Polycrystalline Material Diagnostics Based on the Peak of Polarization Bremsstrahlung Radiation Propagation against the Velocity of Emitting Electrons

O. A. Gorbunova, P. N. Zhukova, and N. N. Nasonov

Laboratory of Radiation Physics, Belgorod State University, ul. Studencheskaya 14, Belgorod, 308007 Russia

e-mail: nnn@bsu.edu.ru

Abstract—We discuss the possibility of using an anomalous peak of polarization bremsstrahlung radiation from relativistic electrons for the diagnostics of polycrystalline materials.

INTRODUCTION

Traditional X-ray diffraction methods for diagnostics of a substance's atomic structure can be supplemented by a method based on polarization bremsstrahlung radiation (PBR) from relativistic electrons of the object under study [1]. One advantage of this new method is the possibility of relatively easy control over the parameters of electron beam, particularly when focusing the beam on a target to increase the spatial resolution of the measurements. The experiments performed so far have demonstrated the efficiency of using PBR to measure the lattice parameters of polycrystalline materials [2]. Analysis has revealed the special promise of polycrystal diagnostics based on coherent peaks of PBR propagating backwards with respect to the velocity of the emitting electron [3]. The interest in using such peaks is justified by the extremely narrow width of their spectrum, which is inversely proportional to the energy square of electron (the width of spectrum of the PBR common peaks is inversely proportional to the energy of electron).

The properties of these peaks and possibilities of using them for diagnostics of the texture of polycrystals are the subject of this work's research.

EXPERIMENTAL

Let us examine the structure of an electromagnetic field excited by a relativistic electron emitting at a velocity $\mathbf{V} = -\mathbf{e}_x V$ with respect to the normal into a semispace $x < 0$ filled with polycrystalline material.

Let us begin with the Maxwell equations for the Fourier image of the electromagnetic field $\mathbf{E}_{\omega\mathbf{k}} = (2\pi)^{-4} \int d^3r dt \mathbf{E}(\mathbf{r},t)\exp(i\omega t - i\mathbf{kr})$. Allowing for the domination of the transverse component of the field of the relativistic electron $\left(\mathbf{E}_{\omega\mathbf{k}} \approx \mathbf{E}^{\mathrm{tr}}_{\omega\mathbf{k}} = \sum_{\lambda=1}^{2} \mathbf{e}_{\lambda\mathbf{k}} E_{\lambda\mathbf{k}},\ \mathbf{k}\mathbf{e}_{\lambda\mathbf{k}} = 0,\ \mathbf{e}_{\lambda\mathbf{k}}\mathbf{e}_{\lambda'\mathbf{k}} = \delta_{\lambda,\lambda'}\right)$, we obtain the initial equation

$$\left(k^2 - \omega^2\right) E_{\lambda\mathbf{k}} + \int d^3k' G(\mathbf{k}'-\mathbf{k}) \sum_{\lambda'=1} (\mathbf{e}_{\lambda\mathbf{k}}\mathbf{e}_{\lambda'\mathbf{k}'}) E_{\lambda\mathbf{k}'} = -\frac{i\omega\, e}{2\pi^2}(\mathbf{e}_{\lambda\mathbf{k}}\mathbf{e}_x)\delta\left(k_x + \frac{\omega}{V}\right), \tag{1}$$

where the function $G(\mathbf{k}'-\mathbf{k})$, which describes the scattering of the magnetic field by the target atoms, is as follows [3]:

$$G(\mathbf{k}'-\mathbf{k}) = \frac{e^2}{2\pi^2 m} F(\mathbf{k}'-\mathbf{k}) \sum_{\alpha} \exp(i(\mathbf{k}'-\mathbf{k})\mathbf{r}_\alpha). \tag{2}$$

In formula (2), $F(\mathbf{k}'-\mathbf{k})$ is the atomic form factor; $\mathbf{r}_\alpha$ denotes the coordinates of the atoms, which are in this case set by the ratio $\mathbf{r}_\alpha = \mathbf{r}_n + \mathbf{r}_{nm} + \mathbf{r}_{nml} + \mathbf{u}_{nml}$, where $\mathbf{r}_n$ is the coordinate of the centre of the nth grain of a polycrystal; $\mathbf{r}_{nm}$ denotes the coordinate of the mth cell in the nth grain; and $\mathbf{r}_{nml}$, $\mathbf{u}_{nml}$ are the equilibrium coordinate and thermal shift, respectively, of the lth atom of the mth cell in the nth grain.

Let us consider solution (1) in the simplest case of an infinite uniform medium. Dividing the function $G(\mathbf{k}'-\mathbf{k})$ into mean and fluctuation components,

$$G(\mathbf{k}'-\mathbf{k}) = \bar{G}(\mathbf{k}'-\mathbf{k}) + \tilde{G}(\mathbf{k}'-\mathbf{k}),\quad \bar{G} = \langle G \rangle = \omega_p^2 \delta(\mathbf{k}'-\mathbf{k}), \tag{3}$$

where $\omega_p = \sqrt{4\pi Z e^2 n_a / m}$ is the plasma frequency, $Z = F(0)$ denotes the number of electrons in an atom,

and n_a stands for the density of the atoms in the target. Solving equation (1) with iterations in powers of $\tilde{G}(\mathbf{k}'-\mathbf{k})$, we obtain the following simple expression for the Fourier-image of the radiated field:

$$E^{\mathrm{rad}}_{\lambda\mathbf{k}} = \frac{i\omega e}{2\pi^2}\frac{1}{k^2-\omega^2\varepsilon(\omega)}\int\frac{d^3k'}{k'^2-\omega^2\varepsilon}\tilde{G}(\mathbf{k}'-\mathbf{k})\left[(\mathbf{e}_{\lambda\mathbf{k}}\mathbf{e}_x)-\frac{(\mathbf{k}'\mathbf{e}_{\lambda\mathbf{k}})(\mathbf{k}'\mathbf{e}_x)}{k'^2}\right]\delta\left(k'_x+\frac{\omega}{V}\right), \tag{4}$$

where $\varepsilon(\omega)=1-\omega_p^2/\omega^2$ is the common dielectric function stemming from (3) in the X-ray range of frequencies.

Using the standard methods in [3], we obtain from (4) the spectral-angle propagation of PBR intensity:

$$\omega\frac{dN}{dtd\omega d\Omega}=\sum_g\omega\frac{dN_g}{dtd\omega d\Omega},\quad \omega\frac{dN_g}{dtd\omega d\Omega}=A_gF_g,\quad A_g=\frac{e^2\omega_p^4|S(\mathbf{g})|^2\exp(-g^2u_T^2)}{16\pi g^3(1+g^2R^2)^2},$$

$$F_g=\left[2\frac{1+\frac{\omega^2}{g^2V^2}\varepsilon^{3/2}V^3\cos\theta}{D^{1/2}}-1-\left(1-2\sqrt{\varepsilon}V\cos\theta(1-\sqrt{\varepsilon}V\cos\theta)\frac{\omega^2}{g^2V^2}\right)\times\frac{\left(1-2\varepsilon V^2(1-\sqrt{\varepsilon}V\cos\theta)\frac{\omega^2}{g^2V^2}\right)^2+4\varepsilon^2V^4(1-\varepsilon V^2)\frac{\omega^4}{g^4V^4}}{D^{3/2}}\right]\sigma\left(\frac{gV}{1-\sqrt{\varepsilon}V\cos\theta}-\omega\right), \tag{5}$$

$$D=\left(1-2(1-\sqrt{\varepsilon}V\cos\theta)\frac{\omega^2}{g^2V^2}\right)^2+4(1-\varepsilon V^2)\left(1-(1-\sqrt{\varepsilon}V\cos\theta)^2\frac{\omega^2}{g^2V^2}\right),$$

where $\mathbf{g}$ are the vectors of the reciprocal lattice, and θ is the angle between the electron velocity and direction of observation for the emitted photon.

Let us consider the next dependence of (5), the dependence of function maximum (5)

$$F_g(\omega_g)\cong\frac{2\gamma_*\sin(\theta/2)}{\sqrt{\cos^2(\theta/2)-(1/4)\gamma_*^{-2}\cos(\theta)}}\equiv F_g|_{\max}, \tag{6}$$

which is reached at point $\omega=\omega_g=gV/\sqrt{2(1-\sqrt{\varepsilon}V\cos\beta\theta)}\approx g/2\sin(\theta/2)$ from the angle of radiation θ. Here, $\gamma_*=\gamma/\sqrt{1+\gamma^2\omega_p^2/\omega^2}$, and γ is the Lorentz factor. It is easy to see that $F_g|_{\max}\propto\gamma_*$ at all values of θ that differ substantially from π. In the vicinity of $\theta=\pi$, formula (6) predicts a sharp rise in the amplitude of the PBR peak in proportion to the square of the electron energy:

$$F_g|_{\max}\cong\frac{4\gamma_*^2}{\sqrt{1+\gamma_*^2(\pi-\theta)^2}}. \tag{7}$$

It is noteworthy that the relative spectral width of the peak $\Delta\omega/\omega$ is described under the considered conditions by the formula $\Delta\omega/\omega\approx(F_g|_{\max})^{-1}$, i.e., it shrinks in inverse proportion to the square of the fast electron energy [3]. The spectral density of the rigidly collimated PBR peak is thus emitted backwards only and grows in proportion to the fourth power of the electron energy [3]. The spectral curves of the coherent peaks emitted at various angles from electrons with an energy of 15 MeV in an aluminium polycrystal are shown in Fig. 1.

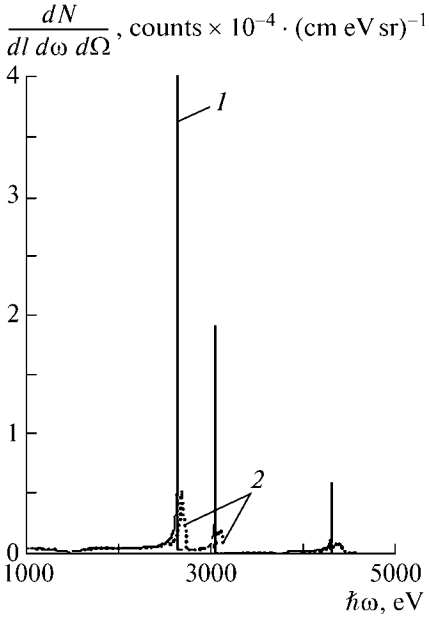

Fig. 1. Coherent peaks of PBR electrons with an energy of 7 MeV from aluminium polycrystal: (*1*) angle of radiation $\theta=180°$, (*2*) 160°.

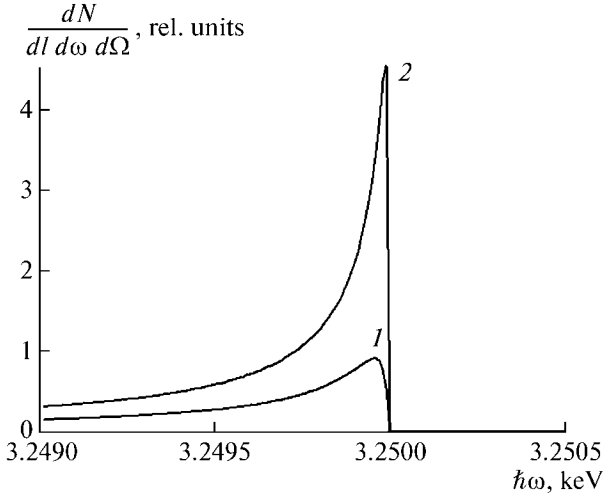

Fig. 2. Suppression of the density effect in an anomalous peak of PBR with $\omega_0 = 30$ eV, $\gamma = 200$, and $\theta = 0°$. Curve *1* represents the Pb spectrum without allowance for the contribution from transient radiation; curve *2* represents the spectrum with the contribution from transient radiation taken into account.

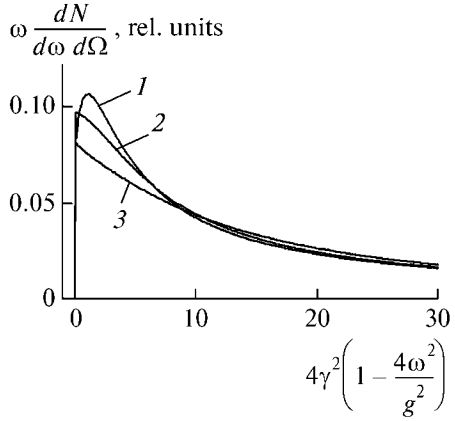

Fig. 3. Influence of multiple scattering (Ψ_S^2 is the square of the scattering angle per length unit, and L is the thickness of the target) and the initial divergence Ψ_0 on the spectrum of the anomalous PBR peak when $\gamma^2\Psi_S^2 L = 1$. Curve *1* represents $\gamma\Psi_0 = 0.1$; curve *2*, $\gamma\Psi_0 = 0.1$; curve *3*, $\gamma\Psi_0 = 2.0$.

Let us consider next effect stemming from (7), that of the saturation of the PBR spectral density with a rise in the electron energy. This phenomenon, known as the density effect, reveals itself in the region of values of the Lorentz factor $\gamma \geq \gamma_* = \omega/\omega_p \approx g/2\omega_p$ and could limit substantially the possibilities of this method for material structure diagnostics. In the case of a copper target, e.g., saturation occurs at an electron energy on the order of 20 MeV. We shall demonstrate that the density effect is almost completely suppressed in the case of a bremsstrahlung from a seminfinite polycrystal.

The predicted effect is determined by the contribution to the process of PBR emission formation from the transient radiation generated by an electron crossing the surface of the target. To solve the problem, we return to equations (1)–(3) and find separate solutions for both sides of the surface with subsequent overlapping of the solutions on the surface. Omitting tedious calculations, we present the ultimate result:

$$\omega\frac{dN^{pb}}{d\omega\, d^2\Omega} \approx \frac{e^2}{16\pi^2}\frac{\omega_p^4}{\omega^4}\sum_{\mathbf{g}}{}'\frac{\pi}{\chi''}\frac{|S(\mathbf{g})|^2\exp(-g^2u_T^2)}{\left(1+g^2R^2\right)^2}\sum_{l=1}^{2}\Phi_l(\omega,\theta)\sigma\left(\frac{g}{2}-\omega\right),$$

$$\Phi_l \approx \frac{2\omega}{g}\frac{\left(\frac{g^2}{4\omega^2}-1-\frac{1}{4}\theta^2\right)^2+\frac{1}{4}\rho_l^2\left(\frac{g^2}{4\omega^2}-1+\frac{1}{4}\theta^2\right)}{\left(\left(\frac{g^2}{4\omega^2}-1-\frac{1}{4}\theta^2\right)^2+\frac{1}{2}\rho_l^2\left(\frac{g^2}{4\omega^2}-1+\frac{1}{4}\theta^2\right)+\frac{1}{16}\rho_l^4\right)^{\frac{3}{2}}}, \tag{8}$$

where $\rho_1^2 = \gamma^{-2}$, $\rho_2^2 = \gamma^{-2} + \omega_p^2/\omega^2 \approx \gamma^{-2} + 4(\omega_p^2/g^2)$, $\sigma(x) = 1$ at $x > 0$ and at $\sigma(x) = 0$ at $x < 0$.

In the region of low electron energies where $\rho_1^2 \approx \rho_2^2 \approx \gamma^{-2}$, the contribution from the transient radiation is low and (18) describes normal PBR propagation. In the region of high energies ($\rho_2^2 \approx \omega_p^2/\omega^2 \approx 4(\omega_p^2/g^2)$, however, the main contribution to the formation of PBR emission comes from the scattering of transient radiation. The density effect is completely suppressed, as follows from Fig. 2, which shows the spectral distributions from the collimated PBR ($\theta^2 < \gamma^{-2}$), constructed with and without allowance for the contribution from the transient radiation.

It is important to determine the effect of the multiple scattering of emitting electrons on the properties of the anomalous PBR peak. The results of [4] show that the contribution from the diffracted bremsstrahlung is not registered in the region of electron energies up to 100 MeV, and to solve the problem all we have to do is average the PBR cross section (with respect to the

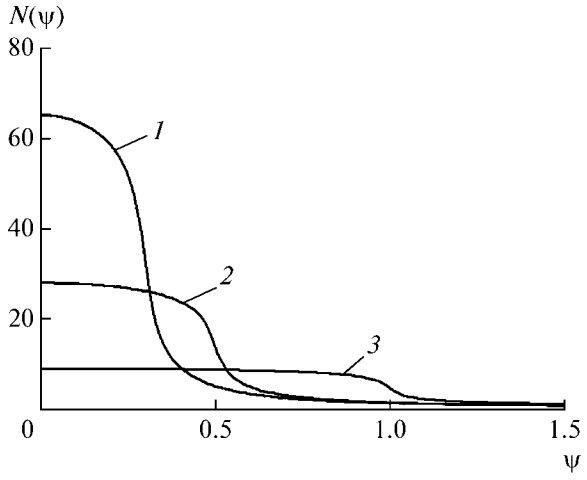

Fig. 4. Orientation dependence of the PBR reflex emission: (*1*) angular width of the texture $\theta = 0.3$; (*2*) $\theta = 0.5$; (*3*) $\theta = 1.0$ rad.

angles of scattering), which is calculated on a linear path. On the basis of formula (5), which is observed in the vicinity of $\theta = \pi$, and assuming the local temporal distribution of electrons over the scattering angles to be Gaussian, we obtain an expression for the collimated PBR spectrum ($\theta^2 < \gamma^{-2}$) that is convenient for numerical calculation. Analysis shows that the width of the spectrum of the coherent peak of backwards PBR is primarily determined by the initial divergence of the electron beam. The curves in Fig. 3 show the relatively minor influence divergence has on the spectrum even when the electron beams are of low quality.

CONCLUSIONS

We have demonstrated the possibilities for using the anomalous PBR peak in the diagnostics of texture in a polycrystal. Figure 4 shows orientational emission curves for a collimated peak of backwards PBR. The texture was simulated using a rectangular distribution of microblocks with respect to the vector's orientation angle. We can see that the angular width of the curves corresponds perfectly to the distribution width of the microblocks. This allows us to measure the parameters of the texture of polycrystals with relative ease. In addition, we can measure the distribution in various planes by rotating the target around the beam axis.

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