ISSN 0012-2661, Differential Equations, 2022, Vol. 58, No. 2, pp. 195–206. © Pleiades Publishing, Ltd., 2022.
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PARTIAL DIFFERENTIAL EQUATIONS

Convective Heat Transfer Between a Moving Solid Spherical Particle and a Viscous Gas

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Received May 29, 2020; revised January 21, 2022; accepted February 7, 2022

Abstract—An approximate solution of a boundary value problem for the convective heat transfer equation is obtained by the method of matched asymptotic expansions for small Péclet and Reynolds numbers. When solving the stationary system of gasdynamic heat transfer equations including the system of Navier–Stokes equations linearized in the velocity, the convective heat transfer equation, and the Poisson equation, it is assumed that the temperature dependences of the viscosity, thermal conductivity, and density of a gaseous medium are power-law.

DOI: 10.1134/S0012266122020069

INTRODUCTION

Mathematical studies of transport processes occurring in aerodisperse systems are caused by both theoretical and practical demands and currently represent a vast and rapidly developing area of gas dynamics. The transfer phenomenon is a nonequilibrium process caused by perturbations of the system that violate the state of its thermodynamic equilibrium. When aerodisperse systems having different temperatures interact, they exchange energy (heat transfer). The significance of the heat transfer process in nature and production is determined by the fact that the properties of bodies most significantly depend on their thermal state, which, in turn, is itself determined by the heat transfer conditions. These conditions have a significant impact on the state of matter evolution processes and on the mechanical, thermal, magnetic, and other properties of bodies.

A dimensionless parameter $\Theta(T)$ is introduced in the mathematical description of a heat transfer process. It characterizes the relative temperature difference, which is understood as the ratio of the difference between the average temperature T_S of the particle surface and the temperature T_∞ of the region away from it to the temperature T_∞; i.e., $\Theta(T) = (T_S - T_\infty)/T_\infty$. The relative temperature difference is considered small if $\Theta(T) \ll 1$ and significant otherwise. In the first case, the viscous gaseous medium is said to be isothermal and in the second case, it is said to be nonisothermal.

If the condition $\Theta(T) \ll 1$ is satisfied, then the solution of the system of gasdynamic equations describing the heat transfer process is greatly simplified. In particular, the molecular transfer coefficients (viscosity; thermal conductivity) and the gaseous medium density can be considered constant values, and the system of gasdynamic equations itself breaks down into hydrodynamic equations (the system of Navier–Stokes equations) and thermal equations (the Laplace equation and the Poisson equation).

In the present paper, we consider convective heat transfer due to the joint action of two mechanisms of heat transfer—convection and heat conduction. In this case, the general heat transfer equation in a viscous nonisothermal gaseous medium in the time-invariant approximation has the form [1, Ch. V, p. 232]

$$\rho(x)c_p(x)\big(\mathbf{U}(x)\nabla\big)T(x) = \operatorname{div}\big(\lambda(x)\nabla T(x)\big) + \Phi(x) + Q(x),$$

where $\mathbf{U}(x)$ is the mass velocity vector, $T(x)$ is the temperature, $\rho(x)$ is the density, $c_p(x)$ is the specific heat capacity under constant pressure, $\lambda(x)$ is the thermal conductivity coefficient, $\Phi(x)$ is a dissipative function that takes into account the heating of the medium due to internal friction, $Q(x)$ is the internal heat release per unit volume of the medium, ∇ is the nabla operator, and $x = (x_1, x_2, x_3) \in \mathbb{R}^3$.

When finding a solution of this equation, even with small relative temperature differences, it is necessary to know the mass velocity vector field, for which it is necessary to solve the system of Navier–Stokes equations. The system of Navier–Stokes equations, which is a mathematical expression of the laws of conservation of momentum and mass, belongs to the class of nonlinear second-order partial differential equations. As is well known, significant mathematical difficulties arise in its solution due to the presence of the convective acceleration term. Therefore, the question of not only solving the convective heat equation itself but also finding solutions of the system of equations of hydrodynamics (the system of Navier–Stokes equations) and the solvability of boundary value problems for this system [2] becomes topical.

Owing to the above-indicated problems, in gas dynamics there are approximate methods that allow one to simplify the system of hydrodynamics and adapt it to the nature of special types of physical problems. In the scientific literature, there is an extensive class of such flows in which the convective acceleration term in the Navier-Stokes system of equations can be neglected. Such systems are called velocity-linearized Navier–Stokes systems.

To date, the process of heat transfer between a spherical particle and a gaseous medium has been studied in detail only in the isothermal case (see, e.g., [3, Ch. 4]). Recently, interest has also increased in the study of boundary value problems in which the parameter $\Theta(T) \sim O(1)$ (see, e.g., [4–7]). However, these papers do not take into account the effect of convective heat transfer on the behavior of suspended aerosol particles in aerodisperse systems.

Note that when describing heat transfer, if the characteristic velocity is low (the Péclet and Reynolds numbers are small) and the relative temperature drop in the gas is small, then convective heat transfer $(\rho c_p(\mathbf{U}\nabla)T)$ can be neglected compared with molecular transfer $(\operatorname{div}(\lambda\nabla T))$. The situation changes dramatically when the relative temperature difference in the gas is significant, i.e., $\Theta(T) \sim O(1)$ (for example, a particle is heated by a laser to $700\,^\circ$C and the temperature of the surrounding gaseous medium is $20\,^\circ$C). Then the convective heat transfer is comparable on the order of magnitude to the molecular heat transfer and must be taken into account when describing aerodisperse systems. The present work deals with the study of such a case.

STATEMENT OF THE PROBLEM. MAIN EQUATIONS AND BOUNDARY CONDITIONS

We consider the classical problem for a steady-state heat transport process in a plane-parallel gas flow with velocity $\mathbf{U}_\infty$ ($\mathbf{U}_\infty \| Oz$; the Oz-axis points horizontally) past a solid nonuniformly heated particle of spherical shape of radius R under arbitrary temperature drops. The description is provided in the spherical coordinate system (r, φ, θ). Here the position of the Cartesian coordinate system is fixed with respect to the particle axis Oz passing through its center. The problem is axisymmetric (this means that all unknown functions depend only on the coordinates r and θ) and is stated as follows.

Problem. A viscous nonisothermal gaseous medium occupies an unbounded domain $\Omega_\mathrm{e} = \mathbb{R}^3 \backslash \Omega_\mathrm{i}$, where Ω_i is a spherical domain centered at the zero of the Euclidean space $\mathbb{R}^3$. Find the mass velocity vector field $\mathbf{U}_\mathrm{e}(x) = (U_1^{(\mathrm{e})}(x), U_2^{(\mathrm{e})}(x), U_3^{(\mathrm{e})}(x))$ $(x \in \Omega_\mathrm{e})$ and the temperature distributions $T_\mathrm{e}(x)$ $(x \in \Omega_\mathrm{e})$ and $T_\mathrm{i}(x)$ $(x \in \Omega_\mathrm{i})$ in the gaseous medium and inside the particle satisfying the equations

$$\nabla_k P_\mathrm{e} = \sum_{j=1}^{3} \nabla_j \left[\mu_\mathrm{e} \left(\nabla_j U_k^{(\mathrm{e})} + \nabla_k U_j^{(\mathrm{e})} - \frac{2}{3} \delta_{jk} (\nabla \cdot \mathbf{U}_\mathrm{e}) \right) \right], \quad k = 1, 2, 3, \tag{1}$$

$$\nabla \cdot (\rho_\mathrm{e} \mathbf{U}_\mathrm{e}) = 0, \tag{2}$$

$$\rho_\mathrm{e} c_{p\mathrm{e}} (\mathbf{U}_\mathrm{e} \cdot \nabla T_\mathrm{e}) = \nabla \cdot (\lambda_\mathrm{e} \nabla T_\mathrm{e}), \tag{3}$$

$$\nabla \cdot (\lambda_\mathrm{i} \nabla T_\mathrm{i}) = -q_\mathrm{i}, \tag{4}$$

where $x = (x_1, x_2, x_3) \in \mathbb{R}^3$, $P_\mathrm{e}(x)$ is the pressure, $\nabla = (\nabla_1, \nabla_2, \nabla_3)$ is the vector differential Hamilton operator in Cartesian coordinates, and $\nabla_j \equiv \partial/\partial x_j$, $q_\mathrm{i}(x)$ is a function given in Ω_i that determines the heat source density distribution inside the particle. Here and in the following, the index "e" refers to the domain Ω_e, the index "i" refers to the domain Ω_i, the index "∞" designates the values of physical variables in the unperturbed flow (i.e., far from the particle), and index "S" designates the values of physical variables taken at the average particle surface temperature equal to T_S.

In view of the fact that $\mu_{\rm e}$, $\rho_{\rm e}$, $\lambda_{\rm e}$, and $\lambda_{\rm i}$ are functions of the desired functions $T_{\rm e}(x)$ and $T_{\rm i}(x)$, the system of equations (1)–(4), on the whole, is nonlinear. In the literature, Eq. (1) is referred to as the stationary velocity-linearized Navier–Stokes equation, (2) is the continuity equation, (3) is the heat equation describing the temperature distribution in the domain $\Omega_{\rm e}$, and (4) is the Poisson equation describing the temperature distribution inside the domain $\Omega_{\rm i}$ (inside the particle).

To solve the system of gasdynamic equations, we make the following physical assumptions, which hold in most applied problems (see, e.g., [4–6]).

Assumption 1. When describing the properties of the particle and the gaseous medium, we consider the power-law form of the dependence of dynamic viscosity, thermal conductivity, and density on temperature (see, say, [8, Ch. IX, p. 217 of the Russian translation]),

$$\mu_{\rm e}=\mu_\infty(T_{\rm e}/T_\infty)^\beta,\quad \rho_{\rm e}=\rho_\infty(T_\infty/T_{\rm e}),\quad \lambda_{\rm e}=\lambda_\infty(T_{\rm e}/T_\infty)^\alpha,\quad \lambda_{\rm i}=\lambda_*(T_{\rm i}/T_\infty)^\gamma,$$

where μ_∞, ρ_∞, λ_∞, λ_*, and T_∞ are positive constants. In the indicated power-law dependences, the exponents are within the following limits: $0.5\le\alpha,\beta\le 1$ and $-1\le\gamma\le 1$.

Assumption 2. Just as in the papers [4–6], we assume that the thermal conductivity of the particle is much greater than that of the gas, which is the case for most real gaseous media. This assumption implies that in the particle–gaseous-medium system, the dependence on the angle θ in the viscosity coefficient can be neglected (it is assumed that the angular asymmetry of the temperature distribution is small), and therefore, the viscosity is related only to the temperature $T_{\rm e0}(r)$; i.e., $\mu_{\rm e}(T_{\rm e}(r,\theta))\approx\mu_{\rm e}(T_{\rm e0}(r))$. In this case, $T_{\rm e}(r,\theta)=T_{\rm e0}(r)+\delta T_{\rm e}(r,\theta)$, where $\delta T_{\rm e}(r,\theta)\ll T_{\rm e0}(r)$, while $\delta T_{\rm e}(r,\theta)$ and $T_{\rm e0}(r)$ are determined from the solution of Eqs. (3) and (4). Under this assumption, we can consider the hydrodynamic part separately from the thermal part, with the connection between the two established by means of boundary conditions.

Assumption 3. The particle is formed by a substance that is homogeneous and isotropic in its properties.

In the spherical coordinate system (r,θ,φ), the system of gasdynamic equations describing the velocity and pressure distribution outside the particle has the form [1, pp. 70–71, 232]

$$\frac{\partial P_{\rm e}}{\partial y}=\frac{\partial\sigma_{rr}}{\partial y}+\frac{2}{y}\sigma_{rr}+\frac{1}{y}\frac{\partial\sigma_{r\theta}}{\partial\theta}+\frac{\cot\theta}{y}\sigma_{r\theta}-\frac{\sigma_{\theta\theta}-\sigma_{\varphi\varphi}}{y},\tag{5}$$

$$\frac{1}{y}\frac{\partial P_{\rm e}}{\partial\theta}=\frac{\partial\sigma_{r\theta}}{\partial y}+\frac{3}{y}\sigma_{r\theta}+\frac{1}{y}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}+\frac{\cot\theta}{y}(\sigma_{\theta\theta}-\sigma_{\varphi\varphi}),\tag{6}$$

$$\frac{1}{y^2}\frac{\partial}{\partial y}(y^2\rho_{\rm e}U_r^{(\rm e)})+\frac{1}{y\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\rho_{\rm e}U_\theta^{(\rm e)})=0,\tag{7}$$

and in view of Assumption 1, the heat equations take the form

$$\frac{\rho_\infty c_{p\rm e}R}{\lambda_\infty t_{\rm e}}\left(U_r^{(\rm e)}\frac{\partial t_{\rm e}}{\partial y}+\frac{U_\theta^{(\rm e)}}{y}\frac{\partial t_{\rm e}}{\partial\theta}\right)=\frac{1}{y^2}\frac{\partial}{\partial y}\left(y^2t_{\rm e}^\alpha\frac{\partial t_{\rm e}}{\partial y}\right)+\frac{1}{y^2\sin\theta}\frac{\partial}{\partial\theta}\left(t_{\rm e}^\alpha\sin\theta\frac{\partial t_{\rm e}}{\partial\theta}\right),\tag{8}$$

$$\frac{1}{y^2}\frac{\partial}{\partial y}\left(y^2t_{\rm i}^\gamma\frac{\partial t_{\rm i}}{\partial y}\right)+\frac{1}{y^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta t_{\rm i}^\gamma\frac{\partial t_{\rm i}}{\partial\theta}\right)=-\frac{R^2}{\lambda_*T_\infty}q_{\rm i},\tag{9}$$

where $y=r/R$; $U_r^{(\rm e)}$ and $U_\theta^{(\rm e)}$ are the normal and tangential components of the mass velocity in the spherical coordinate system; $t_{\rm e}=T_{\rm e}/T_\infty$ and $t_{\rm i}=T_{\rm i}/T_\infty$; σ_{rr}, $\sigma_{r\theta}$, $\sigma_{\theta\theta}$, and $\sigma_{\varphi\varphi}$ are the components of the stress tensor in the spherical coordinate system, which are determined by the relations [1, Ch. II, p. 70]

$$\sigma_{rr}=\mu_{\rm e}\left(2\frac{\partial U_r^{(\rm e)}}{\partial y}-\frac{2}{3}\operatorname{div}\mathbf{U}_{\rm e}\right),\qquad \sigma_{\theta\theta}=\mu_{\rm e}\left(\frac{2}{y}\frac{\partial U_\theta^{(\rm e)}}{\partial\theta}+\frac{2}{y}U_r^{(\rm e)}-\frac{2}{3}\operatorname{div}\mathbf{U}_{\rm e}\right),$$

$$\sigma_{\varphi\varphi}=\mu_{\rm e}\left(\frac{2}{y}U_r^{(\rm e)}+\frac{2}{y}\cot\theta U_\theta^{(\rm e)}-\frac{2}{3}\operatorname{div}\mathbf{U}_{\rm e}\right),\quad \sigma_{r\theta}=\mu_{\rm e}\left(\frac{\partial U_\theta^{(\rm e)}}{\partial y}+\frac{1}{y}\frac{\partial U_r^{(\rm e)}}{\partial\theta}-\frac{U_\theta^{(\rm e)}}{y}\right).$$

Boundary conditions of the first and second kind are set for the heat transfer equations: on the particle surface (for $y = 1$), one has the equality of temperatures and the continuity of radial heat fluxes with allowance for radiation, as well as the following standard conditions as $y \to \infty$ and $y \to 0$:

$$\lim_{y\to 1} T_{\mathrm{e}}(y,\theta) = \lim_{y\to 1} T_{\mathrm{i}}(y,\theta), \tag{10}$$

$$\lim_{y\to 1} \lambda_{\mathrm{e}} \frac{\partial T}{\partial y} = \lim_{y\to 1}\left(\lambda_{\mathrm{i}} \frac{\partial T_{\mathrm{i}}}{\partial y} + R\sigma_0\sigma_1 (T_{\mathrm{i}}^4 - T_\infty^4)\right), \tag{11}$$

$$\lim_{y\to\infty} P_{\mathrm{e}} = P_\infty, \quad \lim_{y\to\infty} T_{\mathrm{e}} = T_\infty, \quad \lim_{y\to 0} |T_{\mathrm{i}}| < \infty, \tag{12}$$

$$\lim_{y\to\infty}\left(U_r^{(\mathrm{e})}(y,\theta) - U_\infty\cos\theta\right) = 0, \quad \lim_{y\to\infty}\left(U_\theta^{(\mathrm{e})}(y,\theta) + U_\infty\sin\theta\right) = 0,$$

where σ_0 is the Stefan–Boltzmann constant, σ_1 is the total emissivity of the body, and $U_\infty = |\mathbf{U}_\infty|$. By virtue of the generality of the statement of the problem, the boundary conditions on the particle surface for the normal $U_r^{(\mathrm{e})}(y,\theta)$ and tangential $U_\theta^{(\mathrm{e})}(y,\theta)$ components of the mass velocity $\boldsymbol{U}_{\mathrm{e}}$ are not given here. In the mathematical description of the uniform motion of heated particles in a viscous nonisothermal gaseous medium, we will not be interested in the nature of the forces causing this motion (it can be gravitational, magnetic, thermophoretic, etc.); this will permit extending the method for solving the system of gasdynamic equations to a wider class of physical problems.

We will look for the mass velocity and pressure components in the form of an expansion in Legendre and Gegenbauer polynomials. Using the property of the Legendre and Gegenbauer polynomials, one can readily show that in the case of an axisymmetric flow, to determine the physical variables characterizing it, the consideration can be restricted to the first terms in the expansions (see [3, Ch. 4, p. 156 of the Russian translation]). Therefore, we will assume that

$$\begin{aligned} V_r^{(\mathrm{e})}(y,\theta) &= G(y)\cos\theta, \\ V_\theta^{(\mathrm{e})}(y,\theta) &= -g(y)\sin\theta, \end{aligned} \tag{13}$$

where $V_r^{(\mathrm{e})} = U_r^{(\mathrm{e})}/U_\infty$, $V_\theta^{(\mathrm{e})} = U_\theta^{(\mathrm{e})}/U_\infty$, and $G(y)$ and $g(y)$ are functions depending on the radial coordinate.

The relation between the functions $G(y)$ and $g(y)$ is found from the continuity equation (7) with allowance for the dependence of the density of the gaseous medium on temperature (Assumption 1) and has the form

$$\begin{aligned} g(y) &= G(y) + \frac{y}{2}\big(G'(y) - f(y)G(y)\big), \\ f(y) &= \frac{1}{t_{\mathrm{e}0}(y)} \frac{dt_{\mathrm{e}0}(y)}{dy}. \end{aligned} \tag{14}$$

The defining parameters in the problem are the material constants μ_∞, ρ_∞, and λ_∞ and the constants R, T_∞, and U_∞, which are preserved in the process of flow past the particle. From these parameters, it is possible to piece two dimensionless combinations—the Péclet and Reynolds numbers, which are considered to be small quantities. Since the Péclet number is expressed in terms of the Reynolds number, the Reynolds number ($\mathrm{Re}_\infty = (\rho_\infty R U_\infty)/\mu_\infty$) is used as a small parameter in the problem. The solution of Eqs. (8), (9) will be sought by the method of matched asymptotic expansions (see [9, Ch. 12, p. 291 of the Russian translation]), while we restrict ourselves to terms up to the first order of smallness (as a rule, this is sufficient for physical applications).

The inner and outer asymptotic expansions of the nondimensionalized temperature are sought in the form

$$t_{\mathrm{e}}(y,\theta) = \sum_{n=0}^{\infty} f_n(\varepsilon) t_{\mathrm{e}n}(y,\theta), \quad f_0(\varepsilon) = 1, \tag{15}$$

$$t_{\mathrm{e}}^*(\xi,\theta) = \sum_{n=0}^{\infty} f_n^*(\varepsilon) t_{\mathrm{e}n}^*(\xi,\theta), \tag{16}$$

where $\xi = \varepsilon y$ is the "contracted" radial coordinate [9, Ch. 12.3, p. 291 of the Russian translation]. In this case, it is required that the following relations hold:

$$\frac{f_{n+1}(\varepsilon)}{f_n(\varepsilon)} \to 0 \quad \text{and} \quad \frac{f^*_{n+1}(\varepsilon)}{f^*_n(\varepsilon)} \to 0 \quad \text{as} \quad \varepsilon \to 0. \tag{17}$$

As to the functions $f_n(\varepsilon)$ and $f^*_n(\varepsilon)$, it is only assumed that their order of smallness in ε increases with n (see (17)). The missing boundary conditions for the inner and outer expansions follow from the condition that the asymptotic continuations of both expansions are identical in some intermediate domain,

$$t_{\text{e}}(y \to \infty, \theta) = t^*_{\text{e}}(\xi \to 0, \theta).$$

As shown by the boundary conditions on the particle surface, the asymptotic expansions of the solution inside the particle should be sought in a form similar to (15),

$$t_{\text{i}}(y, \theta) = \sum_{n=0}^{\infty} f_n(\varepsilon) t_{\text{i}n}(y, \theta).$$

Considering the contracted radial coordinate, we have the following equation for the temperature t^*_{e} (it is obtained from the equation of convective heat transfer by replacing y with $\xi = \varepsilon y$):

$$\frac{\Pr_\infty}{t^*_{\text{e}}}\left(V^*_r \frac{\partial t^*_{\text{e}}}{\partial \xi} + \frac{V^*_\theta}{\xi}\frac{\partial t^*_{\text{e}}}{\partial \theta}\right) = \operatorname{div}^*(t^{*\alpha}_{\text{e}} \nabla^* t^*_{\text{e}}); \tag{18}$$

in this case

$$\mathbf{V}^*_{\text{e}} = \mathbf{n}_z + \varepsilon \mathbf{V}^*_{\text{e}} + \ldots, \quad t^*_{\text{e}} \to 1 \quad \text{as} \quad \xi \to \infty,$$

where $\Pr_\infty = \mu_\infty c_p/\lambda_\infty$ is the Prandtl number, ∇^* is the nabla operator obtained from the operator ∇ by the replacement of y with ξ and so on, and $\mathbf{n}_z$ is the unit vector in the direction of the Oz-axis.

SOLUTIONS OF THE HEAT TRANSFER EQUATIONS

First, we find a solution of the convective heat transfer equation. The construction of the solution starts from determining the zero term of the outer expansion (16). Obviously, in this case our boundary value problem is satisfied by the solution

$$t^*_{\text{e}0} = 1. \tag{19}$$

Let us find the zero term of the inner expansion (15). For $\varepsilon = 0$, from Eq. (8) we obtain

$$\Delta t^{1+\alpha}_{\text{e}0} = 0,$$

and the general solution has the form

$$t_{\text{e}0} = \left[K_0 + \frac{\Gamma_0}{y} + \sum_{n=1}^{\infty}\left(K_n y^n + \frac{\Gamma_n}{y^{n+1}}\right) P_n(\cos\theta)\right]^{1/(1+\alpha)}. \tag{20}$$

Here the $P_n(\cos\theta)$ are the Legendre polynomials.

The integration constants K_n and Γ_n are determined from the matching condition; to this end, the outer expansion must be expanded in a series in ξ. Then the values of the constants are found from the requirement that the behavior of the terms in the resulting series as $\xi \to 0$ corresponds to the behavior of terms of the expansion (20) as $y \to \infty$. For the zero approximations, the matching is trivial: $K_0 = 1$, and $\Gamma_n = K_n = 0$ for $n \geq 1$. Consequently,

$$t_{\text{e}0} = \left(1 + \frac{\Gamma_0}{y}\right)^{1/(1+\alpha)}. \tag{21}$$

The integration constant K_0 is found from the boundary conditions on the particle surface $(y=1)$. To determine it, one needs to know the temperature field inside the particle (9). As is shown below, the temperature field $t_{\rm i}(y,\theta)$ can be sought in the form

$$t_{\rm i}(y,\theta)=t_{\rm i0}(y)+\varepsilon t_{\rm i1}(y,\theta), \tag{22}$$

where

$$t_{\rm i0}(y)=\left(B_0+\frac{H_0}{y}-\frac{1}{y}\int\limits_1^y\psi_0(y)\,dy+\int\limits_1^y\frac{\psi_0(y)}{y}\,dy\right)^{1/(1+\gamma)},\quad \psi_0(y)=-\frac{R^2}{2\lambda_*}\frac{1+\gamma}{T_\infty}y^2\int\limits_{-1}^{1}q_{\rm i}\,dx,$$

$$H_0=\int\limits_1^0\psi_0(y)\,dy,\quad \psi_1(y)=-\frac{3R^2}{2\lambda_*T_\infty}y^2\int\limits_{-1}^{+1}q_{\rm i}x\,dx,\quad H_1=\int\limits_1^0\psi_1 y\,dy=\frac{R^2}{3\lambda_*T_\infty}J_1,\quad x=\cos\theta,$$

$$t_{\rm i1}(y,\theta)=\frac{N_0}{t_{\rm i0}^\gamma}+\frac{\cos\theta}{t_{\rm i0}^\gamma}\left[B_1y+\frac{H_1}{y^2}+\frac{1}{3}\left(y\int\limits_1^y\frac{\psi_1(y)}{y^2}\,dy-\frac{1}{y^2}\int\limits_1^y\psi_1(y)y\,dy\right)\right],\quad J_1=\frac{1}{V}\int\limits_V q_{\rm i}z\,dV.$$

Here $\int_V q_{\rm i}z\,dV$ is the dipole moment of the heat source density [7], integration is over the entire particle volume, $V=4\pi R^3/3$, and $z=r\cos\theta$.

By T_S we denote the mean temperature on the particle surface $(T_S=t_{{\rm i}S}T_\infty,\ t_{{\rm i}S}=t_{\rm i0}\ (y=1))$; then from the boundary conditions (10), (11) we obtain

$$\Gamma_0=\left(\frac{T_S}{T_\infty}\right)^{1+\alpha}-1,\quad \frac{\ell^{(S)}}{1+\alpha}\frac{\lambda_{{\rm e}S}}{\lambda_{{\rm i}S}}t_{{\rm e}S}=\frac{R^2}{3\lambda_{{\rm i}S}T_\infty}J_0-\sigma_0\sigma_1\frac{RT_\infty^3}{\lambda_{{\rm i}S}}(t_{{\rm i}S}^4-1),$$

where

$$\lambda_{{\rm e}S}=\lambda_\infty t_{{\rm e}S}^\alpha,\quad \lambda_{{\rm i}S}=\lambda_* t_{{\rm i}S}^\gamma,\quad t_{{\rm e}S}=t_{\rm e0}\quad (y=1),$$
$$\ell^{(S)}=\ell\quad (y=1),\quad \ell(y)=\frac{\Gamma_0}{y+\Gamma_0},\quad J_0=\frac{3}{4\pi R^3}\int\limits_V q_{\rm i}\,dV.$$

Let us find the first approximations to the outer temperature. For the terms in the first approximation of the outer expansion, by virtue of the representation (16) we have

$$t_{\rm e}(\xi,\theta)=1+f_1^*(\varepsilon)t_{\rm e1}^*(\xi,\theta).$$

Let us find the explicit form of the coefficient $f_1(\varepsilon)$. To this end, we pass to the outer variable in the solution (21). It can be seen that $f_1(\varepsilon)=\varepsilon$; then

$$t_{\rm e}(\xi,\theta)=1+\varepsilon t_{\rm e1}^*(\xi,\theta). \tag{23}$$

Substituting the expression (23) for $t_{\rm e}$ into Eq. (18) and retaining terms of the order of ε, we obtain the equation

$${\rm Pr}_\infty\left(x\frac{\partial t_{\rm e1}^*}{\partial\xi}+\frac{1-x^2}{\xi}\frac{\partial t_{\rm e1}^*}{\partial x}\right)=\Delta^*t_{\rm e1}^*\quad\text{as}\quad \xi\to\infty,\quad t_{\rm e1}^*\to 0. \tag{24}$$

The general solution of Eq. (24) has the form

$$t_{\rm e1}^*=\exp\left\{\frac{{\rm Pr}_\infty}{2}x\xi\right\}\left(\frac{\pi}{{\rm Pr}_\infty\xi}\right)^{1/2}\sum_{n=0}^{\infty}L_nK_{n+1/2}\left(\frac{{\rm Pr}_\infty\xi}{2}\right)P_n(x),$$

$$\left(\frac{\pi}{\mathrm{Pr}_\infty\xi}\right)^{1/2}K_{n+1/2}\left(\frac{\mathrm{Pr}_\infty\xi}{2}\right)=\left(\frac{\pi}{\mathrm{Pr}_\infty\xi}\right)^{1/2}\exp\left\{-\frac{\mathrm{Pr}_\infty}{2}\xi\right\}\sum_{m=0}^{n}\frac{(n+m)!}{(n-m)!m!(\mathrm{Pr}_\infty\xi)^m}.$$

Here $\left(\frac{\pi}{\mathrm{Pr}_\infty\xi}\right)^{1/2}K_{n+1/2}\left(\frac{\mathrm{Pr}_\infty\xi}{2}\right)$ is a modified Bessel function [10, p. 398 of the Russian translation]. The integration constants L_n are determined from the matching condition, $L_0=(\mathrm{Pr}_\infty\Gamma_0)/(\pi(1+\alpha))$ and $L_n=0$ as $n\ge 1$. Consequently,

$$t^*_{\mathrm{e}1}(\xi,\theta)=\frac{\Gamma_0}{(1+\alpha)\xi}\exp\left\{-\frac{\mathrm{Pr}_\infty}{2}\xi(1-\cos\theta)\right\}. \tag{25}$$

Let us find the first approximation to the inner expansion (15). It can be seen from formula (25) that $f_1(\varepsilon)=\varepsilon$, and then

$$t_{\mathrm{e}}(y,\theta)=t_{\mathrm{e}0}(y)+\varepsilon t_{\mathrm{e}1}(y,\theta). \tag{26}$$

Substituting the expression (26) for t_{e} into Eq. (8), we arrive at an equation for the function $t_{\mathrm{e}1}(y,\theta)$,

$$\frac{\mathrm{Pr}_\infty}{t_{\mathrm{e}0}(y)}V_r^{(\mathrm{e})}(y,\theta)\frac{\partial t_{\mathrm{e}0}(y)}{\partial y}=\operatorname{div}\left(t^\alpha_{\mathrm{e}0}(y)\nabla t_{\mathrm{e}1}(y,\theta)+\alpha t^{\alpha-1}_{\mathrm{e}0}(y)t_{\mathrm{e}1}(y,\theta)\nabla t_{\mathrm{e}0}(y)\right). \tag{27}$$

To determine the behavior of $t_{\mathrm{e}1}(y\to\infty,\theta)$ (i.e., the boundary condition), we match the two-term inner and outer expansions and obtain

$$t_{\mathrm{e}1}(y\to\infty,\theta)=\frac{\omega_1}{2}(\cos\theta-1),\quad \omega_1=\frac{\Gamma_0\mathrm{Pr}_\infty}{1+\alpha}. \tag{28}$$

It can be seen from Eq. (27) that to find the first approximation $t_{\mathrm{e}1}(y,\theta)$ to the inner expansion, one needs to know the components of the mass velocity field. To determine them, we proceed by analogy with how this was done in [4, 5]. Taking into account the solution (21) and Assumption 2, we conclude that the dependence of dynamic viscosity on temperature has the form

$$\mu_{\mathrm{e}}(y,\theta)=\mu_\infty\left(1+\frac{\Gamma_0}{y}\right)^{\beta(1+\alpha)}. \tag{29}$$

Let us differentiate Eq. (5) with respect to the variable θ and Eq. (6) with respect to the variable y; then, substituting the representations (13) into the resulting equations and taking into account relations (14) and (29), after obvious transformations for the function $G(y)$ on the half-interval $[1,\infty)$ we obtain the third-order linear inhomogeneous differential equation

$$\begin{aligned}&[\ell^3-3\ell^4+3\ell^5-\ell^6]\frac{d^3G(\ell)}{d\ell^3}+\left[2\ell^2-\ell^3(10+\gamma_1)+2\ell^4(7+\gamma_1)-\ell^5(6+\gamma_1)\right]\frac{d^2G(\ell)}{d\ell^2}\\&\quad-\left[6\ell-\ell^2(2-2\gamma_1-\gamma_2)-\ell^3(10+4\gamma_1+\gamma_2+\gamma_3)+\ell^4(6+2\gamma_1+\gamma_3)\right]\frac{dG(\ell)}{d\ell}\\&\quad+\gamma_3\ell^2(2-\ell)G(\ell)=\frac{d}{\Gamma_0}\ell(1-\ell)^{\gamma_4^*},\end{aligned} \tag{30}$$

where

$$\gamma_1=\frac{1-\beta}{1+\alpha},\quad \gamma_1=2\frac{1+\beta}{1+\alpha},\quad \gamma_3=\frac{2+2\alpha-\beta}{(1+\alpha)^2},\quad \gamma_4^*=\frac{\beta}{1+\alpha}-1,\quad d=\mathrm{const},$$
$$1<y<\infty,\quad \ell(y)=\frac{\Gamma_0}{y+\Gamma_0},$$

with the boundary condition

$$\lim_{y\to\infty}G(y)=1. \tag{31}$$

In what follows, the homogeneous equation (30) is Eq. (30) with zero right-hand side. Let us find the solution of the homogeneous equation (30). The point $\ell = 0$ is a regular singular point of this equation, and so we seek its solution in the form of the generalized power series [10, Ch. V, p. 101 of the Russian translation; 11, Ch. IV, p. 146 of the Russian translation]

$$G(\ell) = \ell^{\rho} \sum_{n=0}^{\infty} \alpha_n \ell^n, \quad \alpha_0 \neq 0. \tag{32}$$

Substituting the series (32) into the homogeneous equation (30) and equating the coefficient of y^{ρ} with zero, we obtain the determining equation $\rho(\rho-3)(\rho+2) = 0$ with the roots $\rho_1 = 3$, $\rho_2 = 0$, and $\rho_3 = -2$. The largest of these roots is associated with the solution

$$G_1(y) = \ell^3 \sum_{n=0}^{\infty} \alpha_n^{(1)} \ell^n, \quad \alpha_0^{(1)} \neq 0. \tag{33}$$

Substituting the series (33) into the homogeneous equation (30) and using the method of undetermined coefficients, we arrive at the following recursive formula for the coefficients $\alpha_n^{(1)}$ $(n \geq 1)$:

$$\begin{aligned}\alpha_n^{(1)} = \frac{1}{n(n+3)(n+5)} \Big\{ &(n+2)\big[(n+1)(3n+10+\gamma_1) - 2 + 2\gamma_1 + \gamma_2\big]\alpha_{n-1}^{(1)} \\ &- \Big[(n+1)\big(n(3n+11+2\gamma_1) + 10 + 4\gamma_1 + \gamma_2 + \gamma_3\big) + 2\gamma_3\Big]\alpha_{n-2}^{(1)} \\ &+ \Big[n\big((n-1)(n+4+\gamma_1) + 6 + \gamma_3 + 2\gamma_1\big) + \gamma_3\Big]\alpha_{n-3}^{(1)}\Big\}.\end{aligned}$$

The second solution $G_3(\ell)$ of the homogeneous equation (30) corresponding to the root $\rho_2 = 0$ has the form

$$G_3(\ell) = \sum_{n=0}^{\infty} \alpha_n^{(3)} \ell^n + \omega_3^* G_1(\ell) \ln\left(\frac{\ell}{\ell_0}\right), \quad \alpha_0^{(3)} \neq 0, \quad \ell_0 = \ell \quad (y=1), \tag{34}$$

where the coefficients $\alpha_n^{(3)}$ $(n \geq 3)$ are determined by the recursive formula

$$\begin{aligned}\alpha_n^{(3)} &= \frac{1}{n(n+2)(n-3)} \Big\{ (n-1)\big[(n-2)(3n+1+\gamma_1) - 2 + 2\gamma_1 + \gamma_2\big]\alpha_{n-1}^{(3)} \\ &\quad - \Big[(n-2)\big((n-3)(3n+2+2\gamma_1) + 10 + 4\gamma_1 + \gamma_2\gamma_2\big) + 2\gamma_3\Big]\alpha_{n-2}^{(3)} \\ &\quad + \Big[(n-3)\big((n-4)(n+1+\gamma_1) + 6 + \gamma_3 + 2\gamma_1\big) + \gamma_3\Big]\alpha_{n-3}^{(3)} - \omega_3^* S_{n-3}^{(1)}\Big\}, \\ S_n^{(1)} &= (3n^2 + 16n + 15)\alpha_n^{(1)} - \big(9n^2 + 18n + (10+\gamma_1)(2n+3) + 2\gamma_1 + \gamma_2\big)\alpha_{n-1}^{(1)} \\ &\quad + \big(9n^2 + 7 + (14+2\gamma_1)(2n+1) + 4\gamma_1 + \gamma_2 + \gamma_3\big)\alpha_{n-2}^{(1)} \\ &\quad - \big(3n^2 - 6n + 8 + (6+\gamma_1)(2n-1) + 2\gamma_1 + \gamma_3\big)\alpha_{n-3}^{(1)}.\end{aligned}$$

We do not give the third solution of the homogeneous equation (30) linearly independent with the solutions $G_1(\ell)$ and $G_3(\ell)$ and corresponding to the root $\rho_3 = -2$ of the determining equation here, since it does not satisfy the boundary condition (31).

Based on the form of the right-hand side of the inhomogeneous equation (30), we seek its particular solution in the form

$$G(\ell) = A_2 G_2(\ell), \quad G_2(\ell) = \ell \sum_{n=0}^{\infty} \alpha_n^{(2)} \ell^n + \omega_2^* G_1(\ell) \ln\left(\frac{\ell}{\ell_0}\right), \quad \alpha_0^{(2)} \neq 0, \quad A_2 = \text{const}. \tag{35}$$

Acting in a similar manner, we obtain recursive formulas for the coefficients $\alpha_n^{(2)}$ $(n \geq 3)$,

$$\alpha_n^{(2)} = \frac{1}{(n+1)(n+3)(n-2)}\Big\{n\big[(n-1)(3n+4+\gamma_1) - 2 + 2\gamma_1 + \gamma_2\big]\alpha_{n-1}^{(2)} - \Big[(n-1)\big((n-2)(3n+5+2\gamma_1) + 10 + 4\gamma_1 + \gamma_2 + \gamma_3\big) + 2\gamma_3\Big]\alpha_{n-2}^{(2)} + \Big[(n-2)\big((n-3)(n+2+\gamma_1) + 6 + \gamma_3 + 2\gamma_1\big) + \gamma_3\Big]\alpha_{n-3}^{(2)} - \omega_2^* S_{n-2}^{(1)} + \frac{d}{\Gamma_0}(-1)^n \frac{\gamma_4^*!}{(\gamma_4^* - n)!n!}\Big\}.$$

Here

$$\omega_2^* = \frac{1}{15\alpha_0^{(1)}}\Big[(16 + 6\gamma_1 + 2\gamma_2)\alpha_1^{(2)} - \big(10 + 4\gamma_1 + \gamma_2 + 3\gamma_3 + 3\gamma_4^*(\gamma_4^* - 1)\big)\alpha_0^{(2)}\Big],$$

$$\frac{d}{\Gamma_0} = -6\alpha_0^{(2)}, \quad \alpha_1^{(2)} = -\frac{\alpha_0^{(2)}}{6}(-2 + 2\gamma_1 + \gamma_2 + 6\gamma_4^*), \quad \alpha_2^{(2)} = 1, \quad \alpha_1^{(3)} = 0, \quad \alpha_3^{(3)} = 1,$$

$$\omega_3^* = \frac{\gamma_3}{15\alpha_0^{(1)}}\Big[(16 + 6\gamma_1 + 2\gamma_2)\alpha_1^{(2)} - \big(10 + 4\gamma_1 + \gamma_2 + 3\gamma_3 + 3\gamma_4^*(\gamma_4^* - 1)\big)\alpha_0^{(2)}\Big],$$

$$\alpha_2^{(3)} = \frac{\gamma_3}{4}\alpha_0^{(3)}, \quad \alpha_n^{(k)} = 0 \quad \text{if} \quad n < 0 \quad (k = 1, 2, 3).$$

Thus, the general solution of Eq. (30) has the form

$$G(\ell) = A_1 G_1(\ell) + A_2 G_2(\ell) + A_3 G_3(\ell), \tag{36}$$

where the functions $G_1(\ell)$, $G_2(\ell)$, and $G_3(\ell)$ are given by formulas (33), (35), and (34), respectively, while the coefficients occurring in them are determined by the equalities after these formulas.

Note that the function $G(\ell)$ satisfies Eq. (30) by construction. The series determining the functions $G_k(\ell)$, $k = 1, 2, 3$, converge uniformly for all $1 \leq y < \infty$ $(\ell(y) = \Gamma_0/(y + \Gamma_0))$.

Taking into account the explicit form of the coefficients $\alpha_n^{(1)}$, we introduce the majorizing series

$$\sum_{n=0}^{\infty} \beta_n^{(1)} \ell^n, \tag{37}$$

where the coefficients $\beta_n^{(1)}$ are nonnegative and determined by the recursive formula

$$\beta_n^{(1)} = 3\beta_{n-1}^{(1)} + 3\beta_{n-2}^{(1)} + \beta_{n-3}^{(1)};$$

here $\beta_{-2}^{(1)} = \beta_{-1}^{(1)} = \beta_0^{(1)} = \beta$, and $\beta > 0$ is a sufficiently large number.

Then we have the estimate

$$\beta_n^{(1)} \leq 7^n \beta.$$

Consequently, the series (37) converges for $7\ell < 1$ ($\ell < 1/7$ or $y > 6\Gamma_0$, $\Gamma_0 = (T_s/T_\infty)^{1+\alpha} - 1$). Since the coefficients of the differential equation (30) are analytic for $1 \leq y < \infty$, it follows by the analytic continuation principle that the series (37), which is a solution of this equation, can not have singular points for $1 \leq y < \infty$. Thus, the series (33) converges for all $1 \leq y < \infty$. In a similar manner, we can establish the uniform convergence of the series that determine the functions $G_2(\ell)$ and $G_3(\ell)$.

Thus, our study results in the following assertion.

Theorem 1. *The general solution of Eq.* (30) *has the form* (36), *where the coefficients* A_1, A_2, *and* A_3 *are arbitrary constants and the functions* $G_1(\ell)$, $G_2(\ell)$, *and* $G_3(\ell)$ *are given by formulas* (33), (35), *and* (34).

Note that the constants $\alpha_0^{(1)}$, $\alpha_0^{(2)}$, and $\alpha_0^{(3)}$ are chosen so that the passage to the limit problem on a flow past a sphere occurs for small relative temperature drops. To this end, it is convenient to pass from the variable ℓ to the variable y. Then we have

$$G(y) = A_1G_1(y) + A_2G_2(y) + A_3G_3(y),$$

$$G_1(y) = \frac{1}{y^3}\sum_{n=0}^{\infty} C_n^{(1)}\ell^n, \quad G_2(y) = \frac{1}{y}\sum_{n=0}^{\infty} C_n^{(2)}\ell^n + \omega_2 G_1(y)\ln y, \quad G_3(y) = \sum_{n=0}^{\infty} C_n^{(3)}\ell^n + \omega_3 G_1(y)\ln y,$$

$$C_n^{(1)} = \Gamma_0^3(1-\ell)^3\alpha_n^{(1)}, \quad C_n^{(2)} = \Gamma_0(1-\ell)\alpha_n^{(2)}, \quad C_n^{(3)} = \alpha_n^{(3)},$$

$$\omega_2 = \frac{\omega_2^*}{\ln(\Gamma_0(1-\ell)/\ell)}\ln\left(\frac{\ell}{\ell_0}\right), \quad \omega_3 = \frac{\omega_3^*}{\ln(\Gamma_0(1-\ell)/\ell)}\ln\left(\frac{\ell}{\ell_0}\right).$$

Then the functions $G_1(y)$, $G_2(y)$, and $G_3(y)$ tend to the respective functions for the sphere with small relative temperature drops [2, p. 144; 3, p. 83]. As $\Gamma_0 \to 0$, we have

$$G_1(y) \to \frac{1}{y^3}, \quad G_2(y) \to \frac{1}{y}, \quad G_3(y) \to 1;$$

based on this, it can be seen that $C_0^{(1)} = C_0^{(2)} = C_0^{(3)} = 1$ and $A_3 = 1$.

Knowing the general solution of Eq. (30) and the relationship between the functions $G(y)$ and $g(y)$, we obtain an expression for the components of the vector velocity field $\mathbf{U}_{\mathrm{e}}(x)$,

$$V_r(y,\theta) = \cos\theta\big[A_1G_1(y) + A_2G_2(y) + G_3(y)\big], \tag{38}$$

$$V_\theta(y,\theta) = -\sin\theta\big[A_1G_4(y) + A_2G_5(y) + G_6(y)\big], \tag{39}$$

where

$$G_k(y)\left(1 + \frac{\ell}{2(1+\alpha)}\right)G_{k-3}(y) + \frac{y}{2}G'_{k-3}(y) \quad (k = 4,5,6).$$

Thus, in the present paper, for the stationary system of Navier–Stokes equations linearized in velocity with a power-law dependence of the molecular transfer coefficients (viscosity and thermal conductivity) and the gaseous medium density on temperature, we have obtained an approximate solution that satisfies the boundary condition (31).

Since we have obtained expressions for the mass velocity components, we can solve Eq. (27) with the boundary condition (28). We seek a solution of this equation in the form

$$t_{\mathrm{e}1}(y,\theta) = \zeta(y) + \frac{1}{t_{\mathrm{e}0}^{\alpha}}\tau_{\mathrm{e}}(y)\cos\theta \tag{40}$$

with the boundary conditions

$$\zeta(y\to\infty) \to -\frac{\omega_1}{2}, \quad \tau_{\mathrm{e}}(y\to\infty) \to \frac{\omega_1}{2}. \tag{41}$$

Substituting the representation (40) into Eq. (27) and taking into account the fact that

$$V_r^{(\mathrm{e})}(y,\theta) = \cos\theta\big(A_1G_1(y) + A_2G_2(y) + G_3(y)\big), \quad t_{\mathrm{e}0}(y) = (1+\Gamma_0/y)^{1/(1+\alpha)},$$

for the function τ_{e} we obtain the linear ordinary inhomogeneous differential equation

$$\frac{d^2\tau_{\mathrm{e}}}{dy^2} + \frac{2}{y}\frac{d\tau_{\mathrm{e}}}{dy} - \frac{2}{y^2}\tau_{\mathrm{e}} = -\omega_1\frac{\ell}{y\Gamma_0}\big(A_1G_1(y) + A_2G_2(y) + G_3(y)\big),$$

whose solution can be found in a standard manner.

Thus, we have the following expression for the function $t_{\mathrm{e}1}(y,\theta)$ satisfying the boundary conditions (41):

$$t_{\mathrm{e}1}(y,\theta)=\frac{\omega_1}{2yt_{\mathrm{e}0}^{\alpha}(y)}(N_1-y)+\frac{\cos\theta}{t_{\mathrm{e}0}^{\alpha}}\left[\frac{\Gamma_1}{y^2}+\frac{\omega_1}{3}\left(\tau_3(y)+A_2\frac{\tau_2(y)}{y}-A_1\frac{\tau_1(y)}{y^3}\right)\right]. \tag{42}$$

Here

$$\tau_1(y)=(1-\ell)\sum_{n=0}^{\infty}\frac{\Omega_n^{(1)}\ell^n}{n+1}-\frac{(1-\ell)^4}{6}\sum_{n=0}^{\infty}\frac{\Omega_n^{(3)}\ell^n}{n+4},$$

$$\tau_2(y)=\frac{1}{1-\ell}\left[1+\ell\ln\ell+C_1^{(2)}(\ell^2-\ell\ln\ell)-\sum_{n=0}^{\infty}\frac{C_n^{(2)}\ell^n}{n-1}\left(1-\frac{n-1}{n}\ell\right)\right]$$
$$+(1-\ell)^2\sum_{n=0}^{\infty}\frac{\Omega_n^{(4)}\ell^n}{n+2}+\frac{\omega_2(1-\ell)}{y^2}S_n^{(2)},$$

$$\tau_3(y)=\frac{1}{(1-\ell)^2}\left[\frac{1}{2}-2\ell-\ell^2\ln\ell+C_1^{(3)}(\ell+2\ell^2\ln\ell-\ell^3)+C_2^{(3)}\left(-\frac{1}{2}-\ell^2\ln\ell+2\ell^3\right)\right.$$
$$\left.-\sum_{n=3}^{\infty}\frac{C_n^{(3)}\ell^n}{n-2}\left(1+\frac{n-2}{n}\ell^2-2\frac{n-2}{n-1}\ell\right)\right]+(1-\ell)\sum_{n=0}^{\infty}\frac{\Omega_n^{(6)}\ell^n}{n+1}+\frac{\omega_3}{y^3}(1-\ell)S_n^{(2)},$$

$$\Omega_n^{(1)}=\sum_{k=0}^{n}C_k^{(1)},\quad \Omega_n^{(3)}=\sum_{k=0}^{n}(n-k+1)(n-k+2)(n-k+3)C_k^{(1)},$$

$$\Omega_n^{(2)}=\sum_{k=0}^{n}\frac{\Omega_k^{(1)}}{k+1},\quad \Omega_n^{(6)}=\sum_{k=0}^{n}C_k^{(3)},\quad \Omega_n^{(5)}=\sum_{k=0}^{n}\frac{\Omega_k^{(3)}}{k+4},$$

$$\Theta_n^{(1)}=\Omega_n^{(2)}+\Omega_n^{(1)}\ln y,\quad \Theta_n^{(2)}=\Omega_n^{(5)}+\Omega_n^{(3)}\ln y,\quad \Omega_n^{(4)}=\sum_{k=0}^{n}(n-k+1)C_k^{(2)},$$

$$S_n^{(2)}=\sum_{n=0}^{\infty}\frac{\ell^n}{6(n+4)}\left[(1-\ell)^3\Theta_n^{(2)}-6\frac{n+4}{n+1}\Theta_n^{(1)}\right].$$

Theorem 2. *The temperature distributions $T_{\mathrm{e}}(x)$ $(x\in\Omega_{\mathrm{e}})$ and $T_{\mathrm{i}}(x)$ $(x\in\Omega_{\mathrm{i}}$, where $\Omega_{\mathrm{e}}=\mathbb{R}^3\setminus\Omega_{\mathrm{i}}$ $(\Omega_{\mathrm{i}}$ is a spherical domain centered at the origin of the Euclidean space $\mathbb{R}^3)$ in the gaseous medium and inside the particle satisfying Eqs.* (3), (4) *and the boundary conditions* (10)–(12) *have the form* (22), (42).

The integration constants A_1 and A_2 included in the expressions (38) and (39) and the temperature field $t_{\mathrm{e}}(y,\theta)$ are determined from the boundary conditions for the mass velocity components (normal $U_r^{(\mathrm{e})}$ and tangential $U_\theta^{(\mathrm{e})}$) on the particle surface. These boundary conditions are determined by a specific physical problem. In the mathematical description of the process of heat exchange between a moving heated particle and a viscous nonisothermal gaseous medium (in the coordinate system associated with the particle, we have the problem on a flow past a body), the nature of the forces causing this motion can be gravitational, magnetic, photophoretic, thermophoretic, etc.; this permits one to extend the developed method for solving a system of gasdynamic equations to a wide class of physical problems.

The formulas obtained above can also be used for small relative temperature drops, i.e., when the amount of heating of the surface of the particle is low. In this case, the average particle surface temperature differs slightly from the temperature of the surrounding gaseous medium far from it, and for $\Gamma_0\to 0$ $(y=1)$ we have

$$G_1=1,\quad G_1'=1,\quad G_1''=12,\quad G_2=1,\quad G_2'=-1,\quad G_2''=2,\quad G_3=1,\quad G_3'=0,\quad G_3''=0,$$
$$\tau_1=3/4,\quad \tau_1'=0,\quad \tau_2=3/2,\quad \tau_2'=0,\quad \tau_3=3/2,\quad \tau_3'=0.$$

Qualitative analysis shows (see formula (42)) that the convective heat transfer is proportional to the coefficient $\omega_1 = \Gamma_0 \mathrm{Pr}_\infty/(1+\alpha)$ (ceteris paribus). For most gases, the Prandtl number is of the order of one, and $\Gamma_0 = (T_S/T_\infty)^{(1+\alpha)} - 1$. Therefore, $\omega_1 \sim \Gamma_0$; i.e., the more strongly the particle is heated, the more significant the contribution of convective heat transfer is.

CONCLUSIONS

In this paper, an approximate solution of a stationary system of gasdynamic equations is obtained, which describes convective heat transfer between a moving unevenly heated large spherical solid particle and a viscous nonisothermal gaseous medium for low Péclet and Reynolds numbers. When describing heat transfer, it is assumed that the dependences of the thermal conductivity, viscosity, and density coefficients of the gaseous medium on temperature have a power-law nature. An approximate solution of the velocity-linearized system of Navier–Stokes equations is obtained (Theorem 1). The method of matched asymptotic expansions is used to find the solution of the heat equation describing the temperature field outside the particle, and the solution of the Poisson equation describing the temperature field inside the particle is found by the method of perturbation theory (Theorem 2). The zero approximations to the inner and outer asymptotic expansions are defined by formulas (19) and (21) and the first approximations, by formulas (25) and (42).

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