Contribution of the Second-Order Born Approximation to the Coherent Bremsstrahlung Cross Section by Relativistic Electrons and Positrons in Crystals

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Abstract—Equations for the cross section and polarization of the coherent bremsstrahlung emitted by relativistic electrons and positrons in crystals are obtained taking into account the contribution from the second-order Born approximation. The radiation cross section and polarization in the field of the atomic plane is considered as a function of the charge sign of the particle.

INTRODUCTION

When relativistic electrons move in a crystal at a small angle to a certain crystal axis or plane, coherent effects appear in bremsstrahlung [1–3]. As a result of these effects, the radiation cross section of electrons in the crystal can substantially exceed the corresponding cross section in an amorphous medium. The frequency range in which these effects take place grows rapidly with the electron energy, and, at sufficiently high energy, it becomes necessary to take into account the recoil effect of radiation. There are several methods for describing the process of radiation of relativistic electrons taking into account the recoil effect. These methods are based on Born approximation and various versions of the quasi-classical approximation of quantum electrodynamics [3–8]. The radiation cross section as a function of the sign of the particle charge is of special interest. This dependence appears when the contribution of higher orders of perturbation theory with respect to the interaction of the particle with the external field is taken into account. Therefore, it is very important to analyze this process in order to determine the domain of applicability of various approximate methods used for description of radiation of fast particles in an external field.

The dependence of the radiation cross section on the sign of the particle charge in the case of radiation of high-energy electrons and positrons on an isolated atom is rather slight [9]. However, the situation is different in the case of coherent interaction of relativistic particles with the atoms of a crystal lattice. As a result of the coherent effects, the dependence of the radiation cross section on the sign of the particle charge can become substantially stronger as compared to the same dependence in an amorphous medium. The authors of [10] first paid attention to this fact. They considered the contribution of the second-order Born approximation to the cross section of the coherent bremsstrahlung of relativistic electrons in the field of an atomic plane of a crystal.

The results in [10] are obtained for low frequencies of emitted photons, for which the recoil effect of radiation can be neglected. This paper gives some results of the investigation of contribution of the second-order Born approximation to the coherent radiation of relativistic electrons and positrons in a crystal taking into account the recoil effect of radiation. The equations are derived describing the cross section and polarization of the radiation of particles in an inhomogeneous stationary electric field of an arbitrary structure. The dependence of the cross section and polarization of the coherent radiation in the crystal on the particle charge sign is considered on the basis of these equations. In this paper we use the system of units in which the velocity of light c and Planck's constant $\hbar$ are taken equal to unity.

TECHNIQUE

The cross section of the bremsstrahlung in the external field is described by the equation [9]

$$d\sigma = \frac{e^2}{4(2\pi)^4\omega\varepsilon\varepsilon'}\delta(\varepsilon-\varepsilon'-\omega)\overline{|M|^2}d^3p'd^3k, \qquad (1)$$

where e is the electron charge, (ε, $\mathbf{p}$) and (ε', $\mathbf{p}'$) are the energy and momentum of the initial and final particles, ω and $\mathbf{k}$ are the frequency and wave vector of the emitted photon, $\delta(\varepsilon-\varepsilon'-\omega)$ is delta-function expressing the energy conservation law in radiation, and M is the

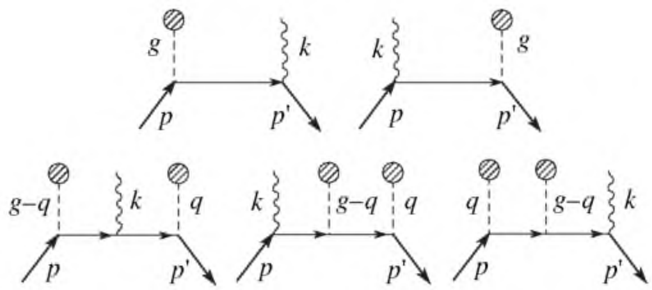

Fig. 1. Feynman diagrams corresponding to the first and second-order Born approximations in the description of bremsstrahlung in the external field.

matrix element of the radiation process. The bar above $|M|^2$ means averaging over the polarization of the initial particles and summation over the polarization of the final particles. According to the Feynman diagram method [9], the squared absolute value of the matrix element in (1) taking into account the contribution of the second-order Born approximation can be written as follows:

$$|M|^2 = |M_1|^2|U_g|^2 + 2U_g \mathrm{Re}\int M_1 M_2^* U_q U_{g-q}^* \frac{d^3q}{(2\pi)^3}, \quad (2)$$

where U_g is the Fourier component of the potential energy of the particle in the external field; $g_\mu = (0, \mathbf{g}) = p_\mu - p'_\mu - k_\mu$ is the 4-momentum imparted to the external field (which is considered as stationary and conservative); p_μ, p'_μ, k_μ are the 4-momenta of the initial and final electron and photon; and M_1 and M_2 are the matrix elements giving the contribution of the first-order and second-order Born approximation (Fig. 1). By explicitly extracting the dependence on the longitudinal and transverse components of the imparted momenta $\mathbf{g}$ and $\mathbf{q}$ in the propagators in M_1 and M_2, one can write:

$$M_1 = \bar{u}'\left[b\hat{e} - \frac{\hat{e}\hat{g}\gamma_0}{2\varepsilon\sigma_g} - \frac{\gamma_0\hat{g}\hat{e}}{2\varepsilon'\tau_g}\right]u, \quad (3)$$

$$M_2 = \bar{u}'\left\{\hat{e}\frac{\hat{p}+m-\hat{g}}{2p\sigma_g}\gamma_0\frac{1-\frac{\hat{q}\gamma_0}{2\varepsilon}}{v\sigma_q} - \frac{1+\frac{\gamma_0\hat{q}'}{2\varepsilon'}}{v'\tau_{q'}}\hat{e}\frac{1-\frac{\hat{q}\gamma_0}{2\varepsilon}}{v\sigma_q} + \frac{1+\frac{\gamma_0\hat{q}'}{2\varepsilon'}}{v'\tau_{q'}}\gamma_0\frac{\hat{p}'+m+\hat{g}}{2p'\tau_g}\hat{e}\right\}u, \quad (4)$$

where e_μ is the polarization vector of the photon, $\hat{p} = p_\mu\gamma^\mu$, γ_μ are Dirac matrices, v and v' are the velocities of the initial and final electrons, and $q'_\mu = g_\mu - q_\mu$. The values b, σ_g, and τ_g in M_1 and M_2 are given by the equations

$$b = g_\parallel\left(\frac{1}{\sigma_g} - \frac{1}{\tau_g}\right), \quad \sigma_g = g_\parallel - \frac{\mathbf{g}^2}{2p},$$
$$\tau_g = g_\parallel + \mathbf{n}_\perp\mathbf{g} + \frac{\mathbf{g}^2}{2p'}, \quad (5)$$

where $\mathbf{n} = \mathbf{p}'/|\mathbf{p}'|$ is the unit vector along the momentum $\mathbf{p}'$, and $\mathbf{n}_\perp$ is the component of this vector perpendicular to $\mathbf{p}$.

The matrix element of the radiation process explicitly depends on the momentum $\mathbf{g}$ imparted to the external field. In this case, the cross section itself can also be expressed in terms of imparted momentum (and angle ϑ between vectors $\mathbf{k}$ and $\mathbf{p}$). This representation is especially convenient for small values of the imparted momentum $g_\perp \ll m$, because it makes it possible to expand the matrix element in power series of $g_\perp$. The transformation to the new variables is described in [1, 3]. In this case, the differential cross section is the following:

$$d\sigma = \frac{e^4}{(2\pi)^4}\frac{\varepsilon'}{\varepsilon}\overline{|M|^2}\frac{\delta}{m}\frac{d\omega}{\omega}\frac{dy}{\sqrt{1-y^2}}d^3g, \quad (6)$$

where $\delta = \omega m^2/2\varepsilon\varepsilon'$. Variable y is related with ϑ as follows:

$$(\varepsilon\vartheta/m)^2 = f + y\sqrt{a}, \quad -1 \le y \le 1, \quad (7)$$

where

$$a = \frac{4g_\perp^2}{m^2\delta}\left(g_\parallel - \delta - \frac{g_\perp^2}{2\varepsilon}\right), \quad f = \frac{1}{\delta}\left(g_\parallel - \delta - \frac{g_\perp^2}{2\varepsilon} + \frac{g_\perp^2\delta}{m^2}\right),$$

$g_\parallel$ and $\mathbf{g}_\perp$ are the components of $\mathbf{g}$ parallel and perpendicular to the momentum $\mathbf{p}$ of the incident particle. Equation (7) gives the possible values of the radiation angle ϑ for given $g_\parallel$ and $g_\perp$. Since it is required that a in the radicand in (7) is positive, one can conclude that

$$g_\parallel \ge \delta + g_\perp^2/2\varepsilon. \quad (8)$$

By neglecting the terms of the order of m^2/ε^2 and m^2/ε'^2, one can bring the expression for M_2 to form [11]

$$M_2 = \bar{u}'\left\{\left[Q_1 - \frac{\omega}{\varepsilon'\tau_g}\left(\hat{e} + \frac{\gamma_0\hat{g}\hat{e}}{2\varepsilon'}\right)\right]\frac{\mathbf{q}_\perp\mathbf{q}'_\perp}{2\varepsilon\sigma_q\sigma_{q'}} + Q_2\right\}u, \quad (9)$$

where Q_1 is the spinor structure from M_1,

$$Q_1 = \hat{e}b - \frac{\hat{e}\hat{g}\gamma_0}{2\varepsilon\sigma_g} - \frac{\gamma_0\hat{g}\hat{e}}{2\varepsilon'\tau_g}$$

and

$$Q_2 = -\frac{\hat{e}\hat{g}\hat{q}_\perp}{4\varepsilon^2\sigma_g\sigma_q} - \frac{\hat{q}'_\perp\hat{g}\hat{e}}{4\varepsilon'^2\tau_g\tau_{q'}} + \frac{\gamma_0\hat{q}'_\perp\hat{e}\hat{q}_\perp\gamma_0}{4\varepsilon\varepsilon'\tau_{q'}\sigma_q}.$$

After summation over polarizations of the final particles and averaging over polarizations of the initial particle, we obtain the following expressions for $\overline{|M_1|^2}$ and $\overline{M_1 M_2^*}$ in the order of m^2/ε^2:

$$\overline{|M_1|^2} = \frac{2}{g_\parallel^2}\left[\left(\frac{\varepsilon}{\varepsilon'}+\frac{\varepsilon'}{\varepsilon}\right)g_\perp^2 - 2m^2b^2\right], \tag{10}$$

$$\overline{M_1 M_2^*} = \frac{\mathbf{q}_\perp \mathbf{q}_\perp}{2\varepsilon\sigma_q\sigma_{q'}}\left\{\overline{|M_1|^2} - \frac{2\omega}{\varepsilon'\tau_g}[2(p'\cdot p - 2m^2)b - \frac{\varepsilon}{\varepsilon'}\frac{\mathbf{g}^2}{\tau_g} + \mathbf{p}'_\perp\mathbf{g}\left(\frac{\varepsilon+\varepsilon'}{\varepsilon'\sigma_g} - \frac{2\varepsilon}{\varepsilon'\tau_g}\right)\right] - 8\frac{\varepsilon}{\varepsilon'}b\right\}, \tag{11}$$

where $p'\cdot p = \varepsilon'\varepsilon - \mathbf{p}'\mathbf{p}$.

Substituting these expressions into (1), after integrating over y and expanding into a series of $g_\perp/m$, we obtain the following equation for the radiation cross section with allowance for the contribution of the second-order Born approximation:

$$d\sigma = \frac{e^2}{4\pi^3}\frac{\varepsilon'}{\varepsilon}\frac{\delta}{m^2}\frac{d\omega}{\omega}\frac{g_\perp^2}{g_\parallel^2}d^3g\left\{F|U_g|^2 + \frac{1}{(2\pi)^3\varepsilon}\left[F + \frac{\omega}{\varepsilon'}\left(1 - 4\frac{\delta}{g_\parallel}\left(1-\frac{\delta}{g_\parallel}\right) + \frac{\omega^2}{2\varepsilon\varepsilon'}\left(1-\frac{\delta}{g_\parallel}\right)\right)\right]I_g\right\}. \tag{12}$$

Here,

$$I_g = U_g\mathrm{Re}\int d^3q\frac{(\mathbf{g}_\perp - \mathbf{q}_\perp)\mathbf{q}_\perp}{(g_\parallel - q_\parallel + i0)(q_\parallel + i0)}U_q U^*_{g-q}, \quad F = 1 + \frac{\omega^2}{2\varepsilon\varepsilon'} - 2\frac{\delta}{g_\parallel}\left(1-\frac{\delta}{g_\parallel}\right). \tag{13}$$

Let us consider some special cases of (12). If $\delta \ll q_{\parallel\mathrm{eff}}$, where $q_{\parallel\mathrm{eff}}$ is the characteristic value of the longitudinal component of momentum $\mathbf{q}$ in (12), we can neglect the fact that U_g and U_{g-q} depend on $g_\parallel$ in (12). After integrating over $g_\parallel$, we see that, in the order of $g_{\parallel\mathrm{eff}}/q_{\parallel\mathrm{eff}}$,

$$d\sigma(g_\perp) = dw(g_\perp)\frac{d^2g_\perp}{(2\pi)^2}\left\{|U_g|^2 - \frac{\varepsilon+\varepsilon'}{2\varepsilon\varepsilon'}U_{g_\perp}\mathrm{Re}\int\frac{d^3q}{(2\pi)^3}\frac{\mathbf{q}'_\perp\mathbf{q}_\perp}{(q_\parallel - i0)^2}U_q U^*_{q'}\right\}, \tag{14}$$

where

$$dw(g_\perp) = \frac{2e^2}{3\pi}\frac{\varepsilon'}{\varepsilon}\left(1 + \frac{3}{4}\frac{\omega^2}{\varepsilon\varepsilon'}\right)\frac{g_\perp^2}{m^2}\frac{d\omega}{\omega}. \tag{15}$$

For $\omega \ll \varepsilon$, Eq. (14) corresponds to the product of the radiation probability $dw/d\omega$ and the elastic scattering cross section of the particle in the external field $d\sigma_{el}$ with allowance for the contribution of the second-order Born approximation

$$d\sigma_{el}(g_\perp) = \frac{d^2g_\perp}{4\pi^2}\left\{|U_g|^2 - \frac{1}{\varepsilon}U_g\mathrm{Re}\int\frac{d^3q}{(2\pi)^3}\frac{\mathbf{q}_\perp\mathbf{q}'_\perp}{(q_\parallel - i0)^2}U_q U^*_{q'}\right\}.$$

In the case of the Coulomb field of a nucleus with a charge $Z|e|$, the latter equation becomes as follows:

$$d\sigma_{el}(g_\perp) = \frac{\pi Z^2 e^4 d\Omega}{\varepsilon^2\vartheta^4}\left\{1 - \frac{e}{|e|}\frac{\pi Z e^2}{2}\vartheta\right\},$$

where the scattering angle $\vartheta \approx g_\perp/p$. This result coincides with the corresponding result [14] obtained by a different method. The equation for $d\sigma_{el}$ in an arbitrary external field is obtained in [15].

Therefore, in the frequency range $\omega \sim \varepsilon$, the factorization theorem for the radiation cross section stating that

$$d\sigma \approx dw(g_\perp)d\sigma_{el}(g_\perp) \tag{16}$$

is valid only to the error governed by the contribution of the second-order Born approximation.

COHERENT RADIATION OF RELATIVISTIC ELECTRONS AND POSITRONS IN A CRYSTAL

Now, let us consider the coherent radiation of electrons and positrons in the continuous potential field of an atomic plane in a crystal when the beam is incident on this plane at a small angle θ. The interaction energy of the particle with a continuous potential of a plane is given by the relationship [12, 13]

$$U(x) = \frac{1}{L_yL_z}\int dydz\sum_{n=1}^{N}u(\mathbf{r}-\mathbf{r}_n). \tag{17}$$

Here, $u(\mathbf{r}-\mathbf{r}_n)$ is the potential energy of interaction with an individual atom of a plane located in the point $\mathbf{r}_n$, L_y and L_z are the dimensions of the atomic plane, and x is the coordinate along the normal to the atomic plane of the crystal (summation in (17) is performed over all N atoms constituting the crystal plane). Choosing the potential of an individual atom as a screened Coulomb potential so that

$$u(r) = \frac{Z|e|e}{r}e^{-r/R},$$

where R is the Thomas–Fermi radius, we find that the Fourier component of the potential energy (17) is

$$U_g = (2\pi)^2\delta(g_z)\delta(g_y)\frac{1}{a_ya_z}u_g, \tag{18}$$

where a_y and a_z are the distances along the y and z axes between the atoms constituting the plane and

$$u_g = \frac{4\pi Z|e|e}{\mathbf{g}^2 + R^{-2}}.$$

Substituting the Fourier component (18) into (12), we obtain the following expression for the radiation cross section:

$$\begin{aligned} d\sigma = {} & Z^2 e^6 16\pi \frac{N}{a_y a_z} \frac{\varepsilon'}{\varepsilon} \frac{\delta}{m^2} \frac{d\omega}{\omega} \frac{dg_x}{\theta^2} \frac{1}{(g_x^2 + R^{-2})^2} \\ & \times \left\{ \left[1 + \frac{\omega^2}{2\varepsilon\varepsilon'} - 2\frac{\delta}{g_x\theta}\left(1 - \frac{\delta}{g_x\theta}\right) \right] \right. \\ & + \frac{e}{|e|} \frac{4\pi Z e^2 R}{\varepsilon a_y a_z \theta^2} \frac{g_x^2 + R^{-2}}{g_x^2 + 4R^{-2}} \\ & \times \left[1 + \frac{\omega^2}{2\varepsilon\varepsilon'} - 2\frac{\delta}{g_x\theta}\left(1 - \frac{\delta}{g_x\theta}\right) \right. \\ & \left.\left. + \frac{\omega}{\varepsilon'}\left(1 - 4\frac{\delta}{g_x\theta}\left(1 - \frac{\delta}{g_x\theta}\right) + \frac{\omega^2}{2\varepsilon\varepsilon'}\left(1 - \frac{\delta}{g_x\theta}\right)\right)\right] \right\}. \end{aligned} \tag{19}$$

Here, we used the fact that in this case $g_{\parallel} \approx \theta g_x$. Therefore, g_x here assumes the values $g_x \geq \delta/\theta$. At $\omega \ll \varepsilon$, Eq. (19) is transformed to the corresponding result obtained in [10].

We can see that the relative contribution of the second-order Born approximation to the cross section of the coherent bremsstrahlung in the field of an atomic plane (together with the dependence of the cross section on the sign of the particle charge) is governed by the parameter

$$\alpha_p = \frac{Ze^2 R}{\varepsilon a^2 \theta^2}, \tag{20}$$

where a is the average distance between the atoms in the crystal plane. It is obvious that the Born expansion of the radiation cross section is valid at $\alpha_p \ll 1$. The parameter α_p is of the same order as the ratio of the square critical angle of planar channeling $\theta_c^2 = Ze^2R/\varepsilon a^2$ [11] to the square angle of the beam incidence θ onto the plane:

$$\alpha_p \sim \theta_c^2/\theta^2. \tag{21}$$

α_p rapidly grows as θ decreases. Therefore, Eq. (19) is valid at

$$\theta_c \ll \theta \ll 1. \tag{22}$$

At $\alpha_p \sim 1$, one should take into account channeling effects and over-barrier movement of the particles relative to the atomic plane [3, 12, 13] and the Born approximation becomes inapplicable.

Equation (19) also shows that, for all frequencies, the radiation cross section of positrons is greater than the radiation cross section of electrons. This result can be intuitively explained. An electron is attracted to the atomic plane and, unlike a positron, spends less time in the domain with a large potential gradient. As a result, the electron emits more weakly than the positron, and this difference becomes stronger as angle θ decreases.

The bremsstrahlung cross section in the crystal is governed by the following relationship [3]:

$$d\sigma = N(d\sigma_{\text{coh}} + d\sigma_{\text{incoh}}),$$

where N is the total number of atoms in the crystal, $d\sigma_{\text{coh}}$ is the coherent part of the radiation cross section resulting from the radiation interference on the atoms located periodically in the crystal, and $d\sigma_{\text{incoh}}$ is the incoherent part caused by the thermal variations in the atom locations in the crystal. The cross section of the incoherent radiation does not depend on the crystal alignment relative to the particle momentum and only slightly (by 5–25% at room temperature) differs from the Bethe–Heitler radiation cross section of a particle on an isolated atom [3].

In the case of interaction of the particle with a set of parallel atomic planes in the crystal, the expression for $d\sigma_{\text{coh}}$ is obtained by summing over $(g_x)_n = \frac{2\pi}{a_x} n$ instead of integrating over dg_x in (19):

$$\int_{\delta/\theta}^{\infty} dg_x \ldots \longrightarrow \frac{2\pi}{a_x} \sum_{g_x \geq \delta/\theta} \ldots,$$

where a_x is the distance between the atomic planes. Figure 2 shows the radiation cross sections of electrons and positrons with the energy of 1 GeV that are incident on a silicon crystal at an angle $\theta = 4 \times 10^{-4}$ rad to the (110) plane. Angle θ is chosen so that the parameter (20) is large enough, but condition (22) is still met. We can see that the difference in the radiation cross sections of electrons and positrons near coherent maxima reaches 10–15%.

The simplifications obtained in deriving (9) make it possible to find the contribution of the second-order Born approximation to the polarization of the particle bremsstrahlung in the external field. It turned out [1] that the radiation of a beam of nonpolarized electrons in a field lacking spherical symmetry, such as the field of an atomic plane in a crystal, exhibits nonzero plane polarization. The degree of the plane polarization of radiation is, by definition, the ratio of the difference of cross sections of the radiation polarized in the directions specified by two mutually perpendicular polarization vectors to their sum:

$$P = \frac{d\sigma_2 - d\sigma_1}{d\sigma_1 + d\sigma_2}. \tag{23}$$

Choosing the polarization vectors as

$$\mathbf{e}^{(1)} = \frac{\mathbf{k}\times\mathbf{e}_\perp}{|\mathbf{k}\times\mathbf{e}_\perp|}, \quad \mathbf{e}^{(2)} = \frac{\mathbf{k}\times\mathbf{e}^{(1)}}{\omega}, \tag{24}$$

where $\mathbf{e}_\perp$ is the unit vector perpendicular to $\mathbf{p}$ and lying in the plane (x, p), we find that the difference of the radiation cross sections for the photons with two mutually perpendicular polarizations emitted by a beam of nonpolarized electrons (positrons) is

$$d\sigma_2 - d\sigma_1 = \frac{e^2}{4\pi^3}\frac{\varepsilon'}{\varepsilon}\frac{\delta}{m^2}\frac{d\omega}{\omega}d^3g\frac{g_x^2-g_y^2}{g_\parallel^2}\left\{\frac{\delta^2}{g_\parallel^2}|U_g|^2 + \frac{1}{(2\pi)^3\varepsilon}\left[\frac{\delta^2}{g_\parallel^2}+\frac{\omega}{\varepsilon'}\left(\frac{\delta}{g_\parallel}+2\left(1-\frac{\delta}{g_\parallel}\right)^2\right)\right]I_g\right\}. \tag{25}$$

For the radiation in the continuous potential of an atomic plane, we have

$$d\sigma_2 - d\sigma_1 = Z^2e^6 16\pi\frac{N}{a_ya_z}\frac{\varepsilon'}{\varepsilon}\frac{\delta}{m^2}\frac{d\omega}{\omega}\frac{dg_x}{\theta^2}\frac{1}{(g_x^2+R^{-2})^2}$$
$$\times\left\{\left(\frac{\delta}{g_x\theta}\right)^2 + \frac{e}{|e|}\frac{4\pi Ze^2R}{\varepsilon a_ya_z\theta^2}\frac{g_x^2+R^{-2}}{g_x^2+4R^{-2}} \tag{26}$$
$$\times\left[\left(\frac{\delta}{g_x\theta}\right)^2+\frac{\omega}{\varepsilon'}\left(\frac{\delta}{g_x\theta}+2\left(1-\frac{\delta}{g_x\theta}\right)^2\right)\right]\right\}.$$

We can see, therefore, that the coherent radiation in these conditions is polarized mainly along vector $\mathbf{e}^{(2)}$, that is, perpendicular to the atomic plane, and the difference of the radiation cross sections for the photons polarized along $\mathbf{e}^{(2)}$ and $\mathbf{e}^{(1)}$ is greater for positrons as compared to the radiation of electrons. At the same time, for the parameter range in which the Born approximation is applicable, the polarization of incoherent radiation is rather small [16]. The bottom plot of Fig. 2 shows the degree of polarization of total (coherent and incoherent) radiation of electrons and positrons in the crystal as a function of the emitted photon energy. We can see that in this case the contribution of the second-order Born correction can also result in a substantial dependence of the radiation polarization on the charge sign of the emitting particles.

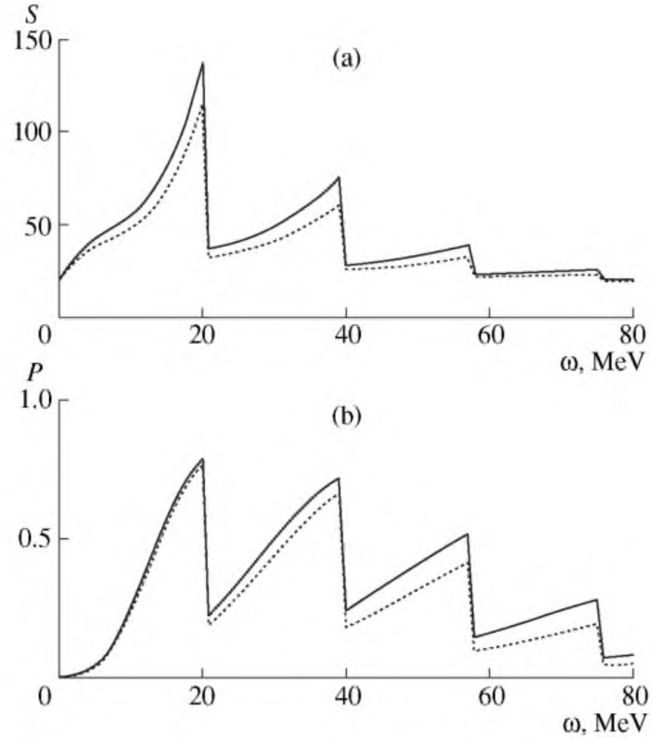

Fig. 2. Reduced cross section $S = (m^2/NZ^2e^6)(\omega d\sigma/d\omega)$ (a) and polarization (b) of the bremsstrahlung of a positron (solid line) and an electron (dashed line) with the energy of 1 GeV incident at an angle $\theta = 4\times10^{-4}$ rad to the (110) plane of a silicon crystal.

CONCLUSIONS

The results obtained show that the bremsstrahlung cross section of relativistic electrons and positrons in the external field with allowance for the second-order Born approximation is given by Eq. (12) in the entire frequency range of the emitted photons. The dependence of the contribution of the second-order Born approximation to the radiation cross section on the emitted photon energy for $\omega \sim \varepsilon$ differs from the corresponding dependence of the contribution of the first-order Born approximation to the radiation cross section. In particular, this means that the factorization theorem for the radiation cross section for $\omega \sim \varepsilon$ is valid only to the error governed by the contribution of the second-order Born approximation.

Allowance for the contribution of the second-order Born approximation to the radiation cross section results in the dependence of the cross section on the sign of the particle charge in the entire frequency range of the emitted photons. For the radiation of a relativistic particle in the field of an isolated atom, this dependence is rather slight. A substantial increase in the dependence of the radiation cross section on the sign of the particle charge is possible in the case of radiation in a crystal. This increase takes place, for example, when the beam is incident at a small angle θ to one of the crystal planes. The dependence of the cross section on the charge sign in the entire frequency range of the emitted photons is governed by parameter (21). As the angle θ decreases, this parameter rapidly grows. At $\theta \sim \theta_c$, the effects associated with channeling should be taken into account. This phenomenon cannot be described within the Born approximation; therefore, Eq. (19) is valid at $\theta_c \ll \theta \ll 1$.

Radiation of relativistic electrons and positrons in a crystal was experimentally studied in [17–20]. However, in those papers, most attention was paid to the investigation of the radiation characteristics in conditions when particle channeling in the crystal takes place. It was shown that, in this case, the radiation cross sections of electrons and positrons differ substantially. There were no detailed experimental studies of the dependence of the radiation cross section on the charge sign when (19) is applicable. Only several measurements of the orientation dependence of the radiation cross sections of electrons and positrons under radiation collimation are available [17, 18]. The results of these measurements are in qualitative agreement with most predictions of Eq. (19).

ACKNOWLEDGMENTS

This work was supported by the Ministry of Science and Education of the Russian Federation within the program Development of Scientific Potential of Higher Educational Institutions (project RNP.2.1.1.1.3263), by the Russian Foundation for Basic Research (project no. 05-02-16512), and by a grant of Belgorod State University.

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