Generation of Circularly Polarized Photons by Relativistic Electrons Moving in a Crystal

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Abstract—The relativistic-electron (relativistic-positron) acceleration arising in its scattering on the potential of a crystal atomic string is highly anisotropic, which causes polarization of accompanying radiation. The possibility of developing an efficient source of circularly polarized photons by using electrons or positrons of energies attainable at many operating accelerators is demonstrated and analyzed.

1. INTRODUCTION

For a long time, coherent bremsstrahlung from relativistic electrons in oriented crystals [1] has been successfully employed to obtain quasimonochromatic linearly polarized photons of high energy, which are extensively used in elementary-particle physics and in the physics of photonuclear reactions [2].

Circularly polarized gamma radiation is of no less interest; however, no efficient sources of such radiations have been developed so far. Previously, the birefringence of photons in an oriented crystal was proposed as a means for transforming a linear polarization into a circular polarization [3] (advances in these realms are surveyed in [4]). Unfortunately, this means is sufficiently efficient only in the region of quite high photon energies (on the order of 10^2 GeV or higher).

In order to generate circularly polarized radiation, Lapko *et al.* [5] proposed using a high degree of circular polarization of the electron acceleration—that is, the rotation that the vector of the electron acceleration executes in the transverse plane as an electron is scattered on the potential of an atomic string in a crystal. The directions of the circular polarization of the acceleration (as well as the direction of the circular polarization of emitted photons) are different for particles moving on the right and the left of the atomic-string axis. The contributions to the yield of radiation from these particle groups can be separated under the conditions of nondipole radiation. Bearing this in mind, the authors of [5] proposed separating photons of specific circular-polarization direction by collimating radiation at an angle with respect to the incidence plane preset by the string axis and the primary-particle momentum. An important modification of the method was proposed in [6] on the basis of the coherent interference of radiation generated by a particle on different strings lying in the same atomic plane in a crystal of finite thickness. This modification allows one to reduce sharply the width of the emitted-photon spectrum.

The method proposed in [5, 6] requires high energies of emitting particles (a few tens of GeV or even higher) and a collimation of the radiation, but the latter presents a technical problem at such energies.

The generation of circularly polarized photons by a relativistic-positron flux reflected from a crystal surface (a crystal surface represents a plane of atomic strings on which the radiation is produced) was considered in [7]. The approach proposed in [7] is advantageous in that it provides the possibility of using positrons of moderately low energy (on the order of 10^2 MeV). A significant drawback of the method is that it requires crystals with an atomically pure surface of an area about a few tens of square centimeters: such surfaces can hardly be obtained at present.

In this article, we analyze the possibility of generating circularly polarized photons as the result of dipole radiation from electrons and positrons in a thin crystal. Interest in this problem is provoked primarily by the following two circumstances:

(i) Electron and positron beams of energies on the order of 10^2 MeV, which can be achieved at many operating accelerators, can be used for this.

(ii) As in the approach proposed in [5], it is possible to obtain a high degree of circular polarization of the radiation.

In our calculations, we use the system of units where $\hbar = c = 1$.

2. GENERAL RELATIONS

Let us consider coherent radiation from relativistic particles entering a crystal with an incident momentum parallel to the atomic planes at a small angle with respect to the axis of atomic strings lying in these planes. Our analysis will be based on the general semi-

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classical expression [8] for the polarization matrix of the spectral density of radiation from a particle in an external field:

$$\frac{dE_{jk}}{d\omega dO} = \frac{e^2\omega^2}{4\pi^2}\int dt_1 dt_2 L_{jk} \times \exp\{i\omega'[t_1 - t_2 - \mathbf{n}\cdot(\mathbf{r}_1 - \mathbf{r}_2)]\}, \tag{1}$$

$$L_{jk} = \frac{(\varepsilon+\varepsilon')^2}{4(\varepsilon')^2}(\mathbf{e}_j\cdot\mathbf{v}_1)(\mathbf{e}_k\cdot\mathbf{v}_2) - \frac{\omega^2}{4(\varepsilon')^2}[(\mathbf{e}_j\cdot\mathbf{v}_2)(\mathbf{e}_k\cdot\mathbf{v}_1) - (\mathbf{v}_1\cdot\mathbf{v}_2) + v^2].$$

Here, $\mathbf{r}_{1,2} = \mathbf{r}(t_{1,2})$ is the trajectory of the radiating particle, $\mathbf{v} = d\mathbf{r}/dt$, $\mathbf{e}_{j,k}$ are the polarization vectors, ε is the particle energy, ω is the photon energy, $\varepsilon' = \varepsilon - \omega$, $\varepsilon'\omega' = \varepsilon\omega$, and $\mathbf{n}$ is a unit vector in the direction of the radiation.

In the case of dipole radiation considered here, the angle of particle scattering on the crystal must be smaller than the specific radiation angle m/ε, where m is the electron mass. Performing integration by parts in expression (1) and setting the particle velocity to $\mathbf{v}(t) \approx \mathbf{v} = \text{const}$ in the resulting expression, we then arrive at

$$\frac{dE_{jk}}{d\omega dO} = \frac{e^2}{4\pi^2(1-\mathbf{n}\cdot\mathbf{v})^2} \times \int_0^T dt\int d\tau G_{jk}\exp\{i\omega'(1-\mathbf{n}\cdot\mathbf{v})\tau\}, \tag{2}$$

$$G_{jk} = \frac{(\varepsilon+\varepsilon')^2}{4(\varepsilon')^2}(\mathbf{e}_j\cdot\mathbf{b}(t+\tau/2))(\mathbf{e}_k\cdot\mathbf{b}(t-\tau/2)) - \frac{\omega^2}{4(\varepsilon')^2}(\mathbf{e}_j\cdot\mathbf{b}(t-\tau/2))(\mathbf{e}_k\cdot\mathbf{b}(t+\tau/2)) + \frac{\omega^2}{4(\varepsilon')^2}\mathbf{w}(t+\tau/2)\cdot\mathbf{w}(t-\tau/2),$$

$$\mathbf{b} = \mathbf{w} + \frac{\mathbf{v}}{1-\mathbf{n}\cdot\mathbf{v}}\mathbf{n}\cdot\mathbf{w},$$

where T is the target thickness, while the particle acceleration $\mathbf{w}$ can be expressed, in accordance with the relativistic equation of motion, in terms of the crystal potential $\varphi(\mathbf{r})$ as

$$\mathbf{w} = -\frac{e}{\varepsilon}(\nabla - \mathbf{v}(\mathbf{v}\cdot\nabla))\varphi(\mathbf{r}). \tag{3}$$

In the case being considered, the potential $\varphi(\mathbf{r})$ can be represented as

$$\varphi(\mathbf{r}) = \bar{\varphi}(x) + \sum_{l\geq 1}\varphi_l(x)\cos(g_l y), \tag{4}$$

where $\bar{\varphi}(x)$ is the averaged potential of the atomic planes; the two-dimensional oscillating component is generated by atomic strings lying in these planes, the string axes being aligned with $\mathbf{e}_z$; and $g_l = 2\pi l/a$, a being the distance between the neighboring strings. The explicit expressions for $\bar{\varphi}(x)$ and $\varphi_l(x)$ are different for radiating positrons and electrons and will be presented below in performing specific calculations.

According to Eqs. (3) and (4), the trajectory of the radiating particle represents a superposition of a smooth trajectory of the motion in the averaged potential $\bar{\varphi}(x)$ of the atomic planes and small-scale oscillations caused by periodic irregularities of the potential of a plane that are induced by the atomic strings. The characteristic radiation frequencies associated with these two types of motion belong to different ranges. The high-frequency radiation component of interest is due to the scattering of a fast particle on the atomic strings; therefore, only the quickly oscillating component of the acceleration, $\tilde{\mathbf{w}}(t)$, which is controlled by the potential $\sum_l \varphi_l(x)\cos(g_l y)$, must be substituted into the integrand on the right-hand side of (2). Let us specify the initial particle velocity by the expression

$$\mathbf{v} = \mathbf{e}_z\left(1 - \frac{1}{2}\gamma^{-2} - \frac{1}{2}\boldsymbol{\psi}^2\right) + \boldsymbol{\psi}, \quad \mathbf{e}_z\cdot\boldsymbol{\psi} = 0,$$

$$\varepsilon = m\gamma.$$

The quantity $\tilde{\mathbf{w}}(t+\tau)$ can then be represented as

$$\tilde{\mathbf{w}}(t+\tau) = \sum_{l\geq 1}(\tilde{\mathbf{w}}_l[x(t+\tau)]\exp[ig_l\psi_y(t+\tau)] + \text{c.c.}), \tag{5}$$

where the coefficients can easily be determined from Eqs. (3) and (4). Since the trajectory $x(t+\tau)$ is a smooth function against the quickly oscillating exponential, it can be assumed that $\tilde{\mathbf{w}}_l[x(t+\tau)] \approx \tilde{\mathbf{w}}_l[x(t)]$ when we substitute (5) into (2). After some simple algebra, Eqs. (2)–(5) yield the expression

$$\frac{dE}{d\omega} = \frac{e^4\eta}{8\varepsilon\psi_y^2(1-\eta)}\sum_{l\geq 1}\frac{1}{g_l^2}\left\{1 + (1-\eta)^2 - 4\frac{\eta}{\eta_l}\left(1 - \frac{\eta}{(1-\eta)\eta_l}\right)\right\}\left\langle\int_0^T dt\left[\left(\frac{d\varphi_l}{dx}\right)^2 + g_l^2\varphi_l^2\right]\right\rangle\sigma\left(\frac{\eta_l}{1-\eta_l} - \eta\right) \tag{6}$$

for the radiation spectrum and the relation

$$\frac{dE}{d\omega}\xi_2 = \frac{e^4\eta[1+(1-\eta)^2]}{4\varepsilon\psi_y^2(1-\eta)}\sum_{l\geq 1}\frac{1}{g_l^2}\left(1 - \frac{2\eta}{(1-\eta)\eta_l}\right) \times \left\langle\int_0^T dt\varphi_l\frac{d\varphi_l}{dx}\right\rangle\sigma\left(\frac{\eta_l}{1+\eta_l} - \eta\right) \tag{7}$$

for the second Stokes coefficient ξ_2, which characterizes the degree of circular polarization of emitted photons.

In (6) and (7), $\eta = \omega/\varepsilon$; $\eta_l = 2\varepsilon g_l \psi_y / m^2$ is the ratio of a characteristic energy in the radiation of the lth harmonic to the radiating-particle energy ε; $\sigma(x)$ is the Heaviside function; and angular brackets denote averaging over the trajectories of the particle flux, which are determined by the point x_0 at which the particles enter the crystal and their angles of incidence, ψ_x. In deriving formulas (6) and (7), we have used the following representations of unit vector $\mathbf{n}$ and the polarization vectors $\mathbf{e}_j$:

$$\mathbf{n} = \mathbf{e}_z\left(1 - \frac{1}{2}\mathbf{\Theta}^2\right) + \mathbf{\Theta}, \quad \mathbf{e}_z \cdot \mathbf{\Theta} = 0, \quad dO = d^2\Theta,$$

$$\mathbf{e}_1 \approx [\mathbf{n} \times \mathbf{e}_x] \approx \mathbf{e}_y - \Theta_y \mathbf{e}_z,$$

$$\mathbf{e}_2 \approx [\mathbf{n} \times \mathbf{e}_1] \approx -\mathbf{e}_x + \Theta_x \mathbf{e}_z.$$

Expressions (6) and (7) form the basis for the ensuing analysis of the spectral and polarization properties of the radiation from the positron (electron) flux in the crystal of small thickness T.

3. RADIATION FROM RELATIVISTIC POSITRONS

The averaged potential of a plane for positrons, $\bar{\varphi}(x)$, is close to a parabolic potential with the well center occurring in the middle between the neighboring atomic planes. The functions $\bar{\varphi}(x)$ and $\varphi_l(x)$ appearing in the total potential (4) are given by

$$\bar{\varphi}(x) = \varphi^{(0)}\left(\frac{2x}{c}\right)^2,$$

$$\varphi^{(0)} = \frac{2\pi ZeR}{ba}\exp\left(\frac{u^2}{2R^2}\right)\left[1 - \Phi\left(\frac{u}{\sqrt{2}R}\right)\right], \qquad (8)$$

$$\varphi_l(x) = \varphi_l^{(0)}\exp\left(-\kappa_l\frac{c}{2}\right)\cosh(\kappa_l x), \quad \varphi_l^{(0)} = \frac{8\pi Ze}{\kappa_l ba},$$

where $\kappa_l^2 = g_l^2 + R^{-2}$, R is the radius of the electron screening of an atom (we use here the simplest statistical atomic model with exponential screening), c is the distance between the neighboring atomic planes, b is the distance between string atoms, u is the root-mean-square amplitude of thermal vibrations of the atoms, and $\Phi(x)$ is the error function.

Considering the fundamental aspect of the problem, we neglect the angular spread of radiating positrons of the beam. In this case, the equation of motion for particles entering the region of the averaged crystal potential (8) with a momentum parallel to the atomic planes has the simple solution

$$x(t) = x_0\cos(\omega_0 t), \qquad (9)$$

where x_0 is the coordinate of the point at which a positron enters the crystal ($-c/2 \le x_0 \le c/2$) and $\omega_0^2 = 8e\varphi_l^{(0)}/\varepsilon c^2$.

Let us now proceed to analyze the coefficient $F_l = \left\langle \int_0^T dt \varphi_l \frac{d\varphi_l}{dx} \right\rangle$ from (7) (the answer to the question of whether it is possible in principle to obtain circularly polarized photons by the method being considered depends crucially on this coefficient). From the expression

$$F_l = \frac{1}{2}\kappa_l \varphi_l^{(0)^2} e^{-\kappa_l c}\left\langle \int_0^T dt \sinh(2\kappa_l x_0)\cos\omega_0 t \right\rangle, \qquad (10)$$

it follows that $F_l(-x_0) = -F(x_0)$, whence we can see that positrons moving along trajectories that are symmetric with respect to the center of the planar channel emit photons whose circular polarizations have opposite directions (at each instant, these particles have accelerations whose circular components are of equal magnitudes, but they are of opposite signs). Moreover, it follows from (10) that, for a fixed trajectory, the sign of the coefficient F_l is conserved only within the time interval that does not exceed a quarter of the period of positron oscillations in the channel. Therefore, the required crystal thickness T is determined by the condition

$$T = \pi/2\omega_0, \qquad (11)$$

whence it follows that the sign of the coefficient F_l depends on the sign of the impact parameter x_0 exclusively and that the sign of x_0 determines unambiguously the direction of positron scattering on the potential of a plane (for instance, a particle is deflected to the left of the channel plane when $x_0 > 0$).

Owing to these two important circumstances, a coincidence scheme simultaneously recording a radiated photon and the direction in which the positron that has emitted this photon leaves the crystal makes it possible to single out the contribution to the radiation yield from positrons having trajectories characterized by a fixed sign of x_0. Since experiments studying radiation from relativistic particles in crystals are usually conducted in the single-particle mode, the application of a coincidence scheme is not expected to require much more complex experimental facilities.

In practice, a system determining the direction of positron escape from the crystal can be implemented by using two thin semiconductor plates (Fig. 1) that are transparent to hard photons, but which detect positrons by ionization losses.

In calculating the integral in (10) with respect to t, we must take into account a fast (exponential) decrease of the potential harmonics $\varphi_l(x)$ with increasing distance from the atomic planes. It is reasonable to perform averaging over x_0 separately for the intervals

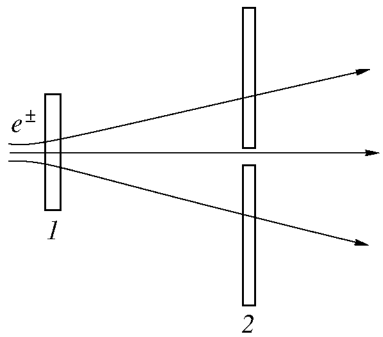

Fig. 1. Coincidence scheme: (*1*) crystal target and (*2*) semiconductor plates.

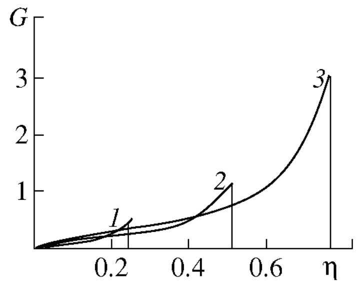

Fig. 2. Universal spectrum of radiation: η_1 = (*1*) 0.3, (*2*) 1, and (*3*) 3.

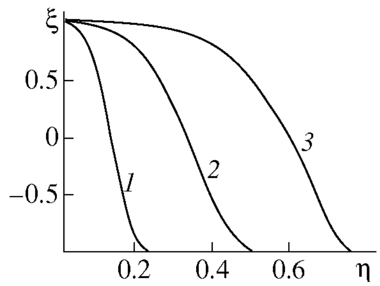

Fig. 3. Degree of radiation polarization as a function of the relative photon energy: η_1 = (*1*) 0.3, (*2*) 1, and (*3*) 3.

$0 < x_0 < c/2$ and $-c/2 < x_0 < 0$. After some simple algebra, we obtain

$$F_l \approx \pm\frac{\varphi_l^{(0)^2}}{4\omega_0 c}\sqrt{\frac{\pi}{2\kappa_l c}}, \qquad (12)$$

where, for example, the plus sign corresponds to particles from the interval $0 < x_0 < c/2$.

Since the coefficients F_l decrease fast with increasing number l of a harmonic ($F_l \approx l^{-5/2}$), we can restrict our consideration to the first harmonic. For the radiation spectrum and the degree of circular polarization, the eventual analytic expressions following from (6) and (7) are given by

$$\frac{dE^{(p)}}{d\eta} = A^{(p)}G(\eta)\sigma\left(\frac{\eta_1}{1+\eta_1} - \eta\right),$$

$$A^{(p)} = \frac{\sqrt{\pi}e^2(e\varphi_1^{(0)})^2}{16\varphi_1^2\omega_0}\frac{g_1^2+\kappa_1^2}{g_1^2}\frac{1}{(2\kappa_1 c)^{3/2}},$$

$$G = \frac{\eta}{1-\eta}\left\{1 + (1-\eta)^2 - 4\frac{\eta}{\eta_1}\left[1 - \frac{\eta}{(1-\eta)\eta_1}\right]\right\}, \qquad (13)$$

$$\xi_2^{(p)} = \pm\xi(\eta)\sigma\left(\frac{\eta_1}{1+\eta_1} - \eta\right),$$

$$\xi = \frac{2g_1\kappa_1}{g_1^2+\kappa_1^2}\frac{(1+(1-\eta)^2)\left(1 - \frac{2\eta}{(1-\eta)\eta_1}\right)}{1+(1-\eta)^2 - 4\frac{\eta}{\eta_1}\left(1 - \frac{\eta}{(1-\eta)\eta_1}\right)}.$$

Since $2g_1\kappa_1/(g_1^2 + \kappa_1^2) \approx 1$, the dependences of G and ξ_2 on the emitted-photon energy ω are characterized by single parameter η_1. Figure 2 shows the curves representing the spectral dependence $G(\eta)$ at various values of η_1. The corresponding curves for $\xi(\eta)$ in Fig. 3 demonstrate the possibility of obtaining quasi-monochromatic photons with a high degree of circular polarization within the approach being discussed.

To conclude this section, we present an expression for estimating the total number of radiated photons:

$$N^{(p)} \cong \frac{e^2}{8m\sqrt{Rc}}\left(\frac{\pi Z e^2}{abg_1\kappa_1\psi_y}\right)^{3/2}\frac{g_1^2+\kappa_1^2}{\kappa_1^2}Q(\eta_1), \qquad (14)$$

$$Q(\eta_1) = \frac{1}{\sqrt{\eta_1}}\left[\frac{8}{\eta_1} + \frac{\eta_1(2+\eta_1)}{2(1+\eta_1)^2} + \left(1 - \frac{4}{\eta_1} - \frac{8}{\eta_1^2}\right)\ln(1+\eta_1)\right].$$

As can be seen from (14), the energy dependence of $N^{(p)}(\varepsilon)$ is absorbed in the function $Q(\eta_1)$. The coefficient of this function in (14) is determined exclusively by the parameters of the crystal and by the angle ψ_y specifying the orientation of the positron velocity with respect to the atomic strings. By way of example, we indicate that, for positrons radiating photons in a silicon crystal, $N^{(p)} \approx 10^{-5}Q(\eta_1)$. It should be emphasized that $Q(\eta_1)$ is a nonmonotonic function of η_1: $Q(\eta_1) \approx 4\sqrt{\eta_1}/3$ for $\eta_1 \ll 1$ and $Q(\eta_1) \approx (\ln\eta_1)/\sqrt{\eta_1}$ for $\eta_1 \gg 1$. It can easily be shown that an increase in the number of emitted photons with increasing positron energy in the low-η_1 region, where the effect of quantum recoil is immaterial in radiation, is associated with the increase in the crystal thickness T determined from (11). A decrease in $N^{(p)}$ with increasing positron energy at high values of η_1 is due to the suppression of the radiation yield by the quantum-recoil effect. Figure 4 shows the dependence $Q(\eta_1)$, which permits choosing optimum values for the positron energy and for the orientation angle ψ_y.

4. RADIATION FROM RELATIVISTIC ELECTRONS

An analysis of the possibility of using electron beams to generate circularly polarized photons by the proposed method is of the greatest practical interest. In the case of emitting electrons, the atomic strings are located at the center of the planar channel at the bottom of the potential well. The properties of the potential (4) are then determined by the formulas

$$\bar{\varphi}(x) = \varphi^{(0)}\left(1 - \frac{2|x|}{c}\right)^2, \quad \varphi_l(x) = \varphi_l^{(0)} e^{-\kappa_l |x|}, \qquad (15)$$

where the coefficients $\varphi^{(0)}$ and $\varphi_l^{(0)}$ coincide with those given in (8) for $|x| \le c/2$.

Taking into account the sign of the electron charge, we conclude that the potential $\bar{\varphi}(x)$ (15) is essentially anharmonic for channeling electrons; as a result, the period of electron oscillations in the channel depends sharply on the impact parameter x_0. In the case of positron channeling in a crystal of thickness T given by (11), the sign of x_0 determines unambiguously both the direction of positron escape from the crystal and the sign of circular polarization of the emitted photon, while, in the case being considered, these features depend on the sign and on the absolute value of x_0 in a rather complicated way at a preset crystal thickness T.

In the case being discussed, the shape of the spectrum and the photon-energy dependence of the degree of the circular polarization of emitted photons are given by the expressions

$$\frac{dE^{(2)}}{d\eta} = \frac{e^2(e\varphi^{(1)})^2 T}{8\psi_y^2} \frac{g_1^2 + \kappa_1^2}{\kappa_1^2} G(\eta)\sigma\left(\frac{\eta_1}{1+\eta_1} - \eta\right) f_1(T),$$
$$\xi_2^2 = \pm\xi(\eta)\sigma\left(\frac{\eta_1}{1+\eta_1} - \eta\right) f_2(T), \qquad (16)$$

which are similar to (13). The $\pm$ signs in (16) correspond to the directions of electron escape from the crystal. The functions $f_1(T)$ and $f_2(T)$, which determine the crystal-thickness dependence of the radiation yield and of the degree of circular polarization, can be represented as

$$f_1 = \frac{1}{Tc}\int_{S^\pm} dx_0 \int_0^T dt\, e^{-2\kappa_1 |x(t, x_0)|},$$
$$f_2 = \frac{1}{Tc}\int_{S^\pm} dx_0 \int_0^T dt\, \mathrm{sgn}[x(t, x_0)] e^{-2\kappa_1 |x(t, x_0)|} / f_1. \qquad (17)$$

Integration in (17) with respect to x_0 is performed within the regions $S^\pm$ corresponding to a certain direction of electron escape from the crystal and consisting of a set of bands of x_0 values, which can easily be inferred from the trajectory of electron motion in the channel. From (3) and (15), it follows that this trajectory is given by

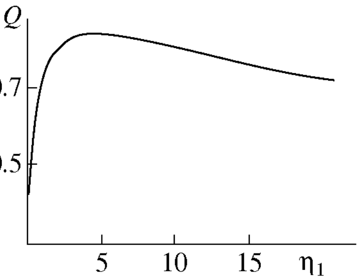

Fig. 4. Radiation yield as a function of the parameter η_1, which is proportional to the particle energy.

$$x(t, x_0) = (-1)^n \frac{c}{2}\left\{1 - \left[1 - \frac{2x_0}{c}\right]\cosh(\omega_0(t - 2nT_0))\right\},$$
$$(2n-1)T_0 \le t \le (2n+1)T_0, \qquad (18)$$
$$T_0 = \frac{1}{\omega_0}\mathrm{arccosh}\left(\frac{1}{1 - 2x_0/c}\right).$$

The function $x(t, x_0)$ is presented for electrons entering the crystal within the impact-parameter range $0 \le x_0 \le c/2$. For $x_0 < 0$, the trajectory is determined from the symmetry conditions, the quantity ω_0 in (18) being coincident with the analogous quantity in (9).

For the functions $f_2(T)$ and $f_1(T)$, expressions convenient for a numerical analysis can be derived from (17) and (18). They can be represented as

$$f_2(T') = \frac{1}{2T'}\sum_{l=0,2}\sum_{k\ge 0}\left\{\int_{y_{l+1+4k}}^{y_{l+4k}} dy \int_{z_1}^{y} dz g(z, y) + \int_{y_{l+2+4k}}^{y_{l+1+4k}} dy \int_{z_2}^{y} dz g(z, y)\right\} f_1^{-1}(T'),$$

$$g(z, y) = \frac{e^{-kz}}{\sqrt{(1-z)^2 - (1-y)^2}},$$

$$f_1(T') = \frac{1}{2T'}\sum_{k\ge 0}\left\{k\int_{y_{1+k}}^{y_k} dy \int_0^y dz g(z, y) + \int_{y_{1+2k}}^{y_{2k}} dy \int_{z_3}^{y} dz g(z, y) + \int_{y_{2+2k}}^{y_{1+2k}} dy \int_0^{z_4} dz g(z, y)\right\},$$

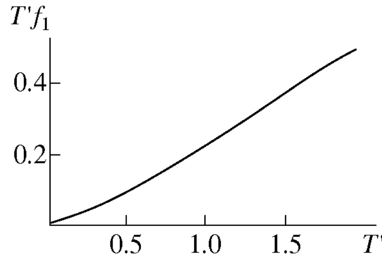

Fig. 5. Yield of radiation from electrons as a function of the parameter $T' = \omega_0 T$, which is proportional to the crystal thickness.

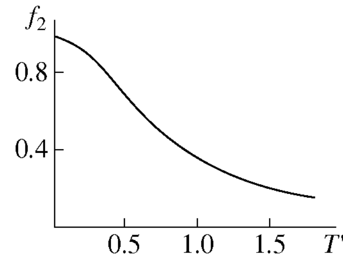

Fig. 6. Degree of circular photon polarization as a function of the parameter $T' = \omega_0 T$, which is proportional to the crystal thickness.

where

$$T' = \omega_0 T, \quad y_m = 1 - \frac{1}{\cosh(T'/m)}, \quad T'_0 = \omega_0 T_0,$$

$$z_1 = z(T' - (l + 4k)T'_0, y),$$

$$z_2 = z((l + 2 + 4k)T'_0 - T', y),$$

$$z_3 = z(T' - 2kT'_0, y),$$

$$\kappa = c\kappa_1, \quad z_4 = z((2 + 2k)T'_0 - T', y),$$

$$z(t', y) = 1 - (1 - y)\cosh t'.$$

Figure 5 displays the function $f_1(T')$ illustrating a nearly linear dependence of the radiation yield on the crystal thickness. As the crystal thickness is increased, the emitting particles are spread fast over the phase space with the result that the degree of radiation polarization decreases monotonically. Figure 6 shows the dependence $f_2(T')$.

Thus, a high degree of circular polarization of photons produced by the proposed method can be expected only in the case of very thin crystals (of thickness $T \leq 1/2\omega_0$). We will now consider the example of how the proposed scheme can be implemented by using an extracted electron beam from the Pakhra synchrotron of the Lebedev Institute of Physics (Russian Academy of Sciences), where the energy of accelerated particles is about 600 MeV and where the angular spread of their beam is about 10^{-4}. With a thin silicon crystal used as a radiator and oriented along the (100) plane, the critical angle of planar channeling is about 2.5×10^{-4}, which exceeds considerably the angular spread of the beam, so that the total phase space of the beam can be used. At an angle of the particle-velocity orientation with respect to the crystal axis, ψ_y, about 10^{-2}, the proposed method makes it possible to obtain a quasimonochromatic flux of photons with an energy of about 150 MeV at the maximum. At the crystal thickness of about 0.5 μm, the spectral density of radiation, $dE/d\omega$, is about 2×10^{-4} in the region of maximum (this is nearly 20 times as great as the radiation density of ordinary bremsstrahlung), the degree of circular polarization of the radiation being about 0.4 at the maximum.

5. CONCLUSIONS

The above analysis of the possibility of generating circularly polarized photons by relativistic charged particles in an oriented crystal leads to the following conclusions:

(i) Owing to an unambiguous relation between the sign of the circular polarization of a photon emitted in the scattering of a relativistic particle on atomic strings lying in a crystal atomic plane and the direction of emitting-particle scattering by this plane, the use of a coincidence scheme would permit separating the yields of radiation having left- and right-hand circular polarization.

(ii) The proposed scheme makes it possible to generate circularly polarized photons with a quasimonochromatic spectrum by using either electrons or positrons moving in the planar-channeling mode in a thin crystal.

(iii) The radiation in question is of a dipole character, so that one can employ particle beams of energy about 10^2 MeV, which can be achieved at many operating accelerators.

(iv) Numerical examples constructed on the basis of the formulas that have been obtained in the present study and which provide a complete quantitative description of the spectral–angular and polarization features of the radiation demonstrate a high efficiency of the proposed method.

(v) Under the conditions of interference, the spectral–angular distribution of the radiation undergoes a significant rearrangement, which depends on the sign of the emitting-particle charge. As a result, the radiation may become much more intense.

(vi) The interference changes sharply the angular dependence of polarization of the resulting radiation.

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