10.1088/1748-0221/12/12/C12057
11. V. V. Syshchenko and E. A. Larikova, J. Surf. Invest.: Xpray, Synchrotron Neutron Tech. **13**, 359 (2019). https://doi.org/10.1134/S1027451019020393
12. N. F. Shul'ga and V. V. Syshchenko, Nucl. Instrum. Methods Phys. Res., Sect. B **452**, 55 (2019). https://doi.org/10.1016/j.nimb.2019.05.066
13. V. V. Syshchenko and E. A. Larikova, J. Surf. Invest.: X-ray, Synchrotron Neutron Tech. **13**, 990 (2019). https://doi.org/10.1134/S1027451019050367
14. I. I. Abbasov, B. M. Bolotovskii, and V. A. Davydov, Sov. Phys. Usp. **29**, 788 (1986). https://doi.org/10.1070/PU1986v029n08ABEH003484
15. V. A. Bazylev and N. K. Zhevago, *Radiation of Fast Particles in Matter and External Fields* (Nauka, Moscow, 1987) [in Russian].
16. A. I. Akhiezer and N. F. Shul'ga, *High-Energy Electrodynamics in Matter* (Nauka, Moscow, 1993; Gordon and Breach, Luxembourg, 1996).
17. R. Singh, T. Reichert, and B. Walasek-Hoehne, arXiv:2104.08487v1 (2021).
18. R. Singh and T. Reichert, arXiv:2107.08689v3 (2021).

Translated by L. Kulman