Features of the Thermocapillary Drift for a Heated Droplet in a Viscous Fluid Placed into an Electromagnetic Field

Yu. I. Yalamov*, N. V. Malaĭ, and E. R. Shchukin*****

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Study of the motion of a droplet in an electromagnetic field is an important and urgent problem [1, 2]. This motion is caused by a nonuniform distribution of temperature along the droplet surface. In this case, additional tangential stresses appear owing to the temperature dependence of the droplet surface-tension coefficient; these stresses are responsible for the ordered motion of the droplet. The nonuniform distribution of temperature can be induced by various factors, for example, by an external constant gradient of temperature [3, 4], a chemical reaction on the droplet surface [5], the presence of surface-active substances in a fluid [6], etc. If the droplet moves due to the nonuniform distribution of inner heat sources, this motion is called photophoretic [7].

In the past few years, interest in the droplet motion for considerable temperature drops in their neighborhood has grown [8–10]. In this paper, in contrast to previous studies, we took into account the exponential temperature dependence of the coefficient of dynamic viscosity in the thermocapillary drift of a droplet and the influence of fluid motion on the temperature distribution.

The analysis carried out in this work showed that, along with the temperature dependence of the coefficient of dynamic viscosity, the convective transport can also substantially influence the thermocapillary drift of droplets heated by inner heat sources. In particular, it was shown that, if the droplets absorb radiation as a blackbody, two qualitatively different motions of the particle are possible: in the direction of propagation of radiation and in the opposite direction. This circumstance is caused by a marked influence of the convective motion of fluid (large Prandtl numbers) on the angular nonuniformity of the temperature distribution in the neighborhood of a droplet for significant radial temperature drops.

* *Moscow Pedagogical University, ul. Radio 10a, Moscow, 105007 Russia*

** *Belgorod State University, Belgorod, Russia*

*** *Institute of High Temperatures Scientific Association (IVTAN), Russian Academy of Sciences, Izhorskaya ul. 13/19, Moscow, 127412 Russia*

PROBLEM FORMULATION

We consider the steady motion of a nonuniformly heated spherical droplet of radius R, density ρ_i, and heat conductivity λ_i in an immiscible viscous incompressible fluid with a density ρ_e and a heat conductivity λ_e filling the whole space. The fluid is at rest at infinity. As a heated particle, we understand that the particle's mean surface temperature considerably exceeds the environment temperature.

The heated surface of the droplet can have a substantial effect on the thermal characteristics of the environment and, thus, on the distribution of velocity fields and pressure in its neighborhood.

Among the parameters of fluid transport, only the viscosity coefficient depends strongly on temperature. Taking the temperature dependence of viscosity into account, we used formula (1) proposed in [3] (for $F_n = 0$, this formula can be reduced to the Reynolds formula [11]):

$$\mu_e = \mu_\infty\left[1 + \sum_{n=1}^{\infty} F_n\left(\frac{T_e}{T_\infty} - 1\right)^n\right]\exp\left\{-A\left(\frac{T_e}{T_\infty} - 1\right)\right\}. \quad (1)$$

Here, A and F_n are constants, T_∞ is the temperature of the fluid far from the heated droplet, $\mu_\infty = \mu_e(T_\infty)$; hereafter, the subscripts e and i refer to the external fluid and the droplet, respectively.

The fluid viscosity is known to decrease with temperature according to the exponential law [11]. Analysis of the available semiempirical formulas showed that expression (1) makes it possible to best describe the change in viscosity in a wide range of temperatures with an arbitrary desired accuracy. For illustration, we list in Table 1 the values of F_n for water ($A = 5.779$, $F_1 = -2.318$, $F_2 = 9.118$, and $T_\infty = 273$ K); μ_{calcd} is the dynamic viscosity calculated from formula (1) and μ_{expt} is the experimental value of the dynamic viscosity. The relative error is less than 2%.

The heat-conductivity coefficient of a droplet is assumed to considerably exceed the heat-conductivity coefficient of the medium; the density (ρ), heat capacity (c_p), and heat conductivity (λ) are considered to be constant values; droplet motion is reasonably slow (small Peclet and Reynolds numbers); the surface-tension coefficient σ is an arbitrary function of temperature [$\sigma = \sigma(T)$]; and the droplet is assumed to retain its spherical shape (this assumption is valid under the condition $\frac{\mu_e U}{R} \ll \frac{\sigma}{R}$, where U is the droplet drift velocity [12]).

It is convenient to introduce a reference system related to the center of the moving droplet. In this case, the problem is reduced to analysis of the steady flow around the droplet by a homogeneous fluid whose velocity at infinity ($\mathbf{U}_\infty$) is to be defined ($\mathbf{U}_\infty = -\mathbf{U}$).

In terms of the above assumptions describing this flow, the dimensionless conservation equations and boundary conditions can be reduced to the form [12]

$$\eta_i \Delta \mathbf{V}_i = \nabla p_i, \quad \operatorname{div}\mathbf{V}_i = 0, \quad \eta = \frac{\mu}{\mu_\infty}; \tag{2}$$

$$\begin{gathered} \nabla p_e = \eta_e \Delta \mathbf{V}_e + 2(\nabla \eta_e \nabla)\mathbf{V}_e + [\nabla \eta_e \times \operatorname{rot}\mathbf{V}_e], \\ \operatorname{div}\mathbf{V}_e = 0; \end{gathered} \tag{3}$$

$$\mathrm{Re}_\infty \mathrm{Pr}_\infty (\mathbf{V}_e \nabla) t_e = \Delta t_e, \quad \Delta t_i = -\frac{q_i R^2}{\lambda_i T_\infty}; \tag{4}$$

$$\begin{gathered} y = 1,\ \lambda_i \frac{\partial t_i}{\partial y} = \lambda_e \frac{\partial t_e}{\partial y},\ V_r^e = V_r^i = 0,\ V_\theta^e = V_\theta^i, \\ \mu_e \left[\frac{\partial V_\theta^e}{\partial y} + \frac{1}{y}\frac{\partial V_r^e}{\partial \theta} - \frac{V_\theta^e}{y} \right] + \frac{1}{y U_\infty} \frac{\partial \sigma}{\partial t_i} \frac{\partial t_i}{\partial \theta} \\ = \mu_i \left[\frac{\partial V_\theta^i}{\partial y} + \frac{1}{y}\frac{\partial V_r^i}{\partial \theta} - \frac{V_\theta^i}{y} \right]; \end{gathered} \tag{5}$$

$$\begin{gathered} y \longrightarrow \infty, \quad \mathbf{V}_e \longrightarrow \cos\theta\, \mathbf{e}_r - \sin\theta\text{""}\mathbf{e}_\theta, \\ t_e \longrightarrow 1, \quad p_e \longrightarrow 1; \end{gathered} \tag{6}$$

$$y \longrightarrow 0, \quad |\mathbf{V}_i| \neq \infty, \quad t_i \neq \infty, \quad p_i \neq \infty. \tag{7}$$

Here, V_r and V_θ are the radial and tangential components of the mass velocity, while $\mathbf{e}_r$ and $\mathbf{e}_\theta$ are the unit vectors in the spherical system of coordinates, respectively; $y = \frac{r}{R}$ is the dimensionless radial coordinate; $\mathrm{Re}_\infty = \frac{\rho_e U_\infty R}{\mu_\infty}$ and $\mathrm{Pr}_\infty = \frac{\mu_\infty c_p}{\lambda_e}$ are the Reynolds and Prandtl numbers, respectively; and $U_\infty = |\mathbf{U}_\infty|$.

Table 1

T, K	μ_{calcd}, Pa s	μ_{expt}, Pa s	$\frac{\lvert\mu_{\text{calcd}} - \mu_{\text{expt}}\rvert}{\mu_{\text{calcd}}} \times 100$, %
279	0.0017525	0.0017525	0.00
293	0.0010089	0.0010015	0.74
313	0.0006433	0.0006513	1.22
333	0.0004581	0.0004630	1.06
353	0.0003556	0.0003509	1.35
363	0.0003199	0.0003113	2.76

The variables are made dimensionless by using the following characteristic values: R (the droplet radius), T_∞, P_∞, μ_∞, and U_∞ $\left(t = \frac{T}{T_\infty}, p = \frac{P}{P_\infty}, \text{ and } \mathbf{V} = \frac{\mathbf{U}}{U_\infty} \right)$.

For $\mathrm{Re}_\infty \ll 1$, the incoming flow exerts only a perturbing action. Therefore, the solution to the equations of hydrodynamics and heat transfer can be sought in the form

$$\begin{aligned} \mathbf{V} &= \mathbf{V}^{(0)} + \mathrm{Re}_\infty \mathbf{V}^{(1)} + \ldots, \\ p &= p^{(0)} + \mathrm{Re}_\infty p^{(1)} + \ldots, \\ t &= t^{(0)} + \mathrm{Re}_\infty t^{(1)} + \ldots. \end{aligned} \tag{8}$$

When finding the force acting on a nonuniformly heated droplet and its thermocapillary-drift velocity, we restrict our consideration to first-order corrections with respect to Re_∞.

The form of boundary conditions (5)–(7) makes it possible to seek the solution as follows:

$$\begin{gathered} V_r^{(0)} = G(y)\cos\theta, \quad V_\theta^{(0)} = -g(y)\sin\theta, \\ p^{(0)} = 1 + h(y)\cos\theta. \end{gathered} \tag{9}$$

Taking into account the inequality $\lambda_e \ll \lambda_i$, we can ignore the dependence of the coefficient of dynamic viscosity on the angle θ in the droplet–fluid system and assume that $\mu_e(t_e(y, \theta)) = \mu_e(t_e^{(0)})$. Using this fact and substituting (8), (9) into Eqs. (2)–(4), we make sure that the variables are separated and obtain, as a result, linear partial differential equations for perturbed values. In finding the distribution of temperature in the vicinity of a heated droplet, we used the method of joining asymptotic expansions [8]. As a result, the following expressions were obtained for the velocity fields and the temperatures outside and inside a particle:

$$V_r^e(y, \theta) = \cos\theta(1 + A_1 G_1 + A_2 G_2),$$

$$V_\theta^e(y,\theta) = -\sin\theta(1 + A_1G_3 + A_2G_4),$$

$$V_r^i(y,\theta) = \cos\theta(A_3 + A_4y^2),$$

$$V_\theta^i(y,\theta) = -\sin\theta(A_3 + 2A_4y^2),$$

$$t_e(y,\theta) = t_e^{(0)} + \mathrm{Re}_\infty t_e^{(1)}, \quad t_i(y,\theta) = t_i^{(0)} + \mathrm{Re}_\infty t_i^{(0)},$$

where

$$G_1 = -\frac{1}{y^3}\sum_{n=0}^{\infty}\frac{\Delta_n^{(1)}}{(n+3)y^n}, \quad G_3 = G_1 + \frac{y}{2}G_1^{\mathrm{I}},$$

$$G_4 = G_2 + \frac{y}{2}G_2^{\mathrm{I}},$$

$$G_2 = -\frac{1}{y}\sum_{n=0}^{\infty}\frac{\Delta_n^{(2)}}{(n+1)y^n}$$

$$-\frac{\alpha}{y^3}\sum_{n=0}^{\infty}\left[(n+3)\ln\frac{1}{y} - 1\right]\frac{\Delta_n^{(1)}}{(n+3)^2y^n},$$

$$t_e^{(1)}(y,\theta) = \frac{\omega}{2y}(1-y) + \left\{\frac{\Gamma}{y^2} + \omega\sum_{k=1}^{3}A_k\tau_k\right\}\cos\theta,$$

$$t_e^{(0)}(y) = 1 + \frac{\gamma}{y},$$

$$t_i^{(0)}(y) = B_0 + \frac{1}{4\pi RT_\infty\lambda_i y}\int_V q_i dV + \int_1^y\frac{\psi_0}{y}dy - \frac{1}{y}\int_1^y\psi_0 dy,$$

$$\omega = \gamma\mathrm{Pr}_\infty, \tag{10}$$

$$t_i^{(1)}(y) = By + \frac{1}{4\pi R^2T_\infty\lambda_i y^2}\int_V q_i z dV$$

$$+\frac{1}{3}\left[y\int_1^y\frac{\psi_1}{y^2}dy - \frac{1}{y^2}\int_1^y\psi_1 y dy\right],$$

$$\tau_1(y) = -\frac{1}{y^3}\sum_{n=0}^{\infty}\frac{\Delta_n^{(1)}}{(n+1)(n+3)(n+4)y^n},$$

$$\tau_3 = \frac{1}{2}, \quad A_3 = 1,$$

$$\tau_2(y) = -\frac{1}{y}\left\{-\frac{1}{2} + \frac{\Delta_1^{(2)}}{6y}\ln y - \sum_{n=2}^{\infty}\frac{\Delta_n^{(2)}}{(n^2-1)(n+2)y^n}\right.$$

$$-\frac{\alpha}{y^2}\sum_{n=0}^{\infty}\left[(n+1)(n+3)(n+4)\ln\frac{1}{y} - 3n^2 - 16n - 19\right]$$

$$\left.\times\frac{\Delta_n^{(1)}}{(n+1)^2(n+3)^2(n+4)^2y^n}\right\},$$

$$\psi_n(y) = -\frac{R^2}{\lambda_i T_\infty}y^2\frac{2n+1}{2}\int_{-1}^{+1}q_iP_n(\cos\theta)d(\cos\theta),$$

$P_n(\cos\theta)$ are the Legendre polynomials, $\gamma = t_s - 1$, $t_s = \frac{T_s}{T_\infty}$, and T_s is the mean temperature of the heated droplet surface determined by the formula

$$\frac{T_s}{T_\infty} = 1 + \frac{1}{4\pi R\lambda_i T_\infty}\int_V q_i dV. \tag{11}$$

In (10), G_k^{I}, G_k^{II}, and G_k^{III} are the first, second, and third derivatives of the corresponding functions with respect to y ($k = 1, 2$). The values of the coefficients $\Delta_n^{(1)}$ and $\Delta_n^{(2)}$ can be obtained using the following recurrence relations:

$$\Delta_n^{(1)} = -\frac{1}{n(n+5)}\sum_{k=1}^{n}[(n+4-k)$$

$$\times\{\alpha_k^{(1)}(n+5-k) - \alpha_k^2\} + \alpha_k^{(3)}]\gamma^k\Delta_{n-k}^{(1)} \quad (n \ge 1),$$

$$\Delta_n^{(2)} = -\frac{1}{(n+3)(n-2)}\left[-6\alpha_n^{(4)}\gamma^n\right. \tag{12}$$

$$+\sum_{k=1}^{n}\{(n+2-k)[(n+3-k)\alpha_k^{(1)} - \alpha_k^{(2)}] + \alpha_k^{(3)}\}\gamma^k\Delta_{n-k}^{(2)}$$

$$\left.+\alpha\sum_{k=0}^{n}\{(2n+5-2k)\alpha_k^{(1)} - \alpha_k^{(2)}\}\gamma^k\Delta_{n-k-2}^{(1)}\right] \quad (n \ge 3).$$

When calculating the coefficients $\Delta_n^{(1)}$ and $\Delta_n^{(2)}$ from formulas (12), it is necessary to take into account the following equalities:

$$\Delta_0^{(1)} = -3, \quad \Delta_0^{(2)} = -1, \quad \Delta_2^{(2)} = 1, \quad \alpha_0^{(3)} = -4,$$

$$\alpha_n^{(1)} = F_n, \quad \alpha_0^{(2)} = 4, \quad \alpha_0^{(1)} = \alpha_0^{(4)} = 1,$$

$$\alpha_n^{(2)} = (4-n)F_n + AF_{n-1}, \quad \alpha_n^{(4)} = \frac{A^n}{n!},$$

$$\alpha = -\frac{\gamma}{15}\{[3(4\alpha_1^{(1)} - \alpha_1^{(2)}) + \alpha_1^{(3)}]\Delta_1^{(2)}$$

$$- [2(3\alpha_2^{(1)} - \alpha_2^{(2)}) + \alpha_2^{(3)}]\gamma - 6\alpha_2^{(4)}\gamma\},$$

$$\alpha_n^{(3)} = 2AF_{n-1} - 2(2+n)F_n,$$

$$\Delta_1^{(2)} = -\frac{\gamma}{4}[6\alpha_1^{(4)} + 2(3\alpha_1^{(1)} - \alpha_1^{(2)}) + \alpha_1^{(3)}].$$

The integration constants $A_1, A_2, A_3, A_4, B_0, B$, and Γ are determined from the corresponding boundary conditions on the droplet surface.

Our prime interest is the solution for the asymmetric part of perturbed values, which will enable us to determine the force and velocity of the thermocapillary-drift. For this purpose, we specify the nature of the thermal sources. The heating of a particle is assumed to take place through the absorption of electromagnetic radiation, and the droplet absorbs the radiation as a blackbody. In this case, the radiation is absorbed in a thin layer of thickness $\delta R \ll R$ adjoining the heated area of the particle surface. The thermal-source density within the layer of thickness δR is determined from the following formula:

$$q_i(r, \theta) = \begin{cases} -\dfrac{I}{\delta R}\cos\theta, & \dfrac{\pi}{2} \le \theta \le \pi, \quad R - \delta R \le r \le R \\ 0, & 0 \le \theta \le \dfrac{\pi}{2}, \end{cases}$$

where I is the incident radiation intensity.

The expression for the total force acting on the particle is obtained by integrating the stress tensor over the droplet surface. This expression is made up of the viscous force $\mathbf{F}_\mu$ and the force $\mathbf{F}_{\text{ph}}$, whose appearance is caused by the nonuniformity of the distribution of thermal-source density in the body of the particle with allowance for the convective terms in the heat-conductivity equation. In the general case, these expressions can be represented in the form

$$\mathbf{F} = \mathbf{F}_\mu + \text{Re}_\infty \mathbf{F}_{\text{ph}}, \tag{13}$$

where

$$\mathbf{F}_\mu = 6\pi R\mu_\infty U_\infty f_\mu \mathbf{e}_z, \quad \mathbf{F}_{\text{ph}} = -6\pi R\mu_\infty f_{\text{ph}}\mathbf{e}_z,$$

$$f_\mu = \frac{2}{3\Delta}\left(N_3 + \frac{\mu_e^s}{3\mu_i^s}N_4\right)\exp\{-A\gamma\},$$

$$\delta = 1 + 2\frac{\lambda_e^s}{\lambda_i^s}, \quad V = \frac{4}{3}\pi R^3,$$

$$\Delta = N_1 + \frac{\mu_e^s}{3\mu_i^s} - \frac{2\rho_e}{3\mu_i^s}\frac{\omega}{\delta\mu_\infty}\frac{\lambda_e^s}{\lambda_i^s}(G_1\Phi_2 - G_2\Phi_1)\frac{\partial\sigma}{\partial t_i},$$

$$\Phi_k = 2\tau_k + \tau_k^{\text{I}}, \quad k = 1, 2,$$

$$f_{\text{ph}} = \frac{4}{9\mu_i^s\Delta}\exp\{-A\gamma\}\frac{G_1}{\lambda_i^s\delta}\xi_{\text{ph}}\frac{\partial\sigma}{\partial t_i},$$

$$\xi_{\text{ph}} = \omega\lambda_e^s\left(1 - \frac{\Phi_1}{G_1}\right) - \frac{RI}{2T_\infty},$$

and $\mathbf{e}_z$ is the unit vector along the z-axis.

In estimating the coefficients f_μ and f_{ph}, it is necessary to take into account that the subscript s designates values of physical quantities taken at a mean droplet-surface temperature T_s, which is determined from formula (11); the functions $\Phi_1, \Phi_2, G_1, G_2, N_1, N_2, N_3$, and N_4 are taken for $y = 1$ [$N_1 = G_1G_2^{\text{I}} - G_2G_1^{\text{I}}$, $N_2 = G_2(2G_1^{\text{I}} + G_1^{\text{II}}) - G_1(2G_2^{\text{I}} + G_2^{\text{II}})$, $N_3 = -G_1^{\text{I}}$, and $N_4 = 2G_1^{\text{I}} + G_1^{\text{II}}$].

In the case when droplet-surface heating is reasonably weak, i.e., when the mean droplet-surface temperature differs insignificantly from the environment temperature at infinity ($\gamma \longrightarrow 0$), the temperature dependence of the viscosity coefficient can be ignored. In this case, $G_1 = 1$, $G_1^{\text{I}} = -3$, $G_1^{\text{II}} = 12$, $G_2 = 1$, $G_2^{\text{I}} = -1$, $G_2^{\text{I}} = 1$, $G_2^{\text{II}} = 2$, $N_1 = 2$, $N_2 = 6$, $N_3 = 3$, $N_4 = 6$, $\tau_1 = -\frac{1}{4}$, $\tau_1^{\text{I}} = \frac{3}{4}$, $\tau_2 = \frac{1}{2}$, and $\tau_2^{\text{I}} = -\frac{1}{2}$.

Setting the total force equal to zero, we obtain the expression for the thermocapillary-drift velocity:

$$\mathbf{U} = -\text{Re}_\infty h_{\text{ph}}\mathbf{e}_z, \quad h_{\text{ph}} = \frac{f_{\text{ph}}}{f_\mu}. \tag{14}$$

Formulas (13) and (14) enable us to estimate the force acting on a spherical droplet heated by an electromagnetic field in a viscous fluid and its thermocapillary-drift velocity. These estimates are made for arbitrary temperature drops between the droplet surface and the region far from this surface with allowance for the temperature dependence of the viscosity coefficient represented in the form of an exponential series and for the influence of fluid motion on the droplet drift.

We consider the expression

$$\xi_{\text{ph}} = \gamma\text{Pr}_\infty\lambda_e^s\left(1 - \frac{\Phi_1}{G_1}\right) - \frac{RI}{2T_\infty}, \tag{15}$$

Table 2

ξ_{ph}	ξ^*_{ph}	I, 10^2 W/cm^2
0	0	0
0.167	−0.0429	1.2
0.346	−0.0882	2.4
0.534	−0.1356	3.7
0.730	−0.1846	5.1
0.934	−0.2355	6.4
1.142	−0.2871	7.8
1.354	−0.3395	9.3
1.566	−0.3921	10.7
1.780	−0.4451	12.2

which contains two terms entering it with opposite signs. Consequently, there are qualitatively different droplet motions along the direction of propagation of radiation and in the opposite direction. This is due to the contribution of convective terms to the total force and velocity entering the heat-conductivity equation [the term proportional to Pr_∞ in formula (4)]. Moreover, the contribution from the former term can be so important that it can be comparable to the major effect (the latter term). From (15) it follows that this term is proportional to the product of the Prandtl number and the relative temperature drop γ. Taking into account that the Prandtl number in a fluid can be large and the motion for considerable temperature drops in the droplet neighborhood is investigated, this effect can be significant in the proper choice of the fluid.

To illustrate the contribution of the fluid motion to the force and velocity of the thermocapillary-drift, we list in Table 2 data relating the values ξ_{ph} and ξ^*_{ph} to the intensity I for large-size mercury droplets with radius $R = 2 \times 10^{-5}$ m moving in water at $T_\infty = 273$ K. The values of ξ_{ph} were estimated from formula (15), while ξ^*_{ph} were estimated from formula (15) for $\gamma = 0$; i.e., no fluid motion was taken into account. The molecular transport coefficients were taken at the mean surface temperature ($T_e = T_s$). In Table 3, we give numerical estimates for the influence of droplet-surface heating and the convective terms in the heat-conductivity equation on the thermocapillary-drift velocity of the droplet. The value of h_{ph} was estimated from formula (14); the value of h^{B}_{ph}, from formula (14) without convective terms (i.e., for $\omega = 0$). The value of h^*_{ph} was determined for low relative temperature drops ($\gamma \longrightarrow 0$), the molecular-transport coefficients being taken at $T_e = T_s$. The coefficient of dynamic viscosity for water is described by the values $A = 5.779$, $F_1 = -2.318$, and $F_2 = 9.118$ in the temperature range from 273 to 363 K with a relative accuracy to within 2%; $Pr_\infty = 12.99$. If we consider the motion of a mercury droplet in glycerin, this effect is especially significant because, for example, the Prandtl number $Pr_\infty = 4753$ at $T_\infty = 303$ K.

From the above numerical estimates it follows that the convective terms should be taken into account in the heat-conductivity equation when the mean temperature of the surface of heated droplets differs significantly from the environment temperature. For low relative temperature drops, this effect must be taken into account for fluids with high Prandtl numbers. In this case, the contribution can be as high as 20%. In a gas, this effect should not be taken into account because the Prandtl number for most gases is on the order of unity.

Table 3

T_s, K	h_{ph}	h^{B}_{ph}	h^{*}_{ph}	h^{B*}_{ph}
273	0	0	0	0
283	-3.032×10^{-4}	7.785×10^{-5}	-2.017×10^{-4}	8.108×10^{-5}
293	-6.658×10^{-4}	1.700×10^{-4}	-2.944×10^{-4}	1.828×10^{-4}
303	-1.080×10^{-3}	2.743×10^{-4}	-3.101×10^{-4}	3.025×10^{-4}
313	-1.538×10^{-3}	3.891×10^{-4}	-2.707×10^{-4}	4.367×10^{-4}
323	-2.039×10^{-3}	5.141×10^{-4}	-1.913×10^{-4}	5.841×10^{-4}
333	-2.575×10^{-3}	6.473×10^{-4}	-8.426×10^{-5}	7.408×10^{-4}
343	-3.143×10^{-3}	7.883×10^{-4}	4.303×10^{-5}	9.060×10^{-4}
353	-3.734×10^{-3}	9.350×10^{-4}	1.885×10^{-4}	1.077×10^{-3}
363	-4.342×10^{-3}	1.086×10^{-3}	3.414×10^{-4}	1.252×10^{-3}

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