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DYNAMIC THEORY X-RAY RADIATION BY RELATIVISTIC ELECTRON IN COMPOSITE TARGET

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Resume. The dynamic theory of coherent X-ray radiation by relativistic electron crossing a three-layer structure consisting of an amorphous substance layer, a layer of vacuum and a layer with artificial periodic structure has been developed. The process of radiation and propagation of X-ray waves in an artificial periodic structure have been considered based on two-wave approximation of dynamic diffraction theory in Laue scattering geometry.

1. Introduction

When a relativistic electron crosses an amorphous plate, the transition radiation (TR) arises near the boundaries of the plate and then the TR photons propagate at a small angle to direction of electron velocity vector [1]. In the case of a monocrystalline plate, the TR will undergo the dynamical diffraction on a system of atomic planes in the crystal and will be reflected in the Bragg direction, forming the diffracted transition radiation (DTR) [2-5]. By analogy with DTR in the crystal, an electron crossing periodic layered medium generates TR whose photons diffract on system of layers of periodical layered medium in the plate forming the DTR in the direction near Bragg direction [6]. Together with DTR in layered media the parametric X-Ray radiation (PXR) [7-9] occurs as a result of diffraction of pseudo-photons of coulomb field of relativistic electron on the system of parallel atomic plane in the crystal or on the system of layers in the multilayered target.

A theory of coherent X-ray radiation of relativistic in the crystal were developed in the network of two wave approximation of dynamical theory of X-ray waves diffraction in the works [10-14]. In the works [10-11] the coherent X-ray radiation was treated in special case of symmetric reflection, when the reflecting system of atomic planes of the crystal is situated parallel to the target surface (in the case of Bragg scattering geometry) or perpendicular (in the case of Laue scattering geometry). In the works [12-14] the dynamic theory of coherent X-ray radiation of relativistic electron in crystal was developed for the general case of asymmetric to relate of the crystal surface reflection of the electron coulomb field, when a system of parallel reflecting atomic planes in the target can be situated at arbitrary angle to the target surface.

Traditionally the radiation of relativistic electron was considered in a separated amorphous, crystalline or multilayer target. A theoretical description of coherent radiation of relativistic electron in composite targets was not consider previously. The experimental research of generation of coherent X-ray radiation [15-19] in composite structures had shown the possibilities of considerable increase of intensity of DTR yield because of increase of target boundaries number.

Recently in [20-22] the theory of coherent X-ray radiation of relativistic electron crossing composite structure “amorphous layer – crystalline layer” and “amorphous layer-vacuum-crystalline layer” was developed in framework of dynamic theory of diffraction. The expressions describing DTR and PXR of relativistic electron in such structures were derived and investigated. The possibility of considerable increase of spectral-angular density of DTR because of constructive interference of TR on the target boundaries.

The present work is devoted to investigation of coherent X-ray radiation of relativistic electron crossing the two-layers structure “amorphous – vacuum – single crystal”. In the framework of two-wave approximation of dynamic diffraction theory of X-ray waves in the single crystal the expressions describing the spectral-angular characteristics of the radiation in such structures are derived. The expressions describing the DTR and PXR spectral-angular densities and their interference in the considered structure have been obtained for general case of asymmetric reflection of the electron coulomb field from the layer with artificial periodic structure. At that, under constructive interference of TR waves from different boundaries of amorphous layer and constructive interference of TR waves from amorphous layer and entrance boundary of artificial periodic structure the spectral-angular density of DTR can be increased by orders of magnitude greater than the value of the spectral-angular density from artificial structure only. The possibility to increase the angular DTR density with increasing the substance density of amorphous layer has been shown.

2. Radiation amplitude

Let us consider the radiation of relativistic electron rectilinearly with a velocity $\mathbf{V}$ crosses a three-layers structure consisted of two amorphous layers and one a layer with artificial periodic structure (see fig. 1.) with thicknesses of c, a and b, respectively. The dielectric susceptibility of the amorphous layers we will designate as χ_c and χ_a respectively, their thicknesses are l_1 and l_2, the period is $T = l_1 + l_2$.

While solving the problem, let us consider an equation for a Fourier image of an electromagnetic field

$$\mathbf{E}(\mathbf{k},\omega) = \int dt\, d^3\mathbf{r}\, \mathbf{E}(\mathbf{r},t)\exp(i\omega t - i\mathbf{k}\mathbf{r}) \,. \tag{1}$$

Since the field of a relativistic particle could, to a good accuracy, be taken as being transverse, the incident $\mathbf{E}_0(\mathbf{k},\omega)$ and diffracted $\mathbf{E}_{\mathbf{g}}(\mathbf{k},\omega)$ electromagnetic waves are determined by two amplitudes with different values of transverse polarization:

$$\begin{aligned}\mathbf{E}_0(\mathbf{k},\omega) &= E_0^{(1)}(\mathbf{k},\omega)\mathbf{e}_0^{(1)} + E_0^{(2)}(\mathbf{k},\omega)\mathbf{e}_0^{(2)}, \\ \mathbf{E}_{\mathbf{g}}(\mathbf{k},\omega) &= E_{\mathbf{g}}^{(1)}(\mathbf{k},\omega)\mathbf{e}_1^{(1)} + E_{\mathbf{g}}^{(2)}(\mathbf{k},\omega)\mathbf{e}_1^{(2)},\end{aligned} \tag{2}$$

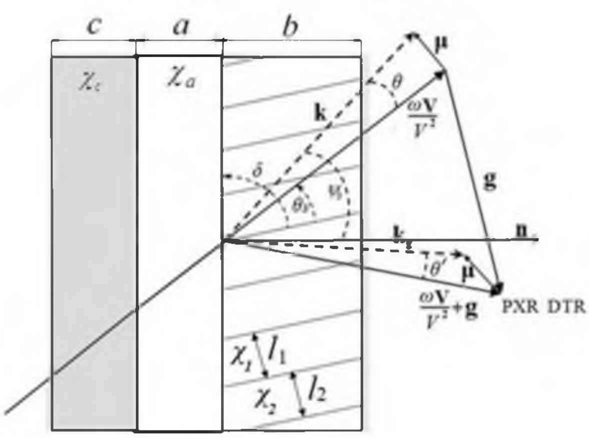

Fig.1. Geometry of the radiation process and the system of the using parameters notations, θ and θ' are the radiation angles, θ_B is Bragg angle, $\mathbf{k}$ and $\mathbf{k}_g$ are wave vectors of incident and diffracted photons.

where the unit vectors of polarization $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_0^{(2)}$ are perpendicular to vector $\mathbf{k}$, and vectors $\mathbf{e}_1^{(1)}$ and $\mathbf{e}_1^{(2)}$ are perpendicular to vector $\mathbf{k}_g = \mathbf{k} + \mathbf{g}$. Vectors $\mathbf{e}_0^{(2)}$, $\mathbf{e}_1^{(2)}$ are situated on the plane of vectors $\mathbf{k}$ and $\mathbf{k}_g$ (π-polarization) and $\mathbf{e}_0^{(1)}, \mathbf{e}_1^{(1)}$ are perpendicular to this plane (σ-polarization); $\mathbf{g}$ is similar to the reciprocal lattice vector in a single crystal medium – it is perpendicular to the layers of the structure and its magnitude is $g = \frac{2\pi}{T} n$, $n = 0, \pm 1, \pm 2, \ldots$

The system of equation for the Fourier transform images of electromagnetic field in two-wave approximation of dynamic theory of diffraction has the following view [23]:

$$\begin{cases} (\omega^2(1+\chi_0) - k^2)E_0^{(s)} + \omega^2 \chi_{-\mathbf{g}} C^{(s)} E_{\mathbf{g}}^{(s)} = 8\pi^2 ie\omega\, \theta V P^{(s)} \delta(\omega - \mathbf{kV}), \\ \omega^2 \chi_{\mathbf{g}} C^{(s)} E_0^{(s)} + (\omega^2(1+\chi_0) - k_{\mathbf{g}}^2)E_{\mathbf{g}}^{(s)} = 0, \end{cases} \tag{3}$$

where $\chi_0 = \chi_0' + i\chi_0''$ is the average dielectric susceptibility, $\chi_{\mathbf{g}}$ and $\chi_{-\mathbf{g}}$ are the coefficients of the Fourier expansion of the dielectric susceptibility of the artificial periodic structure over the vectors $\mathbf{g}$:

$$\chi(\omega, \mathbf{r}) = \sum_{\mathbf{g}} \chi_{\mathbf{g}}(\omega) \exp(i\mathbf{gr}) = \sum_{\mathbf{g}} \left(\chi_{\mathbf{g}}'(\omega) + i\chi_{\mathbf{g}}''(\omega)\right) \exp(i\mathbf{gr}) \cdot \tag{4}$$

The values $C^{(s)}$ and $P^{(s)}$ are defined in the system (3) as

$$C^{(s)} = \mathbf{e}_0^{(s)} \mathbf{e}_1^{(s)}, \; C^{(1)} = 1, \; C^{(2)} = \cos 2\theta_B,$$
$$P^{(s)} = \mathbf{e}_0^{(s)} (\boldsymbol{\mu} / \mu), \; P^{(1)} = \sin\varphi, \; P^{(2)} = \cos\varphi, \tag{5}$$

where $\boldsymbol{\mu} = \mathbf{k} - \omega\mathbf{V}/V^2$ is the virtual photon momentum vector component perpendicular to the particle velocity vector $\mathbf{V}$ ($\mu = \omega\theta / V$, where $\theta \ll 1$ is the angle between $\mathbf{k}$ and $\mathbf{V}$), θ_B is the Bragg angle, φ is the azimuthal angle of incidence of radiation measured from the plane formed by electron velocity vector $\mathbf{V}$ and vector $\mathbf{g}$, the value of the vector $\mathbf{g}$ is shown by expression $g = 2\omega_B \sin\theta_B / V$, ω_B is Bragg's frequency. The angle between vector $\frac{\omega \mathbf{V}}{V^2} + \mathbf{g}$ and diffracted wave vector $\mathbf{k}_{\mathbf{g}}$ is defined as θ'. The equation system (3) under $s = 1$, describes the fields of σ- polarization, and under $s = 2$ the fields of π-polarization

The values χ_0 and $\chi_{\mathbf{g}}$ for considered periodic structure have the following view:

$$\chi_0(\omega) = \frac{l_1\chi_1 + l_2\chi_2}{T}, \qquad \chi_{\mathbf{g}}(\omega) = \frac{\exp(-igl_1) - 1}{igT}(\chi_2 - \chi_1). \tag{6}$$

Further we will use the relations resulted from (6):

$$\chi_0' = \frac{l_1\chi_1' + l_2\chi_2'}{T}, \qquad \chi_0'' = \frac{l_1\chi_1'' + l_2\chi_2''}{T}, \tag{7a}$$

$$\mathrm{Re}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}} = \frac{2\sin\left(\frac{gl_1}{2}\right)}{gT}(\chi_2' - \chi_1'), \qquad \mathrm{Im}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}} = \frac{2\sin\left(\frac{gl_1}{2}\right)}{gT}(\chi_2'' - \chi_1'') \tag{7b}$$

To find the expressions for the wave vector magnitudes k and $k_{\mathbf{g}}$ we solve the dispersion equation for X-waves in periodic structure following from the system (3)

$$(\omega^2(1+\chi_0)-k^2)(\omega^2(1+\chi_0)-k_{\mathbf{g}}^2)-\omega^4\chi_{-\mathbf{g}}\chi_{\mathbf{g}}C^{(s)^2}=0\,, \tag{8}$$

using standard methods of dynamic theory [24]:

$$k=\omega\sqrt{1+\chi_0}+\lambda_0\,,\; k_{\mathbf{g}}=\omega\sqrt{1+\chi_0}+\lambda_{\mathbf{g}}\,. \tag{9}$$

$$\lambda_{\mathbf{g}}^{(1,2)}=\frac{\omega}{4}\left(\beta\pm\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right), \tag{10}$$

$$\lambda_0^{(1,2)}=\omega\frac{\gamma_0}{4\gamma_{\mathbf{g}}}\left(-\beta\pm\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right), \tag{11}$$

where $\beta=\alpha-\chi_0\left(1-\frac{\gamma_{\mathbf{g}}}{\gamma_0}\right)$, $\alpha=\frac{1}{\omega^2}(k_{\mathbf{g}}^2-k^2)$, $\gamma_0=\cos\psi_0$, $\gamma_{\mathbf{g}}=\cos\psi_{\mathbf{g}}$, ψ_0- is the angle between incident wave vector $\boldsymbol{k}$ and vector normal to the plate surface $\mathbf{n}$, ψ_g is the angle between wave vector $\mathbf{k_g}$ and the vector $\mathbf{n}$ (see figure 1). As the dynamical corrections $|\lambda_0|<<\omega$ and $|\lambda_{\mathbf{g}}|<<\omega$, we can show that $\theta\approx\theta'$ (see in figure 1), and hereinafter will use θ in all the occasions.

The lengths $k_a=\omega\sqrt{1+\chi_a}$ and $k_c=\omega\sqrt{1+\chi_c}$ of the wavevector of free photons in amorphous media
can be conveniently written in the form

$$k_a=\omega\left(1+\frac{\chi_0}{2}\right)+\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda'_{\mathbf{g}a}-\frac{\omega\beta}{2}\right),\quad k_c=\omega\left(1+\frac{\chi_0}{2}\right)+\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda'_{\mathbf{g}c}-\frac{\omega\beta}{2}\right), \tag{12}$$

where

$$\lambda'_{\mathbf{g}a}=\lambda_{\mathbf{g}}^*-\frac{\gamma_{\mathbf{g}}}{\gamma_0}\omega\left(\frac{\gamma^{-2}+\theta^2-\chi_a}{2}\right),\lambda'_{\mathbf{g}c}=\lambda_{\mathbf{g}}^*-\frac{\gamma_{\mathbf{g}}}{\gamma_0}\omega\left(\frac{\gamma^{-2}+\theta^2-\chi_c}{2}\right),\; \lambda_{\mathbf{g}}^*=\frac{\omega\beta}{2}+\frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda_0^*,$$

$$\lambda_0^*=\omega\left(\frac{\gamma^{-2}+\theta^2-\chi_0}{2}\right), \tag{13}$$

and the length of the wave vector of a freely emitted photon in the Bragg scattering direction can be as $k_0=\omega\left(1+\frac{\chi_0}{2}\right)+\lambda''_{\mathbf{g}}$, written $\lambda''_{\mathbf{g}}=-\omega\frac{\chi_0}{2}$. $\gamma=1/\sqrt{1-V^2}$ - Лоренц-фактор частицы. Using the notation introduced above and system of equations (3), we can write the expressions for the fields. The field in vacuum in front of the target is represented by pseudophotons of the Coulomb field of a relativistic electron incident on the target:

$$E_0^{(s)vacI}=\frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{1}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(-\chi_0-\frac{2}{\omega}\frac{\gamma_0}{\gamma_{\mathbf{g}}}\lambda_{\mathbf{g}}+\beta\frac{\gamma_0}{\gamma_{\mathbf{g}}}\right)}\delta\left(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^*\right) \tag{14}$$

In amorphous media the field consists of coulomb field of the electron and field of radiated free phonons $E_c^{(s)}$ and $E_a^{(s)}$ of transition radiation

$$E_{c0}^{(s)}=\frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{1}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(-\chi_0+\chi_a-\frac{2}{\omega}\frac{\gamma_0}{\gamma_{\mathbf{g}}}\lambda_{\mathbf{g}}+\beta\frac{\gamma_0}{\gamma_{\mathbf{g}}}\right)}\delta\left(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^*\right)+E_c^{(s)}\delta\left(\lambda_{\mathbf{g}}-\lambda'_{\mathbf{g}c}\right). \qquad (15)$$

$$E_{a0}^{(s)}=\frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{1}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(-\chi_0+\chi_a-\frac{2}{\omega}\frac{\gamma_0}{\gamma_{\mathbf{g}}}\lambda_{\mathbf{g}}+\beta\frac{\gamma_0}{\gamma_{\mathbf{g}}}\right)}\delta\left(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^*\right)+E_a^{(s)}\delta\left(\lambda_{\mathbf{g}}-\lambda'_{\mathbf{g}a}\right). \qquad (16)$$

The solution of system of equations (3) for diffracted field in periodic layered medium we represent in following form:

$$E_{\mathbf{g}}^{(s)\mathrm{medium}}=-\frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{\omega^2\chi_{\mathbf{g}}C^{(s)}}{4\frac{\gamma_0^2}{\gamma_{\mathbf{g}}^2}\left(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(1)}\right)\left(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(2)}\right)}\delta\left(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^*\right)+$$
$$+E_{\mathbf{g}}^{(s)^{(1)}}\delta(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(1)})+E_{\mathbf{g}}^{(s)^{(2)}}\delta(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(2)}), \qquad (17)$$

where $E_{\mathbf{g}}^{(s)^{(1)}}$ и $E_{\mathbf{g}}^{(s)^{(2)}}$ – the diffracted free fields in periodical layered medium.

In vacuum behind periodical structure the diffracted field has the view:

$$E_{\mathbf{g}}^{(s)vacII}=E_{\mathbf{g}}^{(s)Rad}\delta\left(\lambda_{\mathbf{g}}+\frac{\omega\chi_0}{2}\right), \qquad (18)$$

where $E_{\mathbf{g}}^{(s)Rad}$ - is the field of coherent radiation near of the Bragg direction.

From the second equation of system (3) the expression binding the diffracted and incident fields in periodic layered structure:

$$E_0^{(s)\mathrm{medium}}=\frac{2\omega\lambda_{\mathbf{g}}}{\omega^2\chi_g C^{(s)}}E_{\mathbf{g}}^{(s)\mathrm{medium}}. \qquad (19)$$

For definition of the amplitude of radiation field $E_{\mathbf{g}}^{(s)Rad}$ we will use the general conditions on four boundaries of considering three-layer target.

$$\int E_0^{(s)vacI}d\lambda_{\mathbf{g}}=\int E_{c0}^{(s)}d\lambda_{\mathbf{g}},\quad \int E_{c0}^{(s)}e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}c}d\lambda_{\mathbf{g}}=\int E_{a0}^{(s)}e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}c}d\lambda_{\mathbf{g}},$$

$$\int E_{a0}^{(s)}e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}(c+a)}d\lambda_{\mathbf{g}}=\int E_0^{(s)\mathrm{medium}}e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}(c+a)}d\lambda_{\mathbf{g}},$$

$$\int E_{\mathbf{g}}^{(s)\mathrm{medium}}e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}(a+c)}d\lambda_{\mathbf{g}}=0,$$

$$\int E_{\mathbf{g}}^{(s)\mathrm{medium}}e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}(c+a+b)}d\lambda_{\mathbf{g}}=\int E_{\mathbf{g}}^{(s)vacII}e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}(c+a+b)}d\lambda_{\mathbf{g}}. \qquad (20)$$

As the result the expression for the radiation amplitude $E_{\mathbf{g}}^{(s)Rad}=E_{PXR}^{(s)}+E_{DTR}^{(s)}$ contained the contributions of PXR and DTR radiation mechanisms was derived:

$$E_{\mathbf{g}}^{(s)Rad}=E_{DTR}^{(s)}+E_{PXR}^{(s)}, \qquad (21a)$$

$$E_{DTR}^{(s)}=\frac{8\pi^2 ieV\theta P^{(s)}}{\omega}e^{i\left(\frac{\omega\chi_0}{2}+\lambda_g^*\right)\frac{(c+a+b)}{\gamma_g}}\frac{\omega^2\chi_{\mathbf{g}}C^{(s)}}{2\omega\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^{(1)}-\lambda_{\mathbf{g}}^{(2)}\right)}\left(e^{i\frac{\lambda_{\mathbf{g}}^{(1)}-\lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}b}-e^{i\frac{\lambda_{\mathbf{g}}^{(2)}-\lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}b}\right)\times$$

$$\times\left[\left(\frac{1}{\theta^2+\gamma^{-2}-\chi_c}-\frac{1}{\theta^2+\gamma^{-2}}\right)e^{-i\frac{\omega c}{2\gamma_0}\left(\gamma^{-2}+\theta^2-\chi_c\right)-i\frac{\omega a}{2\gamma_0}\left(\gamma^{-2}+\theta^2-\chi_a\right)}+\right.$$

$$\left.+\left(\frac{1}{\theta^2+\gamma^{-2}-\chi_a}-\frac{1}{\theta^2+\gamma^{-2}-\chi_c}\right)e^{-i\frac{\omega a}{2\gamma_0}\left(\gamma^{-2}+\theta^2-\chi_a\right)}+\frac{1}{\theta^2+\gamma^{-2}-\chi_0}-\frac{1}{\theta^2+\gamma^{-2}-\chi_a}\right], \tag{21b}$$

$$E_{PXR}^{(s)}=\frac{8\pi^2 ieV\theta P^{(s)}}{\omega}e^{i\left(\frac{\omega\chi_0}{2}+\lambda_g^*\right)\frac{(c+a+b)}{\gamma_g}}\frac{\omega^2\chi_{\mathbf{g}}C^{(s)}}{2\omega\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^{(1)}-\lambda_{\mathbf{g}}^{(2)}\right)}\left[\left(\frac{1}{\chi_0-\theta^2-\gamma^{-2}}+\frac{\omega}{2\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(1)}\right)}\right)\times\right.$$

$$\left.\times\left(e^{i\frac{\lambda_{\mathbf{g}}^{(1)}-\lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}b}-1\right)-\left(\frac{1}{\chi_0-\theta^2-\gamma^{-2}}+\frac{\omega}{2\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(2)}\right)}\right)\left(e^{i\frac{\lambda_{\mathbf{g}}^{(2)}-\lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}b}-1\right)\right], \tag{21c}$$

3. Radiation amplitude Spectral-angular density of radiation

Let us consider the radiation of a relativistic electron in the case when the second layer is a vacuum ($\chi_a=0$).To clarify and analyze the effects that are not associated with absorption, we consider a simple case of a thin nonabsorbing target ($\chi_0''=\chi_c''=0$). Substituting relations (21b) and (21c) into the well known expression [23] for the spectral–angular density of X-rays

$$\omega\frac{d^2N}{d\omega\, d\Omega}=\omega^2(2\pi)^{-6}\sum_{s=1}^{2}\left|E^{(s)Rad}\right|^2, \tag{22}$$

we obtain the expressions describing the spectral–angular densities of PXR and DTR from a relativistic electron in the target, which can be written in the form convenient for analysis:

$$\omega\frac{d^2N_{\mathrm{PXR}}^{(s)}}{d\omega\, d\Omega}=\frac{e^2}{4\pi^2}\frac{P^{(s)^2}}{|\chi_0'|}\cdot\frac{\Omega^2}{\left(\Omega_0^2+1\right)^2}R_{\mathrm{PXR}}^{(s)}, \tag{23a}$$

where the expression $R_{\mathrm{PXR}}^{(s)}$ describing PXR spectrum has a view

$$R_{\mathrm{PXR}}^{(s)}=4\left(1-\frac{\xi}{\sqrt{\xi^2+\varepsilon}}\right)^2\frac{\sin^2\left(\frac{B^{(s)}}{2}\left(\sigma^{(s)}+\frac{\xi-\sqrt{\xi^2+\varepsilon}}{\varepsilon}\right)\right)}{\left(\sigma^{(s)}+\frac{\xi-\sqrt{\xi^2+\varepsilon}}{\varepsilon}\right)^2}, \tag{23b}$$

$$\omega\frac{d^2N_{\mathrm{DTR}}^{(s)}}{d\omega\, d\Omega}=T_{DTR}^{(s)}=T_1^{(s)}+T_2^{(s)}+T_{\mathrm{int}}^{(s)}, \tag{24a}$$

where

$$T_1^{(s)} = \frac{e^2}{4\pi^2}\frac{P^{(s)2}}{|\chi_0'|}\cdot 4\Omega^2\left(\frac{1}{\Omega_0^2+\frac{\chi_c'}{\chi_0'}}-\frac{1}{\Omega_0^2}\right)^2\sin^2\left(\frac{B^{(s)}}{2}\cdot\frac{c}{b}\cdot\frac{1}{\nu^{(s)}}\left(\Omega_0^2+\frac{\chi_c'}{\chi_0'}\right)\right)R_{DTR}^{(s)}, \quad (24b)$$

$$T_2^{(s)} = \frac{e^2}{4\pi^2}\frac{P^{(s)2}}{|\chi_0'|}\cdot\Omega^2\left(\frac{1}{\Omega_0^2+1}-\frac{1}{\Omega_0^2}\right)^2 R_{DTR}^{(s)}, \quad (24c)$$

$$T_{\text{int}}^{(s)} = \frac{e^2}{4\pi^2}\frac{P^{(s)2}}{|\chi_0'|}\cdot 2\Omega^2\left(\frac{1}{\Omega_0^2+1}-\frac{1}{\Omega_0^2}\right)\left(\frac{1}{\Omega_0^2}-\frac{1}{\Omega_0^2+\frac{\chi_c'}{\chi_0'}}\right)\times$$

$$\times\left[\cos\left(B^{(s)}\cdot\frac{a}{b}\cdot\frac{1}{\nu^{(s)}}\Omega_0^2\right)-\cos\left(B^{(s)}\cdot\frac{a}{b}\cdot\frac{1}{\nu^{(s)}}\Omega_0^2+B^{(s)}\cdot\frac{c}{b}\cdot\frac{1}{\nu^{(s)}}\left(\Omega_0^2+\frac{\chi_c'}{\chi_0'}\right)\right)\right]R_{DTR}^{(s)}, \quad (24d)$$

where the expression describing DTR spectrum has a following view

$$R_{\text{DTR}}^{(s)} = \frac{4\varepsilon^2}{\xi^{(s)}(\omega)^2+\varepsilon}\sin^2\left(\frac{B^{(s)}\sqrt{\xi^{(s)}(\omega)^2+\varepsilon}}{\varepsilon}\right). \quad (24e)$$

The functions $T_1^{(s)}$ and $T_2^{(s)}$ have describing spectral-angular densities of DTR corresponding to waves of the radiation generated in amorphous layer and on the entrance boundary of periodical layered medium and the function $T_{\text{int}}^{(s)}$ describes the interference of these waves.

In expressions (23) – (24) following notations are accepted.

$$\Omega_0^2=\Omega^2+\Gamma^2,\quad \Omega=\frac{\theta}{\sqrt{|\chi_0'|}},\quad \Gamma=\frac{1}{\gamma\sqrt{|\chi_0'|}},\ \chi_0'=\frac{\chi_1'+r\chi_2'}{1+r},\ \nu^{(s)}=\frac{C^{(s)}\left|\sin\left(\frac{\pi n}{1+r}\right)\right|}{\frac{\pi n}{1+r}}\left|\frac{\chi_2'-\chi_1'}{\chi_1'+r\chi_2'}\right|,$$

$$\varepsilon=\frac{\sin(\delta+\theta_B)}{\sin(\delta-\theta_B)},\ B^{(s)}=\frac{1}{2\sin(\delta-\theta_B)}\frac{b}{L_{ext}^{(s)}},\ L_{ext}^{(s)}=\frac{1}{C^{(s)}\omega}\frac{\pi n}{\left|\sin\left(\frac{\pi n}{1+r}\right)\right||\chi_2'-\chi_1'|},\ \varepsilon=\frac{\sin(\delta+\theta_B)}{\sin(\delta-\theta_B)},$$

$$\sigma^{(s)}=\frac{\pi n}{C^{(s)}|\chi_2'-\chi_1'|\left|\sin\left(\frac{\pi n}{1+r}\right)\right|}\left(\theta^2+\gamma^{-2}-\chi_0'\right),\ \xi^{(s)}(\omega)=\frac{2\pi^2n^2}{T^2\omega_B}L_{ext}^{(s)}\left(1-\frac{\omega\left(1-\theta_{//}\sqrt{\frac{T^2\omega_B^2}{\pi^2n^2}-1}\right)}{\omega_B}\right)+\frac{1-\varepsilon}{2\nu^{(s)}},\ r=\frac{l_2}{l_1}. \quad (25)$$

The constructive interference of the waves from different boundaries leads to a considerable increase in the spectral-angular density of DTR under the condition derived from (24b):

$$\frac{B^{(s)}}{2}\cdot\frac{c}{b}\cdot\frac{1}{\nu^{(s)}}\left(\Omega_0^2+\frac{\chi_c'}{\chi_0'}\right)=\frac{\omega c}{4\sin(\delta-\theta_B)}\left(\theta^2+\gamma^{-2}-\chi_c'\right)=(2n+1)\frac{\pi}{2},\quad (n=0,1,2\ldots..). \quad (26)$$

Additionally, DTR spectral-angular density can be increased due to the constructive interference of the waves of TR from the amorphous layer and the entrance boundary of periodical layered target under the condition following from (24d):

$$B^{(s)}\cdot\frac{a}{b}\cdot\frac{1}{\nu^{(s)}}\Omega_0^2=\frac{\omega a}{2\sin(\delta-\theta_B)}\left(\theta^2+\gamma^{-2}\right)=(2m+1)\pi\,,\quad (m=0,1,2.....). \tag{27}$$

When the condition $\left|\frac{\chi_1'+r\chi_2'}{1+r}\right|>|\chi_c'|$ is fulfilled, the interference term $T_{\text{int}}^{(s)}$ can exceed the contribution of each term of TR in the resulting yield of DTR. In special case when averaged dielectric susceptibility of periodical medium and dielectric susceptibility of amorphous medium are equal

$$\frac{\chi_1'+r\chi_2'}{1+r}=\chi_c'\,, \tag{28}$$

from (24) under conditions (26), (27) and (28) we can obtain the expression

$$\omega\frac{d^2N_{\text{DTR}}^{(s)}}{d\omega d\Omega}=9\frac{e^2}{4\pi^2}P^{(s)^2}\theta^2\left(\frac{1}{\theta^2+\gamma^{-2}-\frac{\chi_1'+r\chi_2'}{1+r}}-\frac{1}{\theta^2+\gamma^{-2}}\right)^2R_{DTR}^{(s)}\,, \tag{29}$$

which shows that under these conditions the DTR yield from considered three-layered target will exceed the DTR yield from periodic layered medium 9 times.

The angular density of DTR can be calculated by formula

$$\frac{dN_{DTR}^{(s)}}{d\Omega}=F_{\text{DTR}}^{(s)}(\theta)=F_1^{(s)}(\theta)+F_2^{(s)}(\theta)+F_{\text{int}}^{(s)}(\theta), \tag{30a}$$

where

$$F_{1,2,\text{int}}^{(s)}(\theta)=\frac{T^2\omega_B}{2\pi^2n^2L_{ext}^{(s)}}\int_{-\infty}^{+\infty}T_{1,2,\text{int}}^{(s)}d\xi^{(s)}(\omega). \tag{30b}$$

$$\frac{dN_{DTR}^{(s)}}{d\Omega}=F_{PXR}^{(s)}(\theta)=\frac{T^2\omega_B}{2\pi^2n^2L_{ext}^{(s)}}\frac{e^2}{4\pi^2}\frac{P^{(s)^2}}{|\chi_0'|}\cdot\frac{\Omega^2}{\left(\Omega_0^2+1\right)^2}\int_{-\infty}^{+\infty}R_{\text{PXR}}^{(s)}d\xi^{(s)}(\omega). \tag{30c}$$

4. Numerical calculations

The calculations were fulfilled under conditions (26), (27) and (28) i.e. under condition of ninefold increase in spectral-angular density of DTR.

Parameters of calculation: amorphous medium Germanium (Ge), thickness of amorphous medium is $c=1.2\,\mu m$, the vacuum layer thickness is $a=9.6\,\mu m$; Periodic layered structure is carbon (C) – tungsten (W), thickness of layered structure is $b=1\mu m$, thicknesses of the layers are $l_1=10.64\cdot10^{-4}\,\mu m$ and $l_2=9.36\cdot10^{-4}\mu m$, period $T=l_1+l_2$, the angle between reflecting layers and target surface is $\delta=4.6^o$, energy of relativistic electron is $E=500\,MeV$, the angle between the direction of the electron velocity and reflecting layers is $\theta_B=2.3^o$, Bragg frequency $\omega_B=8\,keV$. The calculations were fulfilled for σ-polarized waves ($s=1$) for the first harmonic $n=1$. In Fig.2 the curves bilt by formulas (24) describing the spectral-angular density of DTR in maximum ($\theta_\perp=1\,mrad$) are represented. One of the curves is plotted for such thicknesses of amorphous and vacuum layers when the conditions of constructive interference (26) and (27) are satisfied. Other curve is plotted for the case of absence of amorphous layer. One can see that the amorphous layer (plate) is able to considerably increase the spectral-angular density of DTR. In Fig.3 the curves describing the angular densities of DTR and PXR are plotted by formulas (30). It is seen that the contribution of PXR under condition of Fig.3 is immaterial because the thickness of layer made of periodical multilayer medium is small ($b = 1\mu m$). The strong oscillations appear in angular density under changes in the observation angle as a result of transition from constructive interference to destructive one in superposition of transition radiation from amorphous layer and inlet surface of lamellar layer.

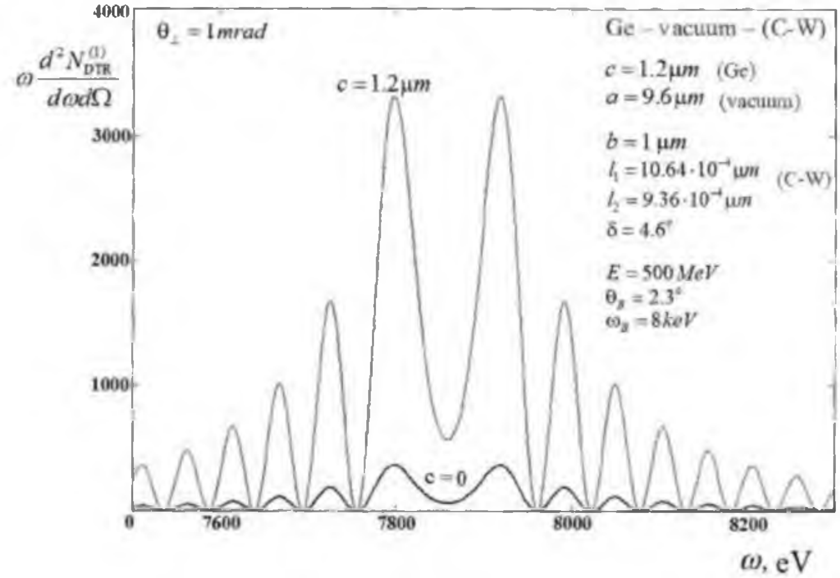

Fig.2 Spectral-angular density of DTR in the three-layer structure (c = 1.2 μm) and in the case of absence of amorphous layer (c=0).

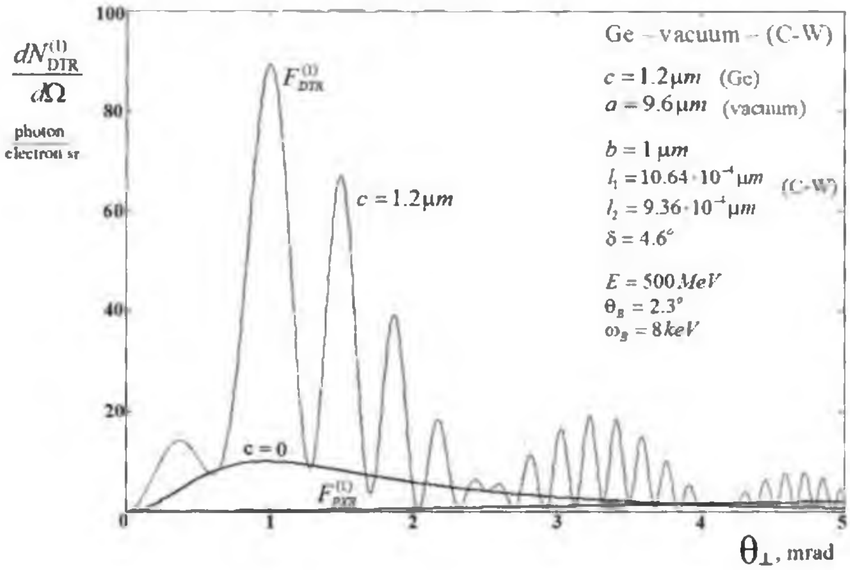

Fig.3. The angular density of DTR by a relativistic electron from three-layered structure and the same in case of absence (c=0) of the amorphous layer. The angular density of PXR under such energy of electrons is considerably lower and independent on thickness of amorphous layer.

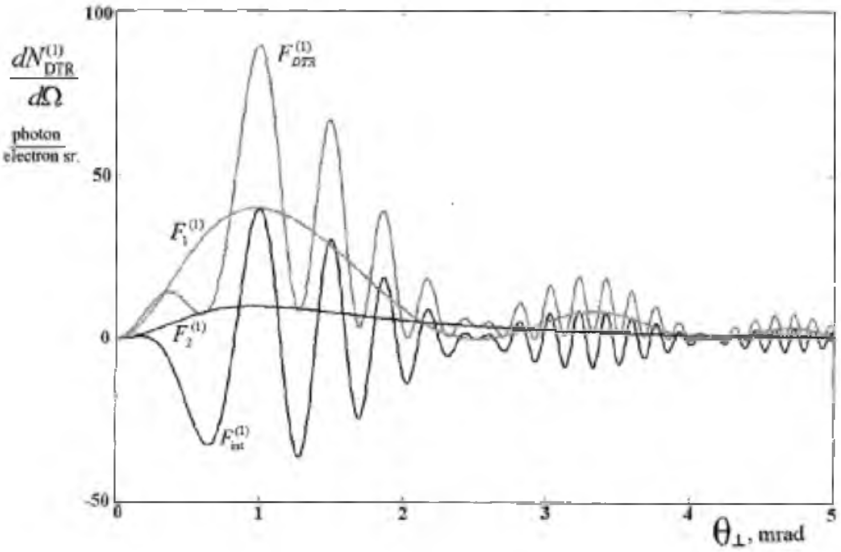

Fig. 4 The contribution of TR from the amorphous layer $F_1^{(1)}$, from the entrance boundary of the multilayered medium $F_2^{(1)}$ and their interference term $F_{int}^{(1)}$ into the angular density of DTR.

Conclusion

A theory of coherent X-ray radiation of relativistic electrons crossing the three-layer structure "amorphous layer- vacuum-periodic layered medium" have been developed within the framework of dynamical diffraction theory. The expressions describing PXR and DTR by relativistic electron have been derived. It is shown that the spectral-angular density of the DTR can be substantially increased due to the constructive interference of TR waves from different boundaries of amorphous layer, as well as due to the constructive interference of waves of transition radiation from the amorphous layer and the entrance boundary of the periodic layered target.

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