DIFFUSIOPHORESIS AND PHOTOPHORESIS
OF HEATED LARGE NONVOLATILE AEROSOL SPHERICAL PARTICLES

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Abstract: This paper describes a theoretical study of the steady motion of a large solid nonvolatile aerosol spherical particle, which contains thermal sources within itself, in a concentration gradient of binary gas mixture components. It is assumed that an average particle surface temperature significantly differs from the temperature of the binary gas mixture surrounding it. Equations of gas dynamics are solved taking into account the power-law dependence of the molecular transfer coefficients (viscosity, thermal conductivity, and diffusion) and the density of the gaseous medium on temperature. Under boundary conditions, diffusion and thermal slip are taken into account. Numerical estimates show that the diffusion and photophoretic forces and velocity substantially depend on the average particle surface temperature.

Keywords: diffusiophoresis and photophoresis, heated large nonvolatile spherical particles.

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INTRODUCTION

In gaseous media, the ordered movement of aerosol particles can occur under the influence of forces of various nature (including molecular), such as diffusiophoretic and photophoretic forces. A diffusiophoretic force causes an ordered motion of particles in two- and multicomponent gaseous media with an inhomogeneous distribution of its components due to external concentration gradients. The velocity acquired by particles as the effect of a diffusiophoretic force is balanced by the force of viscous resistance of the medium is called diffusiophoretic velocity [1–3]. Photophoresis in a gas is particle motion in a field of electromagnetic radiation under the influence of radiometric force [4, 5]. When electromagnetic radiation interacts with a particle inside of it, heat energy is released with bulk density q_i, which is why this particle is nonuniformly heated. The gas molecules surrounding the particle collide with its surface and then are reflected from the heated surface of the particle at a higher velocity speed than from a cold one. Depending on the size, shape, and optical properties of the particle material, as well as the radiation wavelength, both the illuminated and the shady surfaces of the particle may turn out to be hotter. Therefore, both positive (particle motion in the direction of radiation propagation) and negative (particle motion in the opposite direction) photophoresis may occur. The phenomena of diffusiophoresis and photophoresis are almost always observed in thermodynamically nonequilibrium aerodisperse systems

Diffusiophoretic and photophoretic forces can have a significant impact on particle deposition in the channels of heat and mass exchangers and on particle motion in the regions of "bleaching" of dispersed systems, and they

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also can be used for fine cleaning of small volumes of gases, taking aerosol samples, and applying special coatings of given thickness from particles with desired properties, etc. In theoretical works devoted to the study of diffusion and photophoresis, these phenomena are well described at small relative temperature differences in the vicinity of a particle [1–5]. A relative temperature difference is understood as the ratio of the difference between an average particle surface temperature T_{iS} and the temperature of a gaseous medium far from it T_∞ to the latter. The relative temperature difference is considered to be small if inequality $(T_{iS} - T_\infty)/T_\infty \ll 1$ is fulfilled and large if $(T_{iS} - T_\infty)/T_\infty \sim O(1)$ (i stands for the particle, S refers to the physical quantities at an average particle surface temperature, and ∞ means the physical quantities characterizing a gaseous medium in an unperturbed flow). In the latter case, the particle is called heated. Particle surface heating can be caused, for example, by a bulk chemical reaction, radioactive decay of the particle substance, absorption of electromagnetic radiation by a particle, and so on.

If $(T_{iS} - T_\infty)/T_\infty \sim O(1)$, gas-dynamic equations are solved only with account for a dependence of molecular transfer coefficients (viscosity, thermal conductivity, and diffusion) and the density of a gaseous medium on temperature. Moreover, the gaseous medium is considered to be nonisothermal, and the system of gas-dynamic equations describing this medium becomes nonlinear. There are very few papers on particle motion at significant relative temperature differences in gaseous media, with most of those only touching upon the gravitational motion of heated large solid particles [6], thermophoresis of large heated solid particles [7], and photophoresis of heated large solid particles [8]. It is shown in [6–8] that heating a particle surface can have a significant effect on particle motion.

At present, there is increasing interest in studying the behavior of suspended particles in thermodynamically nonequilibrium systems (see, e.g., [9–11]). This work is related to a combined effect of diffusion and photophoretic forces on a heated large nonvolatile spherical particle suspended in a binary gas mixture.

1. FORMULATION OF THE PROBLEM

In a binary gas mixture with density ρ_e, thermal conductivity λ_e, diffusion D_{12}, and dynamic viscosity μ_e, we determine a solid aerosol spherical particle with radius R (subscript e refers to the gas). Inside the particle, there are active nonuniformly distributed heat sources with density q_i. The particle is considered to be large [1]. The gas medium consists of two components whose relative concentrations are denoted as C_1 and C_2. Here $C_1 = n_1/n$, $C_2 = n_2/n$, $n = n_1 + n_2$, $\rho_e = \rho_1 + \rho_2$, $\rho_1 = m_1 n_1$, $\rho_2 = m_2 n_2$, and $m_1 n_1$ and $m_2 n_2$ denote the mass and numerical concentration of molecules of the first and second components of the binary mixture. It is assumed in determining the diffusiophoretic and photophoretic forces and velocity that inequality $C_1 \ll C_2$ is fulfilled.

As the particle is heated, the properties of the gaseous medium and the particle are described while accounting for the power form of the dependences of molecular transfer coefficients on temperature is taken into account [12]:

$$\mu_e = \mu_\infty t_e^\beta, \quad \lambda_e = \lambda_\infty t_e^\alpha, \quad D_{12} = D_\infty t_e^{1+\omega}, \quad \lambda_i = \lambda_{i0} t_i^\gamma.$$

Here $\mu_\infty = \mu_e(T_\infty)$, $\lambda_\infty = \lambda_e(T_\infty)$, $D_\infty = D_{12}(T_\infty)$, $\lambda_{i0} = \lambda_i(T_\infty)$, $t_k = T_k/T_\infty$ ($k = e, i$), $0.5 \leqslant \alpha \leqslant 1.0$, $0.5 \leqslant \beta \leqslant 1.0$, $0.5 \leqslant \omega \leqslant 1.0$, and $-1 \leqslant \gamma \leqslant 1$. The values of thermal conductivity, dynamic viscosity, and diffusion, for example, for a binary air–CO_2 mixture are taken at $\alpha = 0.81$, $\beta = 0.72$, and $\omega = 0.70$ (273 K $\leqslant T_e \leqslant$ 900 K). The relative approximation error is not larger than 5% [12].

In a theoretical study of diffusion and photophoresis, it is assumed that, due to a short thermal relaxation time, the heat and mass transfer process in a particle–binary gas mixture system is quasistationary. Particle motion occurs at small Peclet and Reynolds numbers, with free convection being neglected (the Grashof number is much smaller than unity). The problem is solved using the hydrodynamic method, and the main hydrodynamic variables are determined in solving the hydrodynamic equations with corresponding boundary conditions.

It is convenient to derive expressions for diffusiophoretic and photophoretic forces and velocities in a spherical coordinate system (r, θ, φ), associated with the center of mass of the aerosol particle. Thus, the solution of the problem is reduced to analyzing an infinite plane–parallel gas flow around a particle, and velocity $\boldsymbol{U}_\infty$ of this flow is to be determined ($\boldsymbol{U}_\infty \parallel Oz$). The z axis is directed horizontally along the concentration gradient $\nabla C_{1\infty}$ of the first component of the binary gas mixture, which is constant at infinity. The distributions of velocities, pressures, relative concentrations, and temperatures have axial symmetry with respect to the Oz axis. With the indicated

choice of the origin, the heated particle can be considered as stationary, and the external medium (binary gas) can be viewed as moving at a velocity $\boldsymbol{U}_\infty = -\boldsymbol{U}_{dph}$ ($\boldsymbol{U}_{dph}$ is the velocity of diffusiophoresis and photophoresis) in a direction opposite to the direction of the actual motion of the heated particle.

With account for the assumptions made, we solve the system [13]

$$\operatorname{div}(\lambda_e \nabla T_e) = 0, \qquad \operatorname{div}(\lambda_i \nabla T_i) = q_i; \tag{1.1}$$

$$\operatorname{div}\left(\frac{n^2 m_1 m_2}{\rho_e} D_{12} \nabla C_1\right) = 0, \qquad n = \frac{P_e}{kT_e}; \tag{1.2}$$

$$\frac{\partial}{\partial x_k} P_e = \frac{\partial}{\partial x_j}\left[\mu_e\left(\frac{\partial U_k^{(e)}}{\partial x_j} + \frac{\partial U_j^{(e)}}{\partial x_k} - \frac{2}{3}\delta_k^j \frac{\partial U_n^{(e)}}{\partial x_n}\right)\right], \qquad \frac{\partial}{\partial x_k}(\rho_e U_k^{(e)}) = 0$$

with the following boundary conditions:

$$y \to \infty: \quad U_r^{(e)} = U_\infty \cos\theta, \quad U_\theta^{(e)} = -U_\infty \sin\theta, \quad P_e = P_\infty,$$
$$C_1 = C_0 + |\nabla C_{1\infty}| r \cos\theta, \quad T_e = T_\infty \quad (y = r/R); \tag{1.3}$$

$$y \to 0: \quad T_i \neq \infty; \tag{1.4}$$

$$y = 1: \quad T_e = T_i, \quad -\lambda_e \frac{\partial T_e}{\partial y} = -\lambda_i \frac{\partial T_i}{\partial y} - \sigma_0 \sigma_1 R(T_i^4 - T_\infty^4), \quad \frac{\partial C_1}{\partial y} = 0; \tag{1.5}$$

$$U_r^{(e)} = 0, \quad U_\theta^{(e)} = K_{TS} \frac{\nu_e}{RT_e} \frac{\partial T_e}{\partial \theta} + K_{DS} \frac{D_{12}}{R} \frac{\partial C_1}{\partial \theta}. \tag{1.6}$$

Here x_k are the Cartesian coordinates, $U_k^{(e)}$ denotes the mass velocity components, k is the Boltzmann constant, $\nu_e = \mu_e/\rho_e$ is the kinematic viscosity, $U_\infty = |\boldsymbol{U}_\infty|$, σ_0 is the Stefan–Boltzmann constant, σ_1 is the total emissivity, and K_{TS} and K_{DS} are thermal and diffusion slip coefficients. Under boundary conditions on the particle surface (1.5) and (1.6), the equality of temperatures and the continuity of a radial heat flux with account for thermal radiation, the vanishing of the normal mass velocity component, the thermal and diffusion slip for the tangent mass velocity component, and the impermeability of the particle surface for the radial flow of the first component of the binary gas mixture are taken into account. The standard flow conditions and the concentration gradient of the first component of the binary gas mixture, which is set far from the particle, are taken into account in the boundary condition (1.3) and the finiteness of physical quantities in the particle volume is accounted for in (1.4). As the problem is solved by the hydrodynamic method (gas dynamic equations are solved), the boundary conditions are assumed to be known.

When performing numerical estimates, it is necessary to know the values of the thermal and diffusion slip coefficients. They are determined from solving the Boltzmann equation in the Knudsen layer and, in the general case, depend on the type of the molecular interaction model used, the average particle surface temperature, and the mass ratio of the mixture molecules [11, 14, 15]. If, in the boundary condition $C_{1e} = C_0 + |\nabla C_{1\infty}| r \cos\theta$, we pass on to the dimensionless variable y, then there is a small dimensionless parameter $\varepsilon = R|\nabla C_{1\infty}| \ll 1$ in the problem. It is easy to show that the velocity of a large solid particle in a concentration gradient field is proportional to $|U_{dh}| \sim D_{12}|\nabla C_{1\infty}| \sim (D_{12}/R)\varepsilon$ [1–3]. This means that, when considering diffusionophoresis, we can restrict ourselves to corrections to the first order of smallness with respect to ε. Taking into account the above-given, it is necessary to expand the slip coefficients in a series with respect to a small parameter, and, considering the boundary condition for the tangent mass velocity component and the studies from [14, 15], the role of a zeroth approximation in numerical estimates of the strength and velocity of diffusiophoresis and photophoresis should be played by values $K_{TS}^{(0)} = 1.161$ and $K_{DS}^{(0)} = 0.27$ [1, 14, 15].

The key parameters in the problem are material constants μ_∞, ρ_∞, λ_∞, and D_∞, as well as parameters R, T_∞, $|\nabla C_{1\infty}|$, and U_∞. Reynolds numbers can be made from these parameters ($\mathrm{Re}_\infty = RU_\infty/D_\infty$). Thus, the role of a small parameter is played by $\varepsilon = R|\nabla C_{1\infty}|$ in describing diffusiophoresis and by $\varepsilon = \mathrm{Re}_\infty$ in describing photophoresis.

At $\varepsilon \ll 1$, the solution of the gas-dynamic equations is sought in the form

$$\boldsymbol{V}_e = \boldsymbol{V}^{(0)} + \varepsilon \boldsymbol{V}_e^{(1)} + \dots, \qquad P_e = P_e^{(0)} + \varepsilon P_e^{(1)} + \dots \tag{1.7}$$

$(\boldsymbol{V}_e = \boldsymbol{U}_e/U_\infty)$.

It follows from the boundary conditions that the mass velocity components $V_r^{(e)}$ and $V_\theta^{(e)}$ should be sought for in the form of expansions of Legendre and Gegenbauer polynomials [13]. To determine the total force acting on the particle, it is sufficient to obtain the first terms of these expansions. Then, expressions for the mass velocity components of zero approximation (1.7) take the form

$$V_r^{(e)}(y,\theta) = \cos\theta\, G(y), \qquad V_\theta^{(e)}(y,\theta) = -\sin\theta\, g(y),$$

where $G(y)$ and $g(y)$ are the arbitrary functions dependent on the y coordinate.

2. TEMPERATURE FIELDS OUTSIDE AND INSIDE OF THE PARTICLE
AND THE RELATIVE CONCENTRATION FIELD OF THE FIRST COMPONENT
OF THE BINARY GAS MIXTURE

The diffusiophoretic and photophoretic forces and velocities can be determined one knows temperature fields outside and inside of the particle and the relative concentration distribution of the first component. For this, Eqs. (1.1) and (1.2) are solved. Their solutions are sought for using the perturbation theory [16]:

$$t_e(y,\theta) = t_{e0}(y) + \varepsilon t_{e1}(y,\theta), \qquad t_i(y,\theta) = t_{i0}(y) + \varepsilon t_{i1}(y,\theta). \tag{2.1}$$

Here

$$t_{e0}(y) = \left(1 + \frac{\Gamma_0}{y}\right)^{1/(1+\alpha)}, \quad t_{i0}(y) = \left(B_0 + \frac{H_0}{y} - \frac{1}{y}\int\limits_y^1 \psi_0\, dy + \int\limits_y^1 \frac{\psi_0}{y}\, dy\right)^{1/(1+\gamma)}, \quad V = \frac{4}{3}\pi R^3,$$

$$t_{e1}(y) = \frac{\cos\theta}{t_{e0}^\alpha}\frac{\Gamma_1}{y^2}, \qquad t_{i1}(y) = \frac{\cos\theta}{t_{i0}^\gamma}\left[B_1 y + \frac{H_1}{y^2} + \frac{1}{3}\left(y\int\limits_1^y \frac{\psi_1}{y^2}\, dy - \frac{1}{y^2}\int\limits_1^y \psi_1 y\, dy\right)\right],$$

$$H_0 = \frac{(1+\gamma)R^2}{3\lambda_{i0}T_{e\infty}} J_0, \quad H_1 = \frac{R}{3\lambda_{i0}T_{e\infty}} J_1, \quad J_0 = \frac{1}{V}\int\limits_V q_i\, dV, \quad J_1 = \frac{1}{V}\int\limits_V q_i z\, dV, \quad x = \cos\theta,$$

$$\psi_0 = -\frac{R^2(1+\gamma)}{2\lambda_{i0}T_{e\infty}} y^2 \int\limits_{-1}^{+1} q_i(r,\theta)\, dx, \quad \psi_1 = -\frac{3R^2}{2\lambda_{i0}T_{e\infty}} y^2 \int\limits_{-1}^{+1} q_i(r,\theta)x\, dx \quad \left(t_k = \frac{T}{T_\infty}, \; k = e,i\right),$$

$\int\limits_V q_i z\, dV$ is the dipole moment of density of heat sources [8, 10] ($z = r\cos\theta$ and $dV = r^2\sin\theta\, dr\, d\theta\, d\varphi$); the integration is carried out over the entire volume of the particle.

The solution of the diffusion equation (1.2) is sought in the form

$$C_1(y,\theta) = C_{10}(y) + \varepsilon C_{11}(y,\theta). \tag{2.2}$$

We substitute Eq. (2.2) into the diffusion equation (1.2) and obtain

$$\operatorname{div}\left(t_{e0}^\omega \nabla C_{10}\right) = 0, \qquad \operatorname{div}\left(t_{e0}^\omega \nabla C_{11} + \omega t_{e0}^{\omega-1} t_{e1}\nabla C_{10}\right) = 0. \tag{2.3}$$

Integrating the first equation (2.3) two times, we obtain the following expression satisfying the boundary condition (1.4):

$$C_{10}(y) = C_0 + M_0(t_{e0}^{1+\alpha-\omega} - 1). \tag{2.4}$$

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The solution of the second equation (2.3) is sought in the form

$$C_{11}(y, \theta) = \tau(y) \cos \theta. \tag{2.5}$$

Expressions (2.5), (2.4), and (2.1) are substituted into Eqs. (2.3), which results in

$$y^2 \frac{d^2\tau}{dy^2} + y\left(2 - \frac{\omega l}{1+\alpha}\right)\frac{d\tau}{dy} - 2\tau = -\frac{1+\alpha-\omega}{1+\alpha}\,\omega\,\frac{M_0}{t_{e0}^\omega}\frac{l(2-l)}{y^2}\Gamma_1. \tag{2.6}$$

The homogeneous equation (2.6) is sought in the form

$$y^2 \frac{d^2\tau}{dy^2} + y\left(2 - \frac{\omega l}{1+\alpha}\right)\frac{d\tau}{dy} - 2\tau = 0. \tag{2.7}$$

A point $y = 0$ for Eq. (2.7) is a regular singular point, so its solution is sought in the form of the generalized power series [17]

$$\tau(y) = y^\rho \sum_{n=0}^\infty \Delta_n l^n, \qquad l = \frac{\Gamma_0}{y + \Gamma_0}, \qquad \Delta_0 \neq 0. \tag{2.8}$$

Equation (2.8) is substituted into Eq. (2.7), which results in a governing equation $\rho^2 + \rho - 2 = 0$, whose roots are equal to $\rho_1 = 1$ and $\rho_2 = -2$.

The first solution of Eq. (2.7), which corresponds to a root larger in absolute magnitude, is determined as

$$\Phi_1(y) = \frac{1}{y^2} \sum_{n=0}^\infty \Delta_n^{(1)} l^n \qquad (\Delta_0^{(1)} = 1). \tag{2.9}$$

Equation (2.9) is substituted into Eq. (2.7) and the method of undetermined coefficients is used to obtain the following recursion formula for $\Delta_n^{(1)}$ $(n \geqslant 1)$:

$$\Delta_n^{(1)} = \frac{1}{n(n+1)}\left[(n+1)\left(2n - 2 - \frac{\omega}{1+\alpha}\right)\Delta_{n-1}^{(1)} - (n-2)\left(n-1-\frac{\omega}{1+\alpha}\right)\Delta_{n-2}^{(1)}\right].$$

The second solution of Eq. (2.7) is sought in the form

$$\Phi_2(y) = y \sum_{n=0}^\infty \Delta_n^{(2)} l^n + \frac{a_2}{y^2}\ln(y) \sum_{n=0}^\infty \Delta_n^{(1)} l^n \qquad (\Delta_0^{(2)} = 1).$$

A recursion formula for $\Delta_n^{(2)}$ $(n \geqslant 4)$ is obtained similarly:

$$\Delta_n^{(2)} = \frac{1}{n(n-3)}\left\{(n-2)\left(2n-2-\frac{\omega}{1+\alpha}\right)\Delta_{n-1}^{(2)} - (n-2)\left(n-1-\frac{\omega}{1+\alpha}\right)\Delta_{n-2}^{(2)}\right.$$

$$\left. + \frac{a_2}{2\Gamma_0^3}\sum_{k=0}^{n-3}(n-k-2)(n-k-1)\left[(2k+3)\Delta_k^{(1)} - \left(2k-2-\frac{\omega}{1+\alpha}\right)\Delta_{k-1}^{(1)}\right]\right\},$$

$$\Delta_0^{(1)} = \Delta_0^{(2)} = \Delta_3^{(2)} = 1, \quad \Delta_1^{(2)} = -\frac{\omega}{2(1+\alpha)}, \quad \Delta_2^{(2)} = 1, \quad \frac{a_2}{2\Gamma_0^3} = \frac{\Delta_1^{(2)}}{6}\left(2 - \frac{\omega}{1+\alpha}\right),$$

$$\Delta_n^{(1)} = 0, \quad \Delta_n^{(2)} = 0$$

at $n < 0$.

The partial solution of the heterogeneous equation is written as

$$\Phi_3(y) = -\frac{1+\alpha-\omega}{1+\alpha}\,\omega\,\frac{l(2-l)}{y^2 t_{e0}^\omega}\,M_0\Gamma_1.$$

Thus,

$$C_1(y, \theta) = C_{10}(y) + \varepsilon C_{11}(y, \theta),$$

where $C_{10}(y) = C_0 + M_0(t_{e0}^{1+\alpha-\omega} - 1)$ and $C_1(y, \theta) = \cos\theta(D_1\Phi_2(y) + M_1\Phi_1(y) + \Phi_3(y))$.

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The integration constants included in the expressions for the temperature fields and the concentrations of the first component are determined from the boundary conditions on the particle surface:

$$\Gamma_1 = \frac{Rt_{eS}^\alpha}{\lambda_{iS}T_\infty\delta}\, J_1, \qquad \Gamma_0 = t_{eS}^{1+\alpha} - 1, \qquad M_1 = -\frac{\Phi_2^{\mathrm{I}}(1)}{\Phi_1^{\mathrm{I}}(1)}.$$

Here $\delta = 1 + 2\lambda_{eS}/\lambda_{iS} + (4\sigma_0\sigma_1 R/\lambda_{iS})T_{e\infty}^3 t_{eS}^3$, $\lambda_{iS} = \lambda_{i0}t_{iS}^\gamma$, $\lambda_{eS} = \lambda_{e\infty}t_{eS}^\alpha$, and $\Phi_1^{\mathrm{I}}(1)$, $\Phi_2^{\mathrm{I}}(1)$ denotes the first derivatives from the corresponding functions, taken at $y = 1$, $t_{iS} = t_{i0}$ $(y = 1)$, and $t_{eS} = t_{e0}$ $(y = 1)$.

The average particle surface temperature $T_{iS} = t_{iS}T_\infty$ is calculated by solving the system of transcendental equations

$$t_{iS} = t_{eS}, \quad \frac{l^{(S)}}{1+\alpha}\, t_{eS} = \frac{R^2}{3\lambda_{eS}T_\infty}\, J_0 - \sigma_0\sigma_1 \frac{RT_\infty^3}{\lambda_{eS}}\,(t_{eS}^4 - 1), \quad l^{(S)} = \frac{t_{eS}^{1+\alpha} - 1}{t_{eS}^{1+\alpha}}. \qquad (2.10)$$

The degree of heterogeneity of radiation energy distribution in a particle depends on optical constants of the particle material m_S and diffraction x_a. Then an expression for radiation density transformed into heat can be written as $q_i = 4\pi(n_S a_S/(n_0\lambda_0))I_0 B_S$, where $m_S = n_S + ia_S$, $x_a = 2\pi R/\lambda_0$, n_S is the refractive index, a_S is the absorption coefficient, n_0 is the refractive index of the medium, I_0 and λ_0 denote the radiation intensity and wavelength, and B_S is the coordinate function calculated using the Mie theory [11].

With fulfillment of the inequality $\lambda_e \ll \lambda_i$, which is valid for most gaseous media, the thermal conductivity of the particle is much larger than that of the gas, which means that, in the dynamic viscosity equation, the dependence on angle θ in the particle–gas system can be ignored (assuming that there is weak angular symmetry in the temperature distribution). In view of this, it can be considered that viscosity depends only on temperature $t_{e0}(r)$, i.e., $\mu_e(t_e(r,\theta)) \approx \mu_e(t_{e0}(r))$. This assumption allows one to consider the hydrodynamic part separately from the thermal part (they are related via boundary conditions).

3. SOLUTION OF THE HYDRODYNAMIC PROBLEM.
ANALYSIS OF THE RESULTS OBTAINED

The results of studying the velocity-linearized Navier–Stokes equation in a spherical coordinate system show that, if the thermal conductivity of the particle is much higher than that of the gas (weak angular asymmetry in the temperature distribution), this equation can be reduced to a third-order inhomogeneous differential equation with an isolated singular point whose solution can be sought in the form of generalized power series (see [6]).

Thus, the general expressions for the mass velocity components, which satisfy the boundedness condition of the solution as $y \to \infty$, have the form

$$U_r^{(e)} = U_\infty \cos\theta G(y), \qquad G(y) = A_1 G_1 + A_2 G_2 + G_3,$$

$$U_\theta^{(e)} = -U_\infty \sin\theta g(y), \qquad g(y) = A_1 G_4 + A_2 G_5 + G_6,$$

$$P_e = P_\infty + \frac{\mu_\infty U_\infty}{R}\, t_{e0}^\beta \left\{ \frac{y^2}{2}\frac{d^3 G}{dy^3} + y\left[3 + \frac{\beta-1}{2}\, yf\right]\frac{d^2 G}{dy^2} \right.$$

$$\left. - \left[2 - y^2 f^{\mathrm{I}} - \frac{\beta}{2}\, y^2 f^2 + (\beta-2)yf\right]\frac{dG}{dy} + 2\left[y^2 f^{\mathrm{II}} + yf^{\mathrm{I}}(4 + y\beta f) - \frac{2}{3}\, f\right]G \right\},$$

where

$$f = -\frac{l}{y(1+\alpha)}, \qquad G_k = \left(1 + \frac{l}{2(1+\alpha)}\right)G_{k-3} + \frac{1}{2}\, yG_{k-3}^{\mathrm{I}} \quad (k = 4, 5, 6),$$

f^{I}, f^{II}, G_1^{I}, G_2^{I}, and G_3^{I} denote the first and second derivatives with respect to y from the corresponding functions,

$$G_1(y) = \frac{1}{y^3}\sum_{n=0}^\infty C_n^{(1)} l^n, \qquad G_2(y) = \frac{1}{y}\sum_{n=0}^\infty C_n^{(2)} l^n + \omega_2 \ln(y)G_1(y),$$

$$G_3(y) = \sum_{n=0}^\infty C_n^{(3)} l^n + \omega_3 \ln(y)G_1(y).$$

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The values of $C_n^{(1)}$ $(n \geqslant 1)$, $C_n^{(2)}$ $(n \geqslant 3)$, and $C_n^{(3)}$ $(n \geqslant 4)$ are determined using the recursion formulas

$$C_n^{(1)} = \frac{1}{n(n+3)(n+5)}\{[(n-1)(3n^2+13n+8)+\gamma_1(n+2)(n+3)+\gamma_2(n+2)]C_{n-1}^{(1)}$$

$$-[(n-1)(n-2)(3n+5)+2\gamma_1(n^2-4)+\gamma_2(n-2)+\gamma_3(n+3)]C_{n-2}^{(1)}$$

$$+(n-2)[(n-c1)(n-3)+\gamma_1(n-3)+\gamma_3]C_{n-3}^{(1)}\},$$

$$\Delta_k = (3k^2+16k+15)C_k^{(1)} - [(k-1)(6k+13)+\gamma_1(2k+5)+\gamma_2]C_{kk-1}^{(1)}$$

$$+[3(k-1)(k-2)+2\gamma_1(k-2)+\gamma_3]C_{k-2}^{(1)},$$

$$C_n^{(2)} = \frac{1}{(n+1)(n+3)(n-2)}\Big\{[(n-1)(3n^2+n-6)+\gamma_1 n(n+1)+n\gamma_2]C_{n-1}^{(2)}$$

$$-[\gamma_3(n+1)+(n-1)(n-2)(3n-1)+2\gamma_1 n(n-2)+\gamma_2(n-2)]C_{n-2}^{(2)}$$

$$+(n-2)[(n-1)(n-3)+\gamma_3+\gamma_1(n-3)]C_{n-3}^{(2)}$$

$$+\frac{\omega_2}{\Gamma_0^2}\sum_{k=0}^{n-2}(n-k-1)\Delta_k - 6\frac{(-\gamma_4)(1-\gamma_4)\cdots(n-1-\gamma_4)}{n!}\Big\},$$

$$C_n^{(3)} = \frac{1}{n(n+2)(n-3)}\Big\{(n-1)[3n^2-5n-4+\gamma_1 n+\gamma_2]C_{n-1}^{(3)}$$

$$-[(n-1)(n-2)(3n-4)+2\gamma_1(n-1)(n-2)+\gamma_2(n-2)+n\gamma_3]C_{n-2}^{(3)}$$

$$+(n-2)[(n-1)(n-3)+\gamma_1(n-3)+\gamma_3]C_{n-3}^{(3)}$$

$$+\frac{\omega_3}{2\Gamma_0^3}\sum_{k=0}^{n-3}(n-k-2)(n-k-1)\Delta_k\Big\}.$$

At the same time,

$$C_0^{(1)} = 1, \quad C_0^{(2)} = 1, \quad C_0^{(3)} = 1, \quad C_1^{(3)} = 0, \quad C_2^{(2)} = 1, \quad C_2^{(1)} = -\frac{1}{8}(2\gamma_1+\gamma_2+6\gamma_4),$$

$$\frac{\omega_3}{2\Gamma_0^3} = -\frac{\gamma_3}{60}(10+3\gamma_1+\gamma_2), \quad \gamma_4 = \frac{\beta}{1+\alpha}, \quad \gamma_1 = \frac{1-\beta}{1+\alpha},$$

$$\frac{\omega_2}{\Gamma_0^2} = \frac{1}{15}\Big[\frac{1}{4}(2\gamma_1+\gamma_2+6\gamma_4)(4+3\gamma_1+\gamma_2)+3\gamma_3+3\gamma_4(\gamma_4-1)\Big],$$

$$C_3^{(2)} = \frac{\gamma_3}{4}, \quad C_3^{(3)} = 1, \quad \gamma_2 = 2\frac{1+\beta}{1+\alpha}, \quad \gamma_3 = \frac{2+2\alpha-\beta}{(1+\alpha)^2};$$

at $n < 0$, we have $C_n^{(1)} = 0$, $C_n^{(2)} = 0$, and $C_n^{(3)} = 0$.

Integration constants A_1 and A_2 are determined from the boundary conditions on the surface of the aerosol particle.

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Thus, in a first approximation with respect to ε, we obtained expressions for the temperature fields outside and inside of the aerosol particle, the relative concentration distributions of the first component of the binary gas mixture, and the velocity and pressure distributions in its vicinity.

The resulting force acting on the particle is determined by integrating the stress tensor on the aerosol particle surface [13]:

$$F_z = \int\limits_{(S)} (-P_e \cos\theta + \sigma_{rr} \cos\theta - \sigma_{r\theta} \sin\theta) r^2 \sin\theta \, d\theta \, d\varphi \Big|_{r=R} \tag{3.1}$$

(σ_{rr} and $\sigma_{r\theta}$ are the stress tensor components).

The expressions obtained above are substituted into Eq. (3.1) and integrated, resulting in the expression for the total force

$$\boldsymbol{F} = \boldsymbol{F}_\mu + \boldsymbol{F}_{ph} + \boldsymbol{F}_{dh},$$

where $\boldsymbol{F}_\mu = 6\pi R \mu_\infty U_\infty f_m \boldsymbol{n}_z$ is the force of viscous resistance of the medium, $\boldsymbol{F}_{ph} = -6\pi R \mu_\infty f_{ph} J_1 \boldsymbol{n}_z$ is the photophoretic force, $\boldsymbol{F}_{dh} = -6\pi R \mu_\infty f_{dh} |\nabla C_{1\infty}| \boldsymbol{n}_z$ is the diffusionphoretic force, and $\boldsymbol{n}_z$ is the unit vector in the direction of the Oz axis.

The values of f_m, f_{ph}, and f_{dh} can be estimated using the expressions

$$f_m = \frac{2}{3}\frac{N_2}{N_1}, \qquad f_{ph} = K_{TS}^{(0)} \frac{4\nu_{eS}}{3\delta T_\infty t_{eS}} \frac{G_1(1)}{N_1 \lambda_{iS}},$$

$$f_{dh} = K_{DS}^{(0)} \frac{4D_{12}^{(S)} G_1(1)}{3N_1} \frac{\Phi_2(1)\Phi_1^{\mathrm{I}}(1) - \Phi_1(1)\Phi_2^{\mathrm{I}}(1)}{\Phi_1^{\mathrm{I}}(1)},$$

where $N_1(1) = G_1(1)G_2^{\mathrm{I}}(1) - G_2(1)G_1^{\mathrm{I}}(1)$, $N_2(1) = G_1(1)G_3^{\mathrm{I}}(1) - G_3(1)G_1^{\mathrm{I}}(1)$, and $\nu_{eS} = \nu_{e\infty} t_{eS}^{1+\beta}$.

The total force $\boldsymbol{F}$ is equated to zero, so the following expression is obtained for the diffusiophoretic and photophoretic velocity $\boldsymbol{U}_{dph}$ of the solid large heated spherical particle in the binary gas mixture:

$$\boldsymbol{U}_{dph} = -(h_{ph} J_1 + h_{dh} |\nabla C_{1\infty}|) \boldsymbol{n}_z, \qquad h_{ph} = f_{ph}/f_m, \quad h_{dh} = f_{dh}/f_m.$$

Functions $G_1(y)$, $G_1^{\mathrm{I}}(y)$, $G_1^{\mathrm{II}}(y)$, $G_2(y)$, $G_2^{\mathrm{I}}(y)$, $G_2^{\mathrm{II}}(y)$, $G_3(y)$, $G_3^{\mathrm{I}}(y)$, $G_3^{\mathrm{II}}(y)$, $\Phi_1(y)$, $\Phi_2(y)$, $\Phi_3(y)$, $N_1(y)$, and $N_2(y)$ are obtained at $y = 1$, $G_1^{\mathrm{I}}(y)$, $G_1^{\mathrm{II}}(y)$, etc.; the first and second derivative are obtained from the corresponding functions.

The resulting expressions for the force and velocity of diffusiophoresis and photophoresis can also be used for small relative temperature differences in the vicinity of the particle. In the case where the amount of heating of the particle surface is small, i.e., the average surface temperature slightly differs from the ambient temperature far from the particle ($\Gamma_0 \to 0$), the dependence of the molecular transfer coefficients (viscosity, thermal conductivity, and diffusion) and the density on temperature can be neglected. Then, at $y = 1$, we have $G_1 = 1$, $G_1^{\mathrm{I}} = -3$, $G_1^{\mathrm{II}} = 12$, $G_1^{\mathrm{III}} = -60$, $G_2 = 1$, $G_2^{\mathrm{I}} = -1$, $G_2^{\mathrm{II}} = 2$, $G_2^{\mathrm{III}} = -6$, $G_3 = 1$, $G_3^{\mathrm{I}} = 0$, $G_3^{\mathrm{II}} = 0$, $G_3^{\mathrm{III}} = -0$, $\Phi_1 = 1$, $\Phi_2 = 1$, $\Phi_3 = 0$, $N_1 = 2$, and $N_2 = 3$. In this case, the expressions for the force and velocity of diffusiophoresis and photophoresis match with the known results (see, e.g., [1–4]).

The figure shows the dependence of $f_m^* = f_m/f_m|_{T_{iS}\,=\,273\,\mathrm{K}}$, $f_{ph}^* = f_{ph}/f_{ph}|_{T_{iS}\,=\,273\,\mathrm{K}}$, and $f_{dh}^* = f_{dh}/f_{dh}|_{T_{iS}\,=\,273\,\mathrm{K}}$, which are included into expressions for the resistance force, the photophoretic force, and the diffiophoretic force, on the average temperature T_{iS} for copper particles of a radius $R = 25\cdot10^{-6}$ m, which move in a air–CO_2 binary gas mixture under normal conditions. The numerical estimates obtained using the determined formulas show that f_m^*, f_{ph}^*, and f_{dh}^* strongly depend on the average particle surface temperature.
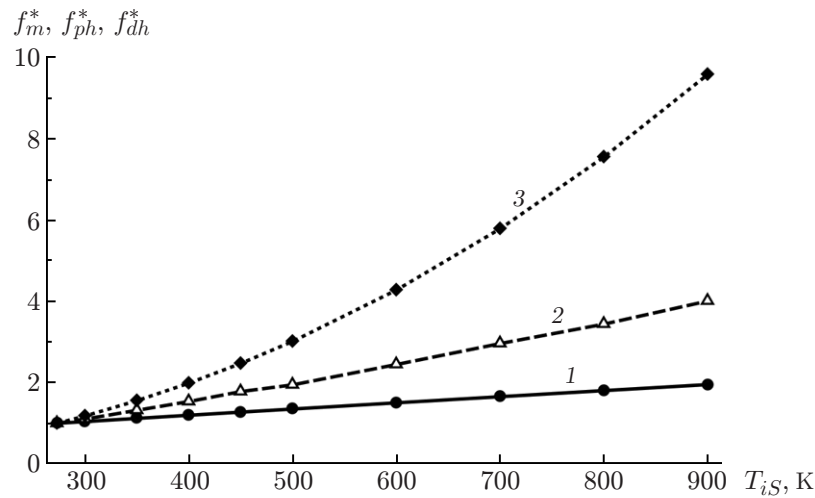
Coefficients f_m^* (*1*), f_{ph}^* (*2*), and f_{dh}^* (*3*) versus the average particle surface temperature T_{iS}.

CONCLUSIONS

In this study, the expressions are obtained that allow one to estimate the force and velocity of diffusiophoresis and photophoresis of large heated nonvolatile aerosol spherical particles when the average temperature of their surface differs significantly from the temperature of the gaseous medium far from them in the case of a power-law dependence of the molecular transfer coefficients (viscosity, thermal conductivity, and diffusion) and the density of the gaseous medium on temperature. Under boundary conditions, thermal and diffusion slip are taken into account. The estimates show that the diffusion and photophoretic forces and velocity substantially depend on the average particle surface temperature. According to the resulting expressions for diffusiophoresis and photophoresis, the behavior of heated large solid spherical particles in thermodynamically nonequilibrium aerodisperse systems, in which a small constant concentration gradient of the binary gas mixture components is maintained using external sources, should be described only with account for the combined effect of these effects, which is due to the dependence of molecular transfer coefficients (thermal conductivity and diffusion) on the average particle surface.

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