ISSN 0147-6882, Scientific and Technical Information Processing, 2021, Vol. 48, No. 5, pp. 342–355. © Allerton Press, Inc., 2021.
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Reduction of Attribute Space Dimensionality: the SOCRATES Method

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Received February 26, 2020

Abstract—The new SOCRATES (ShOrtening CRiteria and ATtributES) method for reducing the dimensionality of attribute space is described. In this method, a large number of initial numerical and/or verbal characteristics of objects are aggregated into a single integral index or several composite indicators with small scales of qualitative estimates. Multiattribute objects are represented as multisets of object properties. The attribute aggregation includes various methods for the transformation of attributes and their scales. Reducing the number of attributes and shortening their scales make it possible to simplify the solution of applied problems, in particular, problems of multicriteria choice and to explain the obtained results.

Keywords: multiattribute objects, multisets, attribute space, dimensionality reduction, aggregation of attributes, composite indicator, multicriteria choice

DOI: 10.3103/S0147688221050063

INTRODUCTION

The problems of strategic and unique choices involving very few compared objects and a very large number of features that characterize their properties, which can reach tens or hundreds, are among the most difficult ones. Examples of such objects are the place for building an airport or power plant, the route for laying a gas or oil pipeline, the scheme of a transport network, the configuration of a complex technical system, etc. In real situations, it is very difficult for decision-makers (DM) and experts to select the best object as well as to rank or classify objects that are described by a large number of attributes, because, as a rule, many objects will be formally incomparable in their characteristics.

Additional difficulties arise in the case of poorly structured problems that combine quantitative and qualitative dependencies, for which the construction of objective models is either impossible in principle or very difficult. The known methods for decision-making [2–5, 7, 14, 15] are extremely effort- and time-consuming in obtaining and processing large amounts of data about objects, DM preferences and/or expert knowledge, and are of little use for solving multicriteria choice problems of high dimensionality.

The following approaches are possible that facilitate the choice in a large attribute space and reduce the information losses: the use of psychologically correct operations for obtaining information from DM and experts, and reduction of the attribute space dimensionality. It has been experimentally established that it is easier for a person, due to the peculiarities of his physical memory, to operate with small amounts of data and to compare objects by a small number of indicators. The results of such operations are more reliable and easier to analyze. For this, it would suffice to describe objects with three to seven indicators. A person makes fewer mistakes when the indicators have verbal scales rather than numerical [3–5, 15]. Reduction of the attribute space dimensionality simplifies the solution of problems of individual and group multicriteria choice by diminishing the number of variables. Practically all applied methods for dimensionality reduction deal with numerical data [1, 2, 16]. Procedures for dimensionality reduction in the spaces of qualitative attributes are presented in [10–13].

This work describes the new SOCRATES (ShOrtening CRiteria and ATtributES) method in which numerous initial characteristics of objects are aggregated into several indicators or a single integral indicator with small scales of verbal assessments. The representation of multiattribute objects as multisets and aggregation of attributes can significantly reduce the complexity of solving the original problem of multicriteria choice and reasonably explain the obtained results.

1. REPRESENTATION AND COMPARISON OF MULTIATTRIBUTE OBJECTS

Let us discuss possible ways of presenting, comparing, and grouping objects that are specified by many numerical and/or verbal attributes and are present in several copies that differ in the values of their characteristics [6–8, 10].

Let the objects $O_1,...,O_q$ be the only ones and be described by the attributes $K_1,...,K_n$ with numerical and/or verbal rating scales $X_i = \{x_i^1,\dots,x_i^{h_i}\}$, $i = 1,...,n$. Traditionally, every object O_p, $p = 1,...,q$ is associated with a vector or tuple $\boldsymbol{x}_p = (x_{p1}^{e_1},\dots,x_{pn}^{e_n})$, $x_{pi}^{e_i}$ is one of the gradations of the attribute K_i on the scale X_i. The vector/tuple $\boldsymbol{x}_p$ is a point of the n-dimension space $X = X_1 \times\dots\times X_n$ formed by the scales of attributes K_1, ..., K_n.

The situation is more complicated when the object O_p is present in several copies $O_p^{\langle s\rangle}$, $p = 1,...,q$, $s = 1,...,t$, which differ in the values of the attributes K_1, ..., K_n. Different versions of the O_p object emerge, for instance, when the object is assessed by t experts by many criteria K_1, ..., K_n, or the characteristics of an object are calculated t times by several methods K_1, ..., K_n, or measured t times using several instruments K_1, ..., K_n. In such cases, the object O_p will be associated not with a single vector/tuple, but with a group of t vectors/tuples $\left\{x_p^{\langle 1\rangle},\dots,x_p^{\langle t\rangle}\right\}$. The vector/tuple $x_p^{\langle s\rangle} = \left(x_{p1}^{\langle s\rangle},\dots,x_{pn}^{\langle s\rangle}\right)$ describes one of the versions $O_p^{\langle s\rangle}$ of the object O_p, and its component $x_{pi}^{\langle s\rangle}$ is the value of the attribute K_i in the version $O_p^{\langle s\rangle}$ of the object O_p equal to $x_p^{e\langle s\rangle}$; $e = 1,...,h$ if all attributes K_1, ..., K_n have the same rating scale $X = \{x^1, ..., x^h\}$ or $x_{pi}^{e_i\langle s\rangle}$, $e_i = 1,...,h_i$ if each attribute K_1, ..., K_n has its own rating scale $X_i = \left\{x_i^1,\dots,x_i^{h_i}\right\}$, $i = 1, ..., n$.

The object O_p is now represented in an n-dimensional attribute space $X = X_1 \times\dots\times X_n$ not by a single point $\boldsymbol{x}_p$ but by an entire group ("cloud") consisting of t points $\left\{\boldsymbol{x}_p^{\langle 1\rangle},\dots,\boldsymbol{x}_p^{\langle t\rangle}\right\}$. Importantly, the group of vectors/tuples $\boldsymbol{x}_p^{\langle 1\rangle},\dots,\boldsymbol{x}_p^{\langle t\rangle}$ representing the object O_p must be treated as an entity. In this case, generally speaking, the individual values of the attributes for various versions of the object O_p (assessments made by different experts, characteristics measured by different methods or instruments) can be both similar and different, and even contradictory, which in turn can lead to incomparability of vectors/tuples, which comprise the group representing one and the same object O_p.

The objects O_1, ..., O_q, each of which exists in several versions $O_p^{\langle s\rangle}$ specified by the vectors/tuples $\boldsymbol{x}_p^{\langle s\rangle}$, and their attributes can be represented by Objects–Attributes matrices $\mathrm{F} = \|x_{pi}\|_{q\times n}$ and $\mathrm{F}^{<\,>} = \left\|x_{pi}^{\langle s\rangle}\right\|_{tq\times n}$. The rows of the matrix F correspond to objects, the columns correspond to attributes, and the x_{pi} elements are the values of the components $x_{pi}^{e_i}$ of vectors/tuple components that define the objects. The number of rows of the matrix $\mathrm{F}^{\langle\,\rangle}$, which has a large dimensionality, is equal to the number of all copies of the objects, the number of columns is equal to the number of attributes, and the elements $x_{pi}^{\langle s\rangle}$ are the values of the components $x_{pi}^{\langle s\rangle}$ of vectors/tuples specifying different versions of the objects.

It is rather difficult to analyze a set of multiattribute objects O_1, ..., O_q, each of which is represented in the attribute space $X = X_1 \times\dots\times X_n$ by its own "cloud" consisting of t various points. Therefore, it is highly desirable in one way or another to simplify the description and to aggregate representation of such multiattribute objects. In the case of numerical attributes K_1,..., K_n, the simplest way is to define each O_p object as a single vector $x_p^{\mathrm{cond}} = \left(x_{p1}^{\mathrm{cond}},\dots,x_{pn}^{\mathrm{cond}}\right)$, whose components are determined by additional formal conditions or meaningful considerations. As an example, it can be the following: a vector that is the center of the group; the vector closest to all vectors in the group or a vector with the total, averaged, or weighted component values of the vectors $\boldsymbol{x}_p^{\langle 1\rangle},\dots,\boldsymbol{x}_p^{\langle t\rangle}$ representing versions of this object. In the case of symbolic, verbal, or mixed attributes K_1,...,K_n, the group of tuples representing copies of any object, even in principle, cannot be replaced by a single tuple with total, averaged, weighted, mixed values of the components, since such operations are mathematically impossible.

A convenient mathematical model for representing objects that are described by many numerical and verbal attributes, is a multiset or a set with repetitions [9]. This model makes it possible to simultaneously take heterogeneous attributes, possible combinations of attribute values, and the presence of different object copies [6–8, 10] into account. When all attributes K_1, ..., K_n have the same rating scale $X = \{x^1, ..., x^h\}$, we associate the object O_p, $p = 1,...,q$ with a multiset of estimates

$$A_p = \left\{k_{A_p}\left(x^1\right)\circ x^1,\dots,k_{A_p}\left(x^h\right)\circ x^h\right\} \tag{1}$$

over the generating set $X = \{x^1,...,x^h\}$ of the scale gradations. Here, the value of the multiplicity function $k_{Ap}(x^e)$ shows the number of times that the grade $x^e \in X$, $e = 1,...,h$ is present in the description of the object O_p.

When each attribute K_i has its own rating scale $X_i = \left\{x_i^1,\dots,x_i^{h_i}\right\}$, $i = 1,...,n$, we introduce a single expanded scale (hyperscale) of attributes: the set $X = X_1\cup\dots\cup X_n = \{x_1^1,\dots,x_1^{h_1};\dots;x_n^1,\dots,x_n^{h_n}\}$, which consists of n groups of attributes and combines all gradations of estimates on the scales of all attributes. Then, the object O_p will correspond to the multiset of estimates

$$A_p = \{k_{Ap}(x_1^1)\circ x_1^1, \ldots, k_{Ap}(x_1^{h_1})\circ x_1^{h_1}; \ldots; k_{Ap}(x_n^1)\circ x_n^1, \ldots, k_{Ap}(x_n^{h_n})\circ x_n^{h_n}\} \quad (2)$$

over the generating set $X = \{x_1^1, \ldots, x_1^{h_1}; \ldots; x_n^1, \ldots, x_n^{h_n}\}$ of attribute scale gradations. Here, the value of the multiplicity function $k_{Ap}(x_i^{ei})$ shows the number of times the estimate $x_i^{e_i} \in X_i$, $e_i = 1, \ldots, h_i$ with regard to the attribute K_i is present in the description of the object O_p. Expression (2) can easily be written in the "usual" form (1), if the following change in variables is carried out in the set $X = \{x_1^1, \ldots, x_1^{h_1}; \ldots; x_n^1, \ldots, x_n^{h_n}\}$: $x_1^1 = x^1, \ldots, x_1^{h_1} = x^{h_1}$, $x_2^1 = x^{h_1+1}, \ldots, x_2^{h_2} = x^{h_1+h_2}$, ..., $x_n^{h_n} = x^h$, $h = h_1 + \ldots + h_n$. Despite the seemingly cumbersome representation of multiattribute objects using multisets, such notation forms are extremely convenient when comparing objects and performing operations, since calculations are carried out in parallel and simultaneously for all elements of all multisets.

A variety of operations on multisets makes it possible to group multiattribute objects in different ways. A group of objects can be formed by specifying multiset $\boldsymbol{J}$ representing the group by the sum $\boldsymbol{J} = \sum_s \boldsymbol{A}_s, k_{\boldsymbol{J}}(x^e) = \sum_s k_{As}(x^e)$, union $\boldsymbol{J} = \cup_s \boldsymbol{A}_s$, $k_{\boldsymbol{J}}(x^e) = \max_s k_{As}(x^e)$, intersection $\boldsymbol{J} = \cap_s \boldsymbol{A}_s$, $k_{\boldsymbol{J}}(x^e) = \min_s k_{As}(x^e)$ of multisets $\boldsymbol{A}_s$ describing the grouped objects or by one of linear operations on multisets $\boldsymbol{A}_s$: $\boldsymbol{J} = \sum_s c_s \boldsymbol{A}_s$, $\boldsymbol{J} = \cup_s c_s\boldsymbol{A}_s$, $\boldsymbol{J} = \cap_s c_s\boldsymbol{A}_s$, $c_s > 0$ is an integer. Upon addition of multisets, all the properties (all values of all attributes) of objects included in the group are aggregated. Upon combining or intersecting the multisets, the best properties (maximum values of all attributes) or, accordingly, the worst properties (minimum values of all attributes) possessed by individual members of the group are amplified.

If there are several versions of the object O_p, all its copies $O_p^{\langle s\rangle}$, $p = 1,\ldots,q$, $s = 1,\ldots,t$ make up a group representing this object. We associate the object O_p with the multiset $\boldsymbol{A}_p = \{k_{Ap}(x^1)\circ x^1, \ldots, k_{Ap}(x^h)\circ x^h\}$ of the form (1), (2), and the version $O_p^{\langle s\rangle}$, with the multiset $\boldsymbol{A}_p^{\langle s\rangle} = \{k_{Ap}^{\langle s\rangle}(x^1)\circ x^1, \ldots, k_{Ap}^{\langle s\rangle}(x^h)\circ x^h\}$ over the set of estimates $X = \{x^1,\ldots,x^h\}$ or $X = \{x_1^1, \ldots, x_1^{h_1}; \ldots; x_n^1, \ldots, x_n^{h_n}\}$. Multiset $\boldsymbol{A}_p$ will be generated as a weighted sum of multisets describing the versions of the object: $\boldsymbol{A}_p = c^{\langle 1\rangle}\boldsymbol{A}_p^{\langle 1\rangle} + \ldots + c^{\langle t\rangle}\boldsymbol{A}_p^{\langle t\rangle}$ where the multiplicity function of the multiset $\boldsymbol{A}_p$ is calculated according to the rule $k_{Ap}(x^e) = \sum_s c^{\langle s\rangle} k_{Ap}^{\langle s\rangle}(x^e)$, and the coefficient $c^{\langle s\rangle}$ characterizes the significance of the copy $O_p^{\langle s\rangle}$ (expert competence, measurement accuracy).

The objects $O_1,\ldots,O_q$ and the values of their attributes represented by the multisets $\boldsymbol{A}_1, \ldots, \boldsymbol{A}_q$ of the form (1) and (2) can be given by matrices Object–Attribute $\mathrm{G} = \|k_{pe}\|_{q\times h}$ and $\mathrm{H} = \|k_{pi}\|_{q\times h}$, $h = h_1 + \ldots + h_n$. The rows of the matrix G correspond to the objects, the columns, to the values of the attribute scale X, and the elements k_{pe} are the values of the multiplicity $k_{Ap}(x^e)$ that characterize elements x^e of multisets specifying the objects. The rows of the matrix H correspond to the objects, the columns, to the values of the attributes hyperscale $X = X_1 \cup \ldots \cup X_n$, and the elements k_{pi} are the values of the multiplicity $k_{Ap}(x_i^{e_i})$ of elements $x_i^{e_i}$ of multisets specifying the objects. Versions of multiattribute objects $O_1,\ldots,O_q$ and the values of their attributes represented by the multisets of the form (1), (2) can be given by Object–Attribute matrices $\mathrm{G}^{\langle\,\rangle} = \|k_{pe}^{\langle s\rangle}\|_{tq\times h}$ and $\mathrm{H}^{\langle\,\rangle} = \|k_{pi}^{\langle s\rangle}\|_{tq\times h}$, $h = h_1 + \ldots + h_n$. The elements of the matrices $\mathrm{G}^{\langle\,\rangle}$, $\mathrm{H}^{\langle\,\rangle}$ are the multiplicity values $k_{Ap}^{\langle s\rangle}(x^e), k_{Ap}^{\langle s\rangle}(x_i^{ei})$ of the elements of multisets $\boldsymbol{A}_p^{\langle s\rangle}$ describing the respective copies of the objects $O_p^{\langle s\rangle}$.

Here is an illustrative example of representing multiattribute objects. There are ten objects $O_1,\ldots,O_{10}$ described by eight attributes $K_1,\ldots,K_8$, each of which takes one of the values on a five-point rating scale $X = \{x^1, x^2, x^3, x^4, x^5\}$. For instance, objects $O_1,\ldots,O_{10}$ are pupils and the attributes $K_1,\ldots,K_8$ of the objects are grades in the following school subjects: K_1 Mathematics, K_2 Physics, K_3 Chemistry, K_4 Biology, K_5 Social science, K_6 History, K_7 Literature, and K_8 Foreign language. The grading scales include: x^1 is 1/very poor, x^2 is 2/poor, x^3 is 3/satisfactory, x^4 is 4/good, and x^5 is 5/excellent. Or it can be that the objects O_1, ..., O_{10} are questions of a public opinion poll on some problem. Attributes of the objects are the answers of K_1, ..., K_8 respondents coded in the following way: x^1 is 1/completely disagree, x^2 is 2/disagree, x^2 is 3/indifferent, x^4 is 4/agree, x^5 is 5/fully agree.

Situations are also possible where each of the objects O_1, ..., O_{10} is present in several copies differing from each other. For instance, pupils are graded in eight subjects K_1, ..., K_8 twice a year for every half-year (semester) or eight respondents K_1, ..., K_8 participate in the poll twice answer the same questions. Therefore, each object is represented by two vectors/tuples of the attributes or two multisets rather than one. This description of an object copy can be considered an individual opinion of some expert and the description of the object "as a whole" is an aggregated collective judgment of two experts.

Table 1. Object–Attribute Matrix $F^{\langle\,\rangle}$

$O\backslash K$	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8
$\boldsymbol{x}_1^{\langle 1\rangle}$	4	5	4	5	4	5	4	5
$\boldsymbol{x}_1^{\langle 2\rangle}$	5	5	5	5	4	4	4	5
$\boldsymbol{x}_2^{\langle 1\rangle}$	4	1	2	1	3	2	2	2
$\boldsymbol{x}_2^{\langle 2\rangle}$	3	2	1	1	4	3	3	2
$\boldsymbol{x}_3^{\langle 1\rangle}$	1	1	3	1	4	1	1	4
$\boldsymbol{x}_3^{\langle 2\rangle}$	1	2	3	1	5	2	1	3
$\boldsymbol{x}_4^{\langle 1\rangle}$	5	3	2	4	4	5	4	5
$\boldsymbol{x}_4^{\langle 2\rangle}$	4	4	3	5	4	5	3	4
$\boldsymbol{x}_5^{\langle 1\rangle}$	4	4	4	4	4	5	4	4
$\boldsymbol{x}_5^{\langle 2\rangle}$	5	5	3	4	4	4	5	4
$\boldsymbol{x}_6^{\langle 1\rangle}$	5	5	4	4	4	5	5	4
$\boldsymbol{x}_6^{\langle 2\rangle}$	4	5	4	4	4	4	5	5
$\boldsymbol{x}_7^{\langle 1\rangle}$	4	1	2	3	3	3	1	2
$\boldsymbol{x}_7^{\langle 2\rangle}$	3	2	1	4	2	4	2	3
$\boldsymbol{x}_8^{\langle 1\rangle}$	4	5	4	2	3	4	5	3
$\boldsymbol{x}_8^{\langle 2\rangle}$	5	4	5	3	4	5	4	4
$\boldsymbol{x}_9^{\langle 1\rangle}$	3	2	3	1	3	3	2	2
$\boldsymbol{x}_9^{\langle 2\rangle}$	4	3	2	2	2	3	3	2
$\boldsymbol{x}_{10}^{\langle 1\rangle}$	5	5	4	5	3	5	5	4
$\boldsymbol{x}_{10}^{\langle 2\rangle}$	3	4	3	4	2	4	2	4

Table 2. Object–Attribute matrix $G^{\langle\,\rangle}$

$O\backslash X$	x^1	x^2	x^3	x^4	x^5
$\boldsymbol{A}_1^{\langle 1\rangle}$	0	0	0	4	4
$\boldsymbol{A}_1^{\langle 2\rangle}$	0	0	0	3	5
$\boldsymbol{A}_2^{\langle 1\rangle}$	2	4	1	1	0
$\boldsymbol{A}_2^{\langle 2\rangle}$	2	2	3	1	0
$\boldsymbol{A}_3^{\langle 1\rangle}$	5	0	1	2	0
$\boldsymbol{A}_3^{\langle 2\rangle}$	3	2	2	0	1
$\boldsymbol{A}_4^{\langle 1\rangle}$	0	1	1	3	3
$\boldsymbol{A}_4^{\langle 2\rangle}$	0	0	2	4	2
$\boldsymbol{A}_5^{\langle 1\rangle}$	0	0	0	7	1
$\boldsymbol{A}_5^{\langle 2\rangle}$	0	0	1	4	3
$\boldsymbol{A}_6^{\langle 1\rangle}$	0	0	0	4	4
$\boldsymbol{A}_6^{\langle 2\rangle}$	0	0	0	5	3
$\boldsymbol{A}_7^{\langle 1\rangle}$	2	2	3	1	0
$\boldsymbol{A}_7^{\langle 2\rangle}$	1	3	2	2	0
$\boldsymbol{A}_8^{\langle 1\rangle}$	0	1	2	3	2
$\boldsymbol{A}_8^{\langle 2\rangle}$	0	0	1	4	3
$\boldsymbol{A}_9^{\langle 1\rangle}$	1	3	4	0	0
$\boldsymbol{A}_9^{\langle 2\rangle}$	0	4	3	1	0
$\boldsymbol{A}_{10}^{\langle 1\rangle}$	0	0	1	2	5
$\boldsymbol{A}_{10}^{\langle 2\rangle}$	0	2	2	4	0

Differing versions of the objects O_1, ..., O_{10} are given by the Object–Attribute matrices $F^{\langle\,\rangle}$ and $G^{\langle\,\rangle}$ (Tables 1, 2). The rows of the matrix $F^{\langle\,\rangle}$ are vectors $\boldsymbol{x}_p^{\langle 1\rangle}, \boldsymbol{x}_p^{\langle 2\rangle}$ of the numerical semester marks of pupils or numerical grades for the answers of university students who evaluated twice the course of lectures. The first rows in the cells of the matrix $F^{\langle\,\rangle}$ are borrowed from [14]. The same grades of the pupil school performance or grades for answers of university students recorded as multisets $\boldsymbol{A}_p$ of numerical or verbal estimates of the form (1) are represented by the rows of the matrix $G^{\langle\,\rangle}$. The annual marks of pupils O_p, $p = 1$, ..., 10, in subjects $K_1,...,K_8$, which have their own scales $X_i = \{x_i^1,\ldots,x_i^5\}$, $i = 1,...,8$, are given by the multiset

$$\boldsymbol{A}_p = \{k_{A_p}(x_1^1)\circ x_1^1,\ldots,k_{A_p}(x_1^5)\circ x_1^5;\ldots;\\ k_{A_p}(x_8^1)\circ x_8^1,\ldots,k_{A_p}(x_8^5)\circ x_8^5\}, \tag{3}$$

the multiplicities of the elements of which form the rows of the Object–Attribute matrix H (Table 3). Matrix $H^{\langle\,\rangle}$ similar to matrix H includes multiplicities of elements of multisets $\boldsymbol{A}_p^{\langle s\rangle}$ describing copies of the objects $O_p^{\langle s\rangle}$ and is too cumbersome to be presented here.

As an example, semester marks of the pupil O_1 are represented in Table 1 by two vectors $\boldsymbol{x}_1^{\langle 1\rangle} = (4, 5, 4, 5, 4, 5, 4, 5)$ and $\boldsymbol{x}_1^{\langle 2\rangle} = (5, 5, 5, 5, 4, 4, 4, 5)$. The annual school performance of the pupil O_1 can be described by the resultant vector $\boldsymbol{x}_1 = (9, 10, 9, 10, 8, 9, 8, 10)$ or averaged vector $\boldsymbol{x}_1^{\text{average}} = (4.5, 5.0, 4.5, 5.0, 4.0, 4.5, 4.0, 5.0)$. However, there are no such numbers in the accepted five-point rating scale $X = \{1, 2, 3, 4, 5\}$. The same marks of the pupil O_1 are represented in Table 2 by the multisets $\boldsymbol{A}_1^{\langle 1\rangle} = \{0\circ x^1, 0\circ x^2, 0\circ x^3, 4\circ x^4, 4\circ x^5\}$ and

Table 3. The Object–Attribute Matrix H

$O\backslash X$	$x_1^1\ x_1^2\ x_1^3\ x_1^4\ x_1^5$	$x_2^1\ x_2^2\ x_2^3\ x_2^4\ x_2^5$	$x_1^1\ x_3^2\ x_3^3\ x_3^4\ x_3^5$	$x_4^1\ x_4^2\ x_4^3\ x_4^4\ x_4^5$	$x_5^1\ x_5^2\ x_5^3\ x_5^4\ x_5^5$	$x_6^1\ x_6^2\ x_6^3\ x_6^4\ x_6^5$	$x_7^1\ x_7^2\ x_7^3\ x_7^4\ x_7^5$	$x_8^1\ x_8^2\ x_8^3\ x_8^4\ x_8^5$
A_1	0 0 0 1 1	0 0 0 0 2	0 0 0 1 1	0 0 0 0 2	0 0 0 2 0	0 0 0 1 1	0 0 0 2 0	0 0 0 0 2
A_2	0 0 1 1 0	1 1 0 0 0	1 1 0 0 0	2 0 0 0 0	0 0 1 1 0	0 1 1 0 0	0 1 1 0 0	0 2 0 0 0
A_3	2 0 0 0 0	1 1 0 0 0	0 0 2 0 0	2 0 0 0 0	0 0 0 1 1	1 1 0 0 0	2 0 0 0 0	0 0 1 1 0
A_4	0 0 0 1 1	0 0 1 1 0	0 1 1 0 0	0 0 0 1 1	0 0 0 2 0	0 0 0 0 2	0 0 1 1 0	0 0 0 1 1
A_5	0 0 0 1 1	0 0 0 1 1	0 0 1 1 0	0 0 0 2 0	0 0 0 2 0	0 0 0 1 1	0 0 0 1 1	0 0 0 2 0
A_6	0 0 0 1 1	0 0 0 0 2	0 0 0 2 0	0 0 0 2 0	0 0 0 2 0	0 0 0 1 1	0 0 0 0 2	0 0 0 1 1
A_7	0 0 1 1 0	1 1 0 0 0	1 1 0 0 0	0 0 1 1 0	0 1 1 0 0	0 0 1 1 0	1 1 0 0 0	0 1 1 0 0
A_8	0 0 0 1 1	0 0 0 1 1	0 0 0 1 1	0 1 1 0 0	0 0 1 1 0	0 0 0 1 1	0 0 0 1 1	0 0 1 1 0
A_9	0 0 1 1 0	0 1 1 0 0	0 1 1 0 0	1 1 0 0 0	0 1 1 0 0	0 0 2 0 0	0 1 1 0 0	0 0 2 0 0
A_{10}	0 0 1 0 1	0 0 0 1 1	0 0 1 1 0	0 0 0 1 1	0 1 1 0 0	0 0 0 1 1	0 1 0 0 1	0 0 0 2 0

$A_1^{\langle 2\rangle} = \{0\circ x^1, 0\circ x^2, 0\circ x^3, 3\circ x^4, 5\circ x^5\}$. The annual school performance of the pupil O_1, considering semiannual marks equally significant: $c^{\langle 1\rangle} = c^{\langle 2\rangle} = 1$, is described by the sum of multisets $A_1^{\langle 1\rangle}$ and $A_1^{\langle 2\rangle}$:

$$A_1 = A_1^{\langle 1\rangle} + A_1^{\langle 2\rangle} = \left\{0\circ x^1,\ 0\circ x^2,\ 0\circ x^3,\ 4\circ x^4,\ 4\circ x^5\right\}$$
$$+ \left\{0\circ x^1,\ 0\circ x^2,\ 0\circ x^3,\ 3\circ x^4,\ 5\circ x^5\right\}$$
$$= \left\{0\circ x^1,\ 0\circ x^2,\ 0\circ x^3,\ 7\circ x^4,\ 9\circ x^5\right\}.$$

This form of record shows that the pupil O_1 over a year received seven marks x^4—good and nine marks x^5—excellent, and had not received any other marks. This result is not directly visible when the annual school performance of the pupil O_1 is represented by the vectors $\boldsymbol{x}_1$ or $\boldsymbol{x}_1^{\text{average}}$. If the marks in Table 1 are symbols, the annual school performance of any pupil O_p cannot be described by any tuple $\boldsymbol{x}_p$ at all.

In Table 3, the annual school performance of the pupil O_1 are associated with the following multiset:

$$A_1 = \{0\circ x_1^1, 0\circ x_1^2, 0\circ x_1^3, 1\circ x_1^4, 1\circ x_1^5; 0\circ x_2^1, 0\circ x_2^2, 0\circ x_2^3, 0\circ x_2^4, 2\circ x_2^5;$$
$$0\circ x_3^1, 0\circ x_3^2, 0\circ x_3^3, 1\circ x_3^4, 1\circ x_3^5; 0\circ x_4^1, 0\circ x_4^2, 0\circ x_4^3, 0\circ x_4^4, 2\circ x_4^5;$$
$$0\circ x_5^1, 0\circ x_5^2, 0\circ x_5^3, 2\circ x_5^4, 0\circ x_5^5; 0\circ x_6^1, 0\circ x_6^2, 0\circ x_6^3, 1\circ x_6^4, 1\circ x_6^5;$$
$$0\circ x_7^1, 0\circ x_7^2, 0\circ x_7^3, 2\circ x_7^4, 0\circ x_7^5; 0\circ x_8^1, 0\circ x_8^2, 0\circ x_8^3, 0\circ x_8^4, 2\circ x_8^5\}.$$

From this it is clear that over a year, the pupil O_1 received in mathematics one mark x^4 – good, one mark x^5– excellent; in physics he received two marks x^5—excellent; in chemistry, one mark x^4—good, one mark x^5—excellent; in biology, two marks x^5—excellent; in social science, two marks x^4—good; in history, one mark x^4—good, one mark x^5—excellent; in literature, two marks x^4—good; and in foreign language, two marks x^5—excellent.

2. REDUCING THE ATTRIBUTE SPACE

Dimensionality reduction for the object description is diminishing the number of indicators that characterize the state or functioning of the objects by some transformations of the initial data, during which the set of initial attributes K_1, ..., K_n is aggregated into smaller sets of intermediate L_1, ..., L_m,... and final N_1, ..., N_l attributes. The transformations of the attributes can formally be recorded as

$$K_1,\ldots,K_n \to L_1,\ldots,L_m \to \ldots \to N_1,\ldots,N_l, \quad (4)$$

where the initial attribute K_i has the scale $X_i = \{x_i^1,\ldots,x_i^{h_i}\}$, $i = 1,\ldots,n$, the intermediate attribute L_j has the scale $Y_j = \{y_j^1,\ldots,y_j^{g_j}\}$, $j = 1, \ldots, m$, and the final attribute N_k has the scale $Z_k = \{z_k^1,\ldots,z_k^{f_k}\}$, $k = 1, \ldots, l$, $l < m < n$. Reduction of the attribute space dimensionality is an informal multistage procedure based on the knowledge, experience, and intuition of a DM/expert who formulates rules for the attribute the transformation, establishes the structure, number, dimension, and conceptual meaning of new indicators.

In the cases where multiattribute objects are represented by vectors/tuples, problem (4) of reduction of the attribute space dimensionality has the form

$$X_1 \times \ldots \times X_n \to Y_1 \times \ldots \times Y_m \to \ldots \to Z_1 \times \ldots \times Z_l. \quad (5)$$

The dimensionality of the respective attribute space is defined then as the cardinality of the direct product of numerical or verbal attribute scale gradations that are components of vectors/tuples. In [10–13], problem (5) is considered as a multicriteria classification problem where the sets containing grades of the initial attributes are classified objects, and the gradations of the composite indicator scale are the classes of solutions [4, 5, 7].

In the cases where multiattribute objects are represented by multisets, problem (4) of reduction of the attribute space dimensionality has the form

$$X_1 \cup \ldots \cup X_n \to Y_1 \cup \ldots \cup Y_m \to \ldots \to Z_1 \cup \ldots \cup Z_l. \quad (6)$$

The dimensionality of the respective attribute space is then defined as the cardinality of the hyperscale, i.e., the union of numeric or verbal attribute scale gradations that are elements of multisets. The SOCRATES method outlined in this work makes it possible to reduce the descriptions of multiattribute objects, which are present in several differing copies and are given by the multisets of numerical and/or verbal characteristics. The method uses two main transformations i.e., shortening the attribute scales and their aggregation. We now consider these transformations in greater detail.

Shortening an attribute scale is a relatively simple transformation of the attribute space and is aimed at decreasing the number of gradations on the attribute scale. For this, several values of some characteristic of an object are combined into one new gradation of the same characteristic. The transition from the initial scales of attributes to scales with a diminished number of gradations is a transformation (6) of the form

$$X_1 \cup \ldots \cup X_n \to Q_1 \cup \ldots \cup Q_n \quad (7)$$

where $X_i = \{x_i^1,\ldots,x_i^{h_i}\}$ is the initial scale and $Q_i = \{q_i^1,\ldots,q_i^{d_i}\}$ is the shortened scale of the ith attribute K_i, $|Q_i| = d_i < h_i = |X_i|, \quad i = 1,\ldots,n.$

When forming (7) shortened scales of the attributes, it is desirable that they consist of a small number (2–4) of gradations that have a well-defined specific content for a DM/expert.

The representation of multiattribute objects is transformed as follows. Let in the attribute space K_1, ..., K_n, the object O_p, $p = 1,\ldots,q$ be given by the multiset $\boldsymbol{A}_p$ (2) over the set $X_1 \cup \ldots \cup X_n$ of initial scale gradations. We use the properties of operations on the multisets [9, 10] and rewrite expression (2) in the form of sums of multisets, i.e.

$$\begin{aligned} \boldsymbol{A}_p &= \boldsymbol{A}_{p1} + \ldots + \boldsymbol{A}_{pn} \\ &= \left\{k_{\boldsymbol{A}p}\left(x_1^1\right)\circ x_1^1,\ldots,k_{\boldsymbol{A}p}\left(x_1^{h_1}\right)\circ x_1^{h_1}\right\} \\ &+ \ldots + \left\{k_{\boldsymbol{A}p}\left(x_n^1\right)\circ x_n^1,\ldots,k_{\boldsymbol{A}p}\left(x_n^{h_n}\right)\circ x_n^{h_n}\right\} \\ &= \sum_{e_1=1}^{h_1}\left\{k_{\boldsymbol{A}p}\left(x_1^{e_1}\right)\circ x_1^{e_1}\right\} + \ldots + \sum_{e_n=1}^{h_n}\left\{k_{\boldsymbol{A}p}\left(x_n^{e_n}\right)\circ x_1^{e_n}\right\}. \end{aligned} \quad (8)$$

When the attribute scales are shortened, gradations $x_i^{e_a}, x_i^{e_b},\ldots,x_i^{e_c}$ of the initial scale $X_i = \{x_i^1,\ldots,x_i^{h_i}\}$for the attribute K_i are combined into the gradation $q_i^{o_i}$ of the shortened scale $Q_i = \{q_i^1,\ldots,q_i^{d_i}\}$. In the reduced attribute space K_1, ..., K_n, which have the scales Q_1,....Q_n, the object O_p will correspond to the multiset

$$\begin{aligned} \boldsymbol{B}_p = \Big\{&k_{\boldsymbol{B}p}\left(q_1^1\right)\circ q_1^1,\ldots,k_{\boldsymbol{B}p}\left(q_1^{d_1}\right)\circ q_1^{d_1};\ldots; \\ &k_{\boldsymbol{B}p}\left(q_n^1\right)\circ q_n^1,\ldots,k_{\boldsymbol{B}p}\left(q_n^{d_n}\right)\circ q_n^{d_n}\Big\} \end{aligned} \quad (9)$$

over the set $Q_1 \cup \ldots \cup Q_n$ of the shortened scale gradations. Multiset $\boldsymbol{B}_p$ (9) can also be written in the equivalent form i.e.,

$$\begin{aligned}\boldsymbol{B}_p &= \boldsymbol{B}_{p1} + \ldots + \boldsymbol{B}_{pn} \\ &= \left\{k_{\boldsymbol{B}p}\left(q_1^1\right)\circ q_1^1, \ldots, k_{\boldsymbol{B}p}\left(q_1^{d_1}\right)\circ q_1^{d_1}\right\} \\ &+ \ldots + \left\{k_{\boldsymbol{B}p}\left(q_n^1\right)\circ q_n^1, \ldots, k_{\boldsymbol{B}p}\left(q_n^{d_n}\right)\circ q_n^{d_n}\right\} \\ &= \sum_{o_1=1}^{d_1}\left\{k_{\boldsymbol{B}p}\left(q_1^{o_1}\right)\circ q_1^{o_1}\right\} + \ldots + \sum_{o_n=1}^{d_n}\left\{k_{\boldsymbol{B}p}\left(q_n^{o_n}\right)\circ q_n^{o_n}\right\}.\end{aligned} \quad (10)$$

The multiplicity of the element $q_i^{o_i}$, $o_i = 1,...,d_i$ of the multiset $\boldsymbol{B}_p$ (9) or (10), which corresponds to the gradation $q_i^{o_i}$ of the shortened scale $Q_i = \{q_i^1, \ldots, q_i^{d_i}\}$, is determined by the rule

$$k_{\boldsymbol{B}p}\left(q_i^{o_i}\right) = k_{\boldsymbol{A}p}\left(x_i^{e_a}\right) + k_{\boldsymbol{A}p}\left(x_i^{e_b}\right) + \ldots + k_{\boldsymbol{A}p}\left(x_i^{e_c}\right), \quad (11)$$

where multiplicities of the elements $x_i^{e_a}, x_i^{e_b}, \ldots, x_i^{e_c}$ of the multiset $\boldsymbol{A}_p$ (2) or (8) are summed, which correspond to the combined gradations of the initial scale $X_i = \{x_i^1, \ldots, x_i^{h_i}\}$for the attribute K_i.

Aggregation of attributes is a more complex transformation of the attribute space and is oriented to diminish the attribute number. For this purpose, several attributes L_a, L_b, ..., L_c are combined into a single new attribute (granule) N_k, which will be named as the composite indicator or composite criterion. Aggregation of several attributes into a composite indicator is the transformation (6) of the form

$$Y_a \cup Y_b \cup \ldots \cup Y_c \to Z_k, \quad (12)$$

where $Y_j = \{y_j^1, \ldots, y_j^{g_i}\}$ is the scale of the initial attribute L_j, $j = a, b, ..., c$; $Z_k = \{z_k^1, \ldots, z_k^{fk}\}$ is the scale of the composite indicator N_k, $k = 1,...,l$, $|Z_k| = f_k \le g_j = |Y_j|$.

The sets of composite indicators and their scales can be formed using different methods for granulation (12), which make it possible to represent each gradation of the composite indicator scale as a combination of gradations of the initial attribute estimates. It is recommended to combine two to four initial attributes in a composite indicator with a small scale of two to four gradations. In practical problems, it is convenient to form the scales of the combined attributes and of the composite indicator so that they have the same number of gradations. i.e., so that $g_a = g_b =...= g_c = f_k = d$, and each gradation of the scale for the composite indicator should consist of similar gradations of the scales for the combined attributes.

The representation of multiattribute objects is transformed as follows. Let in the space of the initial attributes $L_1,...,L_m$ the object O_p, $p = 1,...,q$ be given by the multiset

$$\begin{aligned}\boldsymbol{I}_p = &\left\{k_{\boldsymbol{I}p}\left(y_1^1\right)\circ y_1^1, \ldots, k_{\boldsymbol{I}p}\left(y_1^d\right)\circ y_1^d; \ldots;\right. \\ &\left. k_{\boldsymbol{I}p}\left(y_m^1\right)\circ y_m^1, \ldots, k_{\boldsymbol{I}p}\left(y_m^d\right)\circ y_m^d\right\}\end{aligned} \quad (13)$$

over the set $Y_1 \cup ... \cup Y_m$ of scales gradation where all scales $Y_j = \{y_j^1, \ldots, y_j^d\}$, $j = 1,...,m$ have the same number of gradations d. Taking the fact into account that the order of the elements in the multiset is insignificant [9, 10], we rewrite expression (13) represented as sums of multisets i.e.,

$$\begin{aligned}\boldsymbol{I}_p &= \boldsymbol{I}_{p1} + \ldots + \boldsymbol{I}_{pd} \\ &= \left\{k_{\boldsymbol{I}p}\left(y_1^1\right)\circ y_1^1, \ldots, k_{\boldsymbol{I}p}\left(y_m^1\right)\circ y_m^1\right\} \\ &+ \ldots + \left\{k_{\boldsymbol{I}p}\left(y_1^d\right)\circ y_1^d, \ldots, k_{\boldsymbol{I}p}\left(y_m^d\right)\circ y_m^d\right\} \\ &= \sum_{j=1}^{m}\left\{k_{\boldsymbol{I}p}\left(y_j^1\right)\circ y_j^1\right\} + \ldots + \sum_{j=1}^{m}\left\{k_{\boldsymbol{I}p}\left(y_j^d\right)\circ y_j^d\right\}.\end{aligned} \quad (14)$$

On aggregating the attributes in the reduced space of composite indicators $N_1,...,N_l$, the object O_p will be associated with the multiset

$$\begin{aligned}\boldsymbol{J}_p = &\left\{k_{\boldsymbol{J}p}\left(z_1^1\right)\circ z_1^1, \ldots, k_{\boldsymbol{J}p}\left(z_1^d\right)\circ z_1^d; \ldots;\right. \\ &\left. k_{\boldsymbol{J}p}\left(z_l^1\right)\circ z_l^1, \ldots, k_{\boldsymbol{J}p}\left(z_l^d\right)\circ z_l^d\right\}\end{aligned} \quad (15)$$

over the set $Z_1 \cup ... \cup Z_l$ of the scales gradation where all scales $Z_k = \{z_k^1, \ldots, z_k^d\}$, $k = 1,...,l$ have one and the same number d of gradations. Multiset $\boldsymbol{J}_p$ (15) can also be written in the following equivalent form:

$$\begin{aligned}\boldsymbol{J}_p &= \boldsymbol{J}_{p1} + \ldots + \boldsymbol{J}_{pd} \\ &= \left\{k_{\boldsymbol{J}p}\left(z_1^1\right)\circ z_1^1, \ldots, k_{\boldsymbol{J}p}\left(z_l^1\right)\circ z_l^1\right\} \\ &+ \ldots + \left\{k_{\boldsymbol{J}p}\left(z_1^d\right)\circ z_1^d, \ldots, k_{\boldsymbol{J}p}\left(z_l^d\right)\circ z_l^d\right\} \\ &= \sum_{k=1}^{l}\left\{k_{\boldsymbol{J}p}\left(z_k^1\right)\circ z_k^1\right\} + \ldots + \sum_{k=1}^{l}\left\{k_{\boldsymbol{J}p}\left(z_k^d\right)\circ z_k^d\right\}.\end{aligned} \quad (16)$$

The multiplicity of the element z_k^e, $e = 1,...,d$ in the multiset $\boldsymbol{J}_p$ (15) or (16), which corresponds to the gradation z_k^e of the scale Z_k for the composite indicator N_k, is determined by the rule:

$$k_{\boldsymbol{I}p}\left(z_k^e\right) = k_{\boldsymbol{I}p}\left(y_a^e\right) + k_{\boldsymbol{I}p}\left(y_b^e\right) + \ldots + k_{\boldsymbol{I}p}\left(y_c^e\right), \quad (17)$$

where multiplicities of the elements $y_a^e, y_b^e, \ldots, y_c^e$ of the multiset $\boldsymbol{I}_p$ (13) or (14), which correspond to gradations $y_a^e, y_b^e, \ldots, y_c^e$ of the scales Y_a, Y_b,..., Y^e of the combined attributes L_a, L_b,..., L_c, are summed.

Aggregation of attributes is carried out in stages, step by step. At each step, it is determined which initial attributes should be combined into composite indicators and which should be considered independent final ones. Verbal scales of composite indicators characterize the desired new properties of the objects being compared and have specific semantic content for the DM/expert. By sequentially combining attributes, the DM/expert constructs acceptable intermediate and final indicators. The aggregation tree of the attributes is built from blocks of the same type, which the

DM/expert selects, and in fact is a form of semantic interpretation and granulation of the DM's preferences and/or expert knowledge.

In practical situations of choosing real objects, it is recommended to construct several different schemes of the attribute union combining procedures for shortening the attribute scales and for aggregating them. This decreases the impact of each specific scheme and increases the validity of the obtained results. Depending on the specifics of the practical problem being solved, the last level of the attribute aggregation tree may consist of several final indicators that implement the idea of multicriteria choice or be the only integral indicator that implements the idea of holistic choice [7].

3. ILLUSTRATIVE EXAMPLE

Solving problems of multicriteria choice in reduced spaces of attributes require significantly less DM/expert's labor efforts and allows for a meaningful explanation of the choice made. Here, we show how the SOCRATES method works using the illustrative example from Section 1. Semestrial marks of ten pupils (the objects $O_1,\ldots, O_{10}$) in eight subjects (attributes $K_1,\ldots, K_8$) having their own five-point scales $X_i = \{x_i^1, x_i^2, x_i^3, x_i^4, x_i^5\}$, $i = 1,\ldots,8$, where x_i^1 is 1/very poor, x_i^2 is 2/poor, x_i^3 stands for 3/satisfactory, x_i^4 means 4/good, and x_i^5 is 5/excellent, are presented in Tables 1–3.

Two versions $O_p^{\langle 1\rangle}, O_p^{\langle 2\rangle}$ of the object O_p, $p = 1,\ldots,10$ specified by the vectors/tuples $\boldsymbol{x}_p^{\langle 1\rangle} = (x_{p1}^{\langle 1\rangle},\ldots, x_{p8}^{\langle 1\rangle})$, $\boldsymbol{x}_p^{\langle 2\rangle} = \left(x_{p1}^{\langle 2\rangle},\ldots, x_{p8}^{\langle 2\rangle}\right)$ are the points of an eight-dimensional attribute space $X_1\times\ldots\times X_8$. The length of each vector/tuple is 8, the components of vectors/tuples can take one of five values of the grade x_i^{ei}. The object O_p can be represented by the vector $\boldsymbol{x}_p = (x_{p1},\ldots, x_{p8})$ but it cannot be represented by a tuple. The total number of all possible combinations of components of vectors/tuples (representations of each object versions) is $5^8 = 390\,625$. Operating such a number of vectors/tuples is very difficult. In addition, almost all vectors/tuples, and hence the objects, will be incomparable.

Let us replace the five-point scales of the attributes $X_i = \{x_i^1, x_i^2, x_i^3, x_i^4, x_i^5\}$ by shortened three-point scales $Q_i = \{q_i^0, q_i^1, q_i^2\}$. Here, q_i^0 is 0/high grade, including grades x_i^5–5/excellent and x_i^4–4/good; q_i^1 is 1/middle grade corresponding to the grade x_i^3–3/satisfactory; and q_i^2 is 2/low grade, including grades x_i^2 –2/poor and x_i^1 –1/very poor. We note that if the initial grades were ordered by preference, for example as $x_i^5 \succ x_i^4 \succ x_i^3 \succ x^2 \succ x_i^1$, the new grades will also be ordered in the same way: $q_i^0 \succ q_i^1 \succ q_i^2$.

Then, the object O_p and its versions $O_p^{\langle 1\rangle}, O_p^{\langle 2\rangle}$, $p = 1,\ldots,10$ become tuples $\boldsymbol{q}_p = (q_{p1},\ldots,q_{p8})$, $\boldsymbol{q}_p^{\langle 1\rangle} = \left(q_{p1}^{\langle 1\rangle},\ldots,q_{p8}^{\langle 1\rangle}\right)$, $\boldsymbol{q}_p^{\langle 2\rangle} = \left(q_{p1}^{\langle 2\rangle},\ldots,q_{p8}^{\langle 2\rangle}\right)$, which, as above, are the points of the eight-dimensional attribute space $Q_1 \times\ldots\times Q_8$. The length of each tuple is still 8, but the components of the tuples can take one of three values of the grade $q_i^{o_i}$. The total number of all possible grades in subjects (representations of the object and its copies by components of the tuples) is equal to $3^8 = 6561$, which is almost 60 times less than 390 625, but is still great. At the same time, almost all tuples, and hence the objects, will remain incomparable.

Let us represent each object O_p by the multiset $\boldsymbol{A}_p$ (3) over the set $X = X_1\cup\ldots\cup X_8$ of attribute scale gradations $K_1, \ldots, K_8$. The versions $O_p^{\langle 1\rangle}, O_p^{\langle 2\rangle}$ of the object O_p are specified in the same way. The dimensionality of the attribute space equals $|X| = 5\cdot 8 = 40$. The total number of the possible grades in all subjects (representations of the object and its copies by elements of multisets) is equal to card $\boldsymbol{A}_p = \sum_{xi\in X} k_{\boldsymbol{A}p}\left(x_i^{ei}\right) = 16$, i.e., the cardinality of the multiset $\boldsymbol{A}_p$. Multisets and objects largely remain incomparable. However, it becomes easier to work with them.

On transition from five-point scales of attributes X_i to three-point scales Q_i, $i = 1,\ldots,8$, the object O_p will correspond to the multiset

$$\begin{gathered}\boldsymbol{B}_p = \left\{k_{\boldsymbol{B}p}\left(q_1^0\right)\circ q_1^0, k_{\boldsymbol{B}p}\left(q_1^1\right)\circ q_1^1,\right.\\ k_{\boldsymbol{B}p}\left(q_1^2\right)\circ q_1^2;\ldots;k_{\boldsymbol{B}p}\left(q_8^0\right)\circ q_8^0,\\ \left.k_{\boldsymbol{B}p}\left(q_8^1\right)\circ q_8^1, k_{\boldsymbol{B}p}\left(q_8^2\right)\circ q_8^2\right\}\end{gathered} \tag{18}$$

over the set $Q = Q_1\cup\ldots\cup Q_8$ of the shortened scale gradations of the attributes $K_1,\ldots,K_8$. Multiplicities of the elements multisets $\boldsymbol{B}_p$ (18) make up the rows of the Object–Attribute matrix H_0 (Table 4), which is a reduced (contracted) matrix H (Table 3), and are determined according to the rules (11):

$$\begin{gathered}k_{\boldsymbol{A}p}\left(q_i^0\right) = k_{\boldsymbol{A}p}\left(x_i^5\right) + k_{\boldsymbol{A}p}\left(x_i^4\right),\\ k_{\boldsymbol{B}p}\left(q_i^1\right) = k_{\boldsymbol{A}p}\left(x_i^3\right),\\ k_{\boldsymbol{B}p}\left(q_i^2\right) = k_{\boldsymbol{A}p}\left(x_i^2\right) + k_{\boldsymbol{A}p}\left(x_i^1\right).\end{gathered}$$

In particular, the object O_1 is given by the multiset

$$\begin{gathered}\boldsymbol{B}_1 = \left\{2\circ q_1^0, 0\circ q_1^1, 0\circ q_1^2; 2\circ q_2^0, 0\circ q_2^1, 0\circ q_2^2;\right.\\ 2\circ q_3^0, 0\circ q_3^1, 0\circ q_3^2; 2\circ q_4^0, 0\circ q_4^1, 0\circ q_4^2;\\ 2\circ q_5^0, 0\circ q_5^1, 0\circ q_5^2; 2\circ q_6^0, 0\circ q_6^1, 0\circ q_6^2;\\ \left.2\circ q_7^0, 0\circ q_7^1, 0\circ q_7^2; 2\circ q_8^0, 0\circ q_8^1, 0\circ q_8^2\right\}.\end{gathered}$$

Table 4. Object–Attribute matrix H_0 (shortened scales of the attributes)

$O\backslash Q$	$q_1^0\ q_1^1\ q_1^2$	$q_2^0\ q_2^1\ q_2^2$	$q_1^0\ q_3^1\ q_3^2$	$q_4^0\ q_4^1\ q_4^2$	$q_5^0\ q_5^1\ q_5^2$	$q_6^0\ q_6^1\ q_6^2$	$q_7^0\ q_7^1\ q_7^2$	$q_8^0\ q_8^1\ q_8^2$
$\boldsymbol{B}_1$	2 0 0	2 0 0	2 0 0	2 0 0	2 0 0	2 0 0	2 0 0	2 0 0
$\boldsymbol{B}_2$	1 1 0	0 0 2	0 0 2	0 0 2	1 1 0	0 1 1	0 1 1	0 0 2
$\boldsymbol{B}_3$	0 0 2	0 0 2	0 2 0	0 0 2	2 0 0	0 0 2	0 0 2	1 1 0
$\boldsymbol{B}_4$	2 0 0	1 1 0	0 1 1	2 0 0	2 0 0	2 0 0	1 1 0	2 0 0
$\boldsymbol{B}_5$	2 0 0	2 0 0	1 1 0	2 0 0	2 0 0	2 0 0	2 0 0	2 0 0
$\boldsymbol{B}_6$	2 0 0	2 0 0	2 0 0	2 0 0	2 0 0	2 0 0	2 0 0	2 0 0
$\boldsymbol{B}_7$	1 1 0	0 0 2	0 0 2	1 1 0	0 1 1	1 1 0	0 0 2	0 1 1
$\boldsymbol{B}_8$	2 0 0	2 0 0	2 0 0	0 1 1	1 1 0	2 0 0	2 0 0	1 1 0
$\boldsymbol{B}_9$	1 1 0	0 1 1	0 1 1	0 0 2	0 1 1	0 2 0	0 1 1	0 2 0
$\boldsymbol{B}_{10}$	1 1 0	2 0 0	1 1 0	2 0 0	0 1 1	2 0 0	1 0 1	2 0 0

Hence it is clear that over a year the pupil O_1 received two high marks (excellent and good) in all subjects: mathematics, physics, chemistry, biology, social science, history, literature, and foreign language.

The similar method is used to transform the versions $O_p^{\langle 1\rangle}, O_p^{\langle 2\rangle}$ of the object O_p. The dimensionality of the reduced attribute space is equal to $|Q| = 3{\cdot}8 = 24$, and the total number of grades in all subjects expressed by card $\boldsymbol{B}_p = \sum_{qi\in Q} k_{\boldsymbol{B}}\left(q_i^{e_i}\right) = 16$, i.e., the cardinality of the multiset $\boldsymbol{B}_p$ (18). On shortening the scales of the attributes, the dimensionality of the transformed space is decreased and the total number of grades in subjects remains unchanged. Multisets and the objects still remain incomparable. However, operations therewith are further simplified and facilitated.

Transition from scales X_i to the shortened scales Q_i will be considered zero aggregation scheme of the attributes. We construct different systems of indicators with various aggregation schemes for initial characteristics in order to represent the objects in reduced spaces of the attributes (Fig. 1). For simplicity, we assume that a scale of any new attribute has three gradations of estimates as the scale Q_i. Every gradation of the composite indicator scale includes combinations of the same gradations of estimates on the scales of the initial attributes.

According to the first aggregation scheme (Fig. 1a), all initial attributes K_1, ..., K_8, which have scales Q_i =

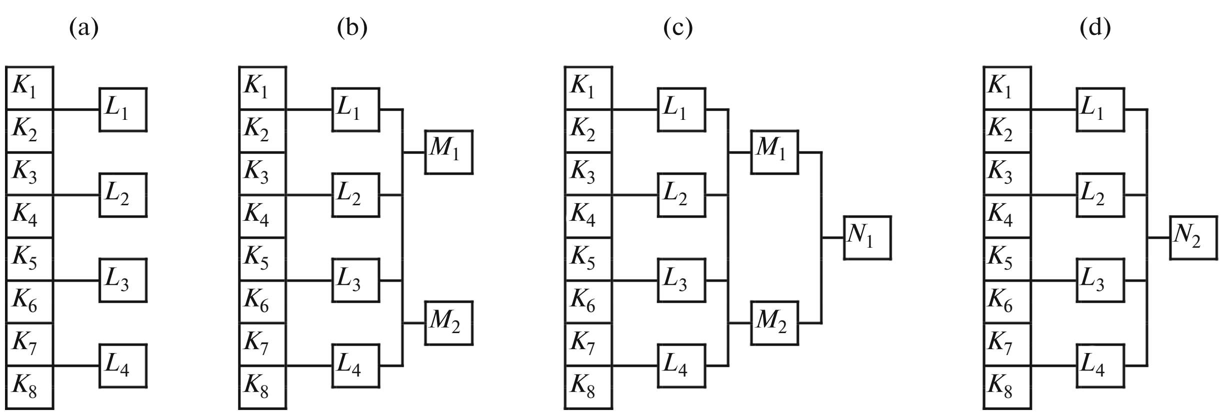

Fig. 1. Aggregation of initial characteristics in composite indicators: (a) first scheme; (b) second scheme; (c) third scheme; and (d) fourth scheme.

Table 5. Object—Attribute matrix H_1 (the first scheme of aggregation)

$O\backslash Y$	$y_1^0\ y_1^1\ y_1^2$	$Y_2^0\ y_2^1\ y_2^2$	$y_1^0\ y_3^1\ y_3^2$	$y_4^0\ y_4^1\ y_4^2$	$l_+(O_p)$	$s(O_p)$	$p(O_p)$	$B(O_p)$
C_1	4 0 0	4 0 0	4 0 0	4 0 0	0.000	48	1–2	25.5
C_2	1 1 2	0 0 4	1 2 1	0 1 3	0.700	24	9	1
C_3	0 0 4	0 2 2	2 0 2	1 1 2	0.684	25	8	4
C_4	3 1 0	2 1 1	4 0 0	3 1 0	0.211	43	4–5	16.6
C_5	4 0 0	3 1 0	4 0 0	4 0 0	0.059	47	3	21
C_6	4 0 0	4 0 0	4 0 0	4 0 0	0.000	48	1–2	26.5
C_7	1 1 2	1 1 2	1 2 1	0 1 3	0.565	27	7	7.5
C_8	4 0 0	2 1 1	3 1 0	3 1 0	0.211	43	4–5	16.5
C_9	1 2 1	0 1 3	0 3 1	0 3 1	0.556	27	10	5.5
C_{10}	3 1 0	3 1 0	2 1 1	3 0 1	0.253	41	6	12

$\{q_i^0, q_i^1, q_i^2\}$, are combined into composite indicators, which are considered final. The attributes K_1 Mathematics and K_2 Physics form the composite indicator. $L_1 = (K_1, K_2)$ Physical and mathematical subjects. The attributes K_3 Chemistry and K_4 Biology form the composite indicator $L_2 = (K_3, K_4)$ Chemical and biological subjects. The attributes K_5 Social science and K_6 History form the composite indicator $L_3 = (K_5, K_6)$ Sociohistorical subjects. The attributes K_7 Literature and K_8 Foreign language form the composite indicator $L_4 = (K_7, K_8)$ Philological subjects. The composite indicators $L_1,...,L_4$ have verbal scales $Y_j = \{y_j^0, y_j^1, y_j^2\}$, j = 1,2,3,4, with the gradation: y_j^0—0/high, including estimates q_a^0, q_c^0; y_j^1—1/middle, including estimates q_a^1, q_c^1; y_j^2—2/low, including estimates $q_a^2 q_c^2$. Here, $a = 1$, $c = 2$ for $j = 1$; $a = 3$, $c = 4$ for $j = 2$; $a = 5$, $c = 6$ for $j = 3$; $a = 7$, $c = 8$ for $j = 4$.

Each object O_p, $p = 1,...,10$ is represented by the multiset

$$\boldsymbol{C}_p = \{k_{\boldsymbol{C}p}(y_1^0)\circ y_1^0, k_{\boldsymbol{C}p}(y_1^1)\circ y_1^1, k_{\boldsymbol{C}p}(y_1^2)\circ y_1^2; \ldots; k_{\boldsymbol{C}p}(y_4^0)\circ y_4^0, k_{\boldsymbol{C}p}(y_4^1)\circ y_4^1, k_{\boldsymbol{C}p}(y_4^2)\circ y_4^2\} \quad (19)$$

over the set $Y = Y_1 \cup ... \cup Y_4$ of grades of estimates upon the indicators $L_1,...,L_4$. Multiplicities of the elements y_j^0, y_j^1, y_j^2 of multisets $\boldsymbol{C}_p$ are the rows of the Object—Attribute matrix H_1 (Table 5) and are determined by the rule (17) for forming the scales of composite indicators $L_1,...,L_4$ from the scales of the attributes $K_1,...,K_8$. In particular, the object O_1 is given by the multiset

$$\boldsymbol{C}_1 = \{4\circ y_1^0, 0\circ y_1^1, 0\circ y_1^2; 4\circ y_2^0, 0\circ y_2^1, 0\circ y_2^2; 4\circ y_3^0, 0\circ y_3^1, 0\circ y_3^2; 4\circ y_4^0, 0\circ y_4^1, 0\circ y_4^2\}$$

Hence, it is clear that over a year the pupil O_1 has received four high marks in physical and mathematical, chemical and biological, socio-historical, and philological subjects.

According to the second aggregation scheme (Fig. 1b), the first step is the same as in the first scheme. At the next step, the attributes L_1 Physical and mathematical subjects and L_2 Chemical and biological subjects form the composite indicator $M_1 = (L_1, L_2)$ Natural science subjects. The attributes L_3 Socio-historical subjects and L_4 Philological subjects form the composite indicator $M_2 = (L_3, L_4)$ Humanities. The composite indicators M_1, M_2 are considered final. They have verbal scales U_r, r = 1,2 with the gradations: u_r^0—0/high, including estimates y_b^0, y_d^0; u_r^1—1/middle, including estimates y_b^1, y_d^1; u_r^2—2/low, including estimates y_b^2, y_d^2. Here, $b = 1$, $d = 2$ for $r = 1$; $b = 3$, $d = 4$ for $r = 2$.

Each object O_p, $p = 1,...,10$ is represented by the multiset

$$\boldsymbol{D}_p = \{k_{\boldsymbol{D}p}(u_1^0)\circ u_1^0, k_{\boldsymbol{D}p}(u_1^1)\circ u_1^1, k_{\boldsymbol{D}p}(u_1^2)\circ u_1^2; k_{\boldsymbol{D}p}(u_2^0)\circ u_2^0, k_{\boldsymbol{D}p}(u_2^1)\circ u_2^1, k_{\boldsymbol{D}p}(u_2^2)\circ u_2^2\} \quad (20)$$

over the set $U = U_1 \cup U_2$ of grades of estimates upon the indicators M_1, M_2. Multiplicities of the elements

Table 6. Object–Attribute matrices

H$_2$ (second aggregation scheme)			H$_3$ (third aggregation scheme)		H$_4$ (fourth aggregation scheme)	
$O\backslash U$	$u_1^0\ u_1^1\ u_1^2$	$u_2^0\ u_2^1\ u_2^2$	$O\backslash Z$	$z_1^0\ z_1^1\ z_1^2$	$O\backslash Z$	$z_2^0\ z_2^1\ z_2^2$
$\boldsymbol{D}_1$	8 0 0	8 0 0	$\boldsymbol{E}_1$	16 0 0	$\boldsymbol{F}_1$	16 0 0
$\boldsymbol{D}_2$	1 1 6	1 3 4	$\boldsymbol{E}_2$	2 4 10	$\boldsymbol{F}_2$	2 4 10
$\boldsymbol{D}_3$	0 2 6	3 1 4	$\boldsymbol{E}_3$	3 3 10	$\boldsymbol{F}_3$	3 3 10
$\boldsymbol{D}_4$	5 2 1	7 1 0	$\boldsymbol{E}_4$	12 3 1	$\boldsymbol{F}_4$	12 3 1
$\boldsymbol{D}_5$	7 1 0	8 0 0	$\boldsymbol{E}_5$	15 1 0	$\boldsymbol{F}_5$	15 1 0
$\boldsymbol{D}_6$	8 0 0	8 0 0	$\boldsymbol{E}_6$	16 0 0	$\boldsymbol{F}_6$	16 0 0
$\boldsymbol{D}_7$	2 2 4	1 3 4	$\boldsymbol{E}_7$	3 5 8	$\boldsymbol{F}_7$	3 5 8
$\boldsymbol{D}_8$	6 1 1	6 2 2	$\boldsymbol{E}_8$	12 3 1	$\boldsymbol{F}_8$	12 3 1
$\boldsymbol{D}_9$	1 3 4	0 6 2	$\boldsymbol{E}_9$	1 9 6	$\boldsymbol{F}_9$	1 9 6
$\boldsymbol{D}_{10}$	6 2 0	5 1 2	$\boldsymbol{E}_{10}$	11 3 2	$\boldsymbol{F}_{10}$	11 3 2

u_r^0, u_r^1, u_r^2 of the multiset $\boldsymbol{D}_p$ are the rows of the Object–Attribute matrix H$_2$ (Table 6) and are determined by the rule (17) for forming the scales of composite indicators M_1, M_2 from the scales of the the attributes $L_1,\ldots,L_4$. Thus, the object O_1 is given by the multiset $\boldsymbol{D}_1 = \{8{\circ}u_1^0, 0{\circ}u_1^1, 0{\circ}u_1^2; 8{\circ}u_2^0, 0{\circ}u_2^1, 0{\circ}u_2^2\}$. Hence, it is clear that over a year the pupil O_1 received eight high marks in natural sciences and humanities.

According to the third aggregation scheme (Fig. 1c) the first and second steps are the same as in the second scheme. At the next step, the attributes M_1 Natural science subjects and M_2 Humanities form the final integral indicator $N_1 = (M_1, M_2)$ Academic progress, which has a verbal scale Z_1 with the gradations z_1^0–0/high, including estimates $u_1^0, u_2^0; z_1^1$–1/middle, including estimates $u_1^1, u_2^1; z_1^2$–2/low, including estimates u_1^2, u_2^2.

Each object O_p, $p = 1,\ldots,10$ is represented by the multiset

$$\boldsymbol{E}_p = \left\{k_{\boldsymbol{E}p}\left(z_1^0\right){\circ}z_1^0, k_{\boldsymbol{E}p}\left(z_1^1\right){\circ}z_1^1, k_{\boldsymbol{E}p}\left(z_1^2\right){\circ}z_1^2\right\} \quad (21)$$

over the set $Z_1 = \{z_1^0, z_1^1, z_1^2\}$ of the gradations of the attribute N_1. Multiplicities of the elements z_1^0, z_1^1, z_1^2 of the multiset $\boldsymbol{E}_p$ are the rows of the Object–Attribute matrix H$_3$ (Table 6) and are determined by the rule (17) for forming the scales of the composite indicator N_1 from the scales of the attributes M_1, M_2. In particular, the object O_1 is given by the multiset $\boldsymbol{E}_1 = \{16{\circ}z_1^0, 0{\circ}z_1^1, 0{\circ}z_1^2\}$. Hence, it is clear that over a year the pupil O_1 received 16 high marks in all subjects.

According to the fourth aggregation scheme (Fig. 1d) the first step is the same as in the first scheme. At the next step, attributes L_1 Physical and mathematical subjects, L_2 Chemical and biological subjects, L_3 Socio-historical subjects, and L_4 Philological subjects together are combined in the final integral indicator $N_2 = (L_1, L_2, L_3, L_4)$ Academic progress, which has a verbal scale Z_2 with the gradations: z_2^0–0/high, including estimates $y_1^0, y_2^0, y_3^0, y_4^0; u_2^1$–1/middle, including estimates $y_1^1, y_2^1, y_3^1, y_4^1; z_2^2$–2/low, including estimates $y_1^2, y_2^2, y_3^2, y_4^2$.

Each object O_p, $p = 1,\ldots,10$ is represented by the multiset

$$\boldsymbol{F}_p = \left\{k_{\boldsymbol{F}p}\left(z_2^0\right){\circ}z_2^0, k_{\boldsymbol{F}p}\left(z_2^1\right){\circ}z_2^1, k_{\boldsymbol{F}p}\left(z_2^2\right){\circ}z_2^2\right\} \quad (22)$$

over the set $Z_2 = \{z_2^0, z_2^1, z_2^2\}$ containing the gradations of the attribute N_2. Multiplicities of the elements z_2^0, z_2^1, z_2^2 of the multiset $\boldsymbol{F}_p$ are the rows of the Object–Attribute matrix H$_4$ (Table 6) and are determined by the rule (17) for forming the scales of the composite indicator N_1 from the scales of the attributes L_1, L_2, L_3, L_4. In particular, the object O_1 is given by the multiset $\boldsymbol{F}_1 = \{16\circ z_2^0, 0\circ z_2^1, 0\circ z_2^2\}$. Hence, it is clear that over a year the pupil O_1 received 16 high marks in all subjects.

The aggregation of the indicators can also be carried out in a different way. For instance, the attributes K_1 Mathematics, K_2 Physics, K_3 Chemistry, and K_4 Biology form the composite indicator $M_3 = (K_1, K_2, K_3, K_4)$ Natural science subjects. The attributes K_5 Social science, K_6 History, K_7 Literature, and K_8 Foreign language form the composite indicator $M_4 = (K_5, K_6, K_7, K_8)$ Humanities. The composite indicators M_3 and M_4 can either be considered final indicators or be further combined into an integral indicator $N_3 = (M_3, M_4)$ Academic progress. Other options for aggregating indicators are also possible. When forming aggregation schemes, it is advisable to combine the initial indicators in a composite indicator in such a way that it has an understandable meaning, and the gradations of its scale consist of a small number of combinations of initial gradations.

Thus, on transition from initial data to the last schemes for the attribute aggregation, the dimensionality of transformed spaces sequentially decreases from 40 to 24, 12, 6, 3, the total number of grades in all subjects expressed by the cardinality of multisets $\boldsymbol{A}_p$ (3), $\boldsymbol{B}_p$ (18), $\boldsymbol{C}_p$ (19), $\boldsymbol{D}_p$ (20), $\boldsymbol{E}_p$ (21), $\boldsymbol{F}_p$ (22) does not change.

Five constructed schemes for the indicator aggregation can be treated as judgments of five independent experts. In this case, any problem of multicriteria choice becomes a collective choice problem, which is solved in various reduced spaces of attributes, and in each space, in addition, by means of several different methods. This ensures a greater validity of the final results.

For illustration, we present the results of ranking the objects O_1, ..., O_{10} by their properties, which were obtained using the PAKS-M technology of multicriteria choice in the attribute space of high dimensionality [10, 11]. First, for each attribute aggregation scheme, collective rankings of the objects were constructed by three methods of group selection: ARAMIS, weighted sum of estimates, and lexicographic ordering [7, 10].

The ARAMIS method enables ranking multiattribute objects, assessed by several experts upon many quantitative and/or qualitative criteria K_1, ..., K_n, without constructing individual rankings of the objects. The objects are ordered in the metric space of multisets by a value of the proximity index $l_+(O_p)$ of the object O_p to the best (possibly hypothetical) object O_+, which has the highest estimates with regard to all criteria according to the judgments of all experts.

The method of weighted sum of estimates makes it possible to rank multiattribute objects by the values of their value function. The value of the object O_p is given by the sum $s(O_p)$ of the products of the number of estimate gradation by the weight of the gradation. In the example above, the high gradation was assigned weight 3, the average gradation had weight 2, and the low gradation had weight 1.

The lexicographic ordering method allows ranking multiattribute objects according to the total number of corresponding estimate gradations. The ranking position $p(O_p)$ of the object O_p is determined first by the number of high grades, then by the number of average grades, then by the number of low grades etc.

For all five schemes for attribute aggregation, the results of data processing by each of the above-mentioned methods proved to be similar. They are presented in Table 5. In other words, the judgments of all five independent experts, based on any of these methods coincided. This resulted from the additivity of rules (11) and (17) for transforming the scales of the attributes. The collective rankings of the objects obtained according to any scheme by the methods ARAMIS $R_{\mathrm{A}}{}^{gr}$, weighted sum of estimates $R_{\Sigma}{}^{gr}$, and the lexicographic ordering $R_{\Lambda}{}^{gr}$, are as follows:

$$R_{\mathrm{A}}^{gr} \Leftrightarrow [O_1, O_6 \succ O_5] \succ [O_4, O_8 \succ O_{10}] \succ [(O_9 \succ O_7) \succ O_3 \succ O_2],$$
$$R_{\Sigma}^{gr} \Leftrightarrow [O_1, O_6 \succ O_5] \succ [O_4, O_8 \succ O_{10}] \succ [O_7, O_9 \succ (O_3 \succ O_2)],$$
$$R_{\Lambda}^{gr} \Leftrightarrow [O_1, O_6 \succ O_5] \succ [O_4, O_8 \succ O_{10}] \succ [O_7 \succ O_3 \succ O_2 \succ O_9].$$

Rankings of the objects using the methods ARAMIS, weighted sum of estimates, and lexicographic ordering can also be interpreted as judgments of some other three experts. Let us combine the opinions of these experts using the Borda method of voting [7], according to which the order of the objects is given by the sum $b(O_p)$ of the Borda scores in the corresponding rankings (Table 5). Generalized group ranking of the objects combining the collective rankings $R_{\mathrm{A}}^{gr}, R_{\Sigma}^{gr}, R_{\Lambda}^{gr}$ has the form

$$R_{\mathrm{B}}^{gr} \Leftrightarrow [O_1, O_6 \succ O_5] \succ [O_4, O_8 \succ O_{10}] \succ [O_7 \succ (O_9 \succ O_3) \succ O_2].$$

Closed objects are enclosed in round brackets, distant groups of objects are enclosed in square brackets.

Thus, the final orderings of the objects obtained in different ways or, which is the same, the collective preferences of many different groups of experts (time periods, aggregation schemes of the attributes, selection methods) almost completely coincide, with the exception of minor differences in the location of the objects in the last group. In all rankings, there are similar groups of good objects O_1, O_6, O_5 with high estimates, middling objects O_4, O_8, O_{10} with middle estimates, and almost coinciding groups of bad objects O_7, O_9, O_3, O_2 with low estimates. According to the aggregated estimates of all experts, the best objects by all features are O_1, O_6, occupied the first place in all rankings. The worst is the object O_2, occupied the last place in three rankings and next to the last place in one ranking. There are clear gaps between the groups of good objects, middling objects, and bad objects. Therefore, we can also consider the grouped ordering of objects as the grouped ordinal classification, where the classes of the objects and the positions of the objects in the classes are given by the corresponding rankings. Exactly the same results for the same illustrative example were obtained in [10] when ranking of the objects $O_1,...,O_{10}$ using a different method for reduction of the attribute space dimensionality.

CONCLUSIONS

The proposed SOCRATES method for reduction of the attribute space dimensionality has a certain universality since it allows one to operate simultaneously with symbolic (qualitative) and numerical (quantitative) data. An attractive feature of the method is that it can be used in combination with various decision-making methods and information processing technologies. And most importantly, the initially available information is not distorted or lost.

The SOCRATES method is easily integrated into the new technologies PAKS [10, 12,13] and PAKS-M [10, 11] for solving multicriteria choice problems in high dimensionality spaces, which provide a better substantiation for choosing the most preferable object. These technologies have important features. On applying them, several schemes with different options for the attribute aggregation are formed, in which the gradations of the composite indicator scale are represented as combinations of gradations of the initial attributes. The posed problem is solving by several methods of multicriteria choice. The DM/expert is given a clear understandable explanation of the obtained results, which helps him to choose the most suitable scheme of the attribute aggregation or apply several schemes together.

When solving the problem of multicriteria choice, the DM/expert may encounter inconsistency and controversy of the obtained results. Such situations are caused by various reasons, in particular, the formal combination of the attributes or the unsuccessful formation of the scales for composite attribute gradations and the integral indicator. The establishment of semantic links between the initial attributes and composite indicators plays an important role in constructing the attribute trees.

Technologies for solving problems of multicriteria choice in spaces of high dimensionality were used in assessing the progress of scientific research, evaluating the effectiveness of activities, rating various organizations, and choosing a prospective computing complex [10–13]. Applying the new SOCRATES method will significantly reduce the complexity and time of solving similar practical problems.

FUNDING

This work was supported by the Russian Foundation for Basic Research (project nos. 17-29-07021, 18-07-00132, 18-07-00280, and 19-29-01047).

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Translated by I. Pertsovskaya