Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

Sub-band detection of small-sized objects during airspace sensing with ultra-short radio pulses

To cite this article: A O Zhukov *et al* 2021 *J. Phys.: Conf. Ser.* **2094** 022013

Sub-band detection of small-sized objects during airspace sensing with ultra-short radio pulses

A O Zhukov[1,2,3,4], E G Zhilyakov[5], I I Oleynik[5], S G Orishchuk[6], P A Fedorov[7] and I N Kartsan[2,8,9,10]

[1] Institute of Astronomy of the Russian Academy of Sciences, Moscow, Russia
[2] FGBNU "Expert and Analytical Center", Moscow, Russia
[3] Russian Technological University, Moscow, Russia
[4] Joint Stock Company "Special Research of Moscow Power Engineering Institute", Moscow, Russia
[5] Federal State Autonomous Educational Institution of Higher Education "Belgorod National Research University", Belgorod, Russia
[6] Scientific and Production Association "Electronic Instrument Engineering" JSC, Moscow, Russia
[7] The Limited Liability Company "Systems on Nod Observation & Information Technologies", Obninsk, Russia
[8] Marine Hydrophysical Institute, Russian Academy of Sciences», Sevastopol, Russia
[9] Reshetnev Siberian State University of Science and Technology, Krasnoyarsk, Russia
[10] Sevastopol State University, Sevastopol, Russia

E-mail: kartsan2003@mail.ru

Abstract. The possibility of ensuring the safety of flights of aircraft, such as helicopters, at low altitudes, where there is a high probability of unauthorized appearance of small-sized objects such as unmanned aerial vehicles, is being considered. The possibility of solving the problem of detecting such objects on the basis of radar soundings in the resonant frequency range of the UHF radio wave range is considered. Sub-band processing of the received signals is proposed, based on the division of the spectral definition area into sub-bands, for adaptation to the frequency response band and noise filtering. The mathematical apparatus of sub-band signal analysis using sub-band matrices has been developed. An optimal solution to the problem of filtering responses in given sub-bands is obtained. A procedure for processing the received signals is given when making decisions about the presence of a response in a given sub-band. Estimates of the probabilities of erroneous decisions are given for a given probability of errors of the first kind.

1. Introduction

Currently, ensuring flight safety is relevant. Especially when flying at low altitudes. The existing onboard optical and radar facilities are not always effective. This is due to the fact that large air and ground objects such as airplanes, helicopters, high-altitude masts, power line towers and tall buildings are fairly well detectable in both optical and radar ranges. When detecting small-sized aerial objects, for example, such as unmanned aerial vehicles (UAVs), problems arise. Currently, small-sized UAVs have become widespread, and their improvement is on the way of reducing the size and expanding the range of tasks performed. Consequently, such objects can pose a threat to flight safety, especially at low altitudes, where the likelihood of their unauthorized appearance is quite high.

The linear dimensions of small-sized UAVs (for example, of the "copter" type) are quite small and can be only two tens of centimeters, and composite materials are used in the structural elements, which reduce radar signature. The use of means of the optical and infrared range does not always allow obtaining the required result. This is determined by their dependence on weather conditions, time of day and very weak thermal contrast of objects [1].

Existing airborne radar facilities, including helicopter ones, operate in the "quasi-optical" frequency range (when the wavelength, of the order of 3 cm, is significantly less than the linear dimensions of objects). At the same time, the main feature in the reflection of radio waves is that the effective scattering surface (ESR) of small objects is very small (about 0.001 - 0.05 m2) [1-2]. Consequently, the detection range of small-sized UAVs is limited by the power (energy) characteristics of radar facilities and today does not exceed a few kilometers (and even then in almost ideal conditions). At the same time, methods based on the Doppler Effect are mainly used (it manifests itself in the presence of the radial velocity of an object due to a change in the frequency of the signal during reflection). However, most of the small-sized UAVs are of low speed, or in general can be practically stationary (hover). In addition, the flight altitude of such objects is small, usually not exceeding hundreds of meters. Under these conditions, detection and recognition must be carried out against the background of the underlying surface. In this case, in the resolved volume of the radar, reflections arise not only from the object itself, but also from the underlying surface (earth) and possibly from other foreign objects. Specific EPR of the underlying surface, in this radio wave range, can reach 0.1 m2. In this case, the ground is a passive interference, and the signal-to-interference ratio in the reflected signal can reach minus 10-15 dB in power (even for locators with good resolution) [1-3]. To solve the problems listed above, new approaches and methods for processing radar information are required, as well as the use of other radio wave ranges where reflections from small-sized UAVs will be more informative.

To detect and recognize small-sized UAVs, it is necessary to use the radio wave range in which the reflective properties of such objects are most fully manifested and where reflections from passive interference are much less pronounced, in this case reflections from the underlying surface. The most acceptable is the UHF radio wave band. This is due to the fact that in this range, "resonant" properties are manifested when signals are reflected. Separate structural elements of small-sized UAVs are capable of forming responses to electromagnetic effects [4]. This is especially evident in cases where the wavelength is commensurate with or a multiple of the linear dimensions of the structural elements of such objects. The dimensions of the reactive structural elements are not known a priori. Therefore, it is impossible to determine in advance the frequency of the probing action. As a way out of this situation, it is natural to use multi-frequency broadband sounding [5-7]. For this, it is possible to use linear frequency modulated (chirp) signals with a wide bandwidth, or ultra-short smooth pulses. Exposure to chirp broadband signals is equivalent to the use of short radio pulses of a certain frequency. Synthesis of such signals is possible using digital methods [8-9].

To ensure broadband, the chirp samples of the signal must be sufficiently short in duration. However, short-term exposure cannot always generate a resonant response; therefore the resulting response corresponds to the concept of the impulse response of a certain system. The duration of the reaction depends on the bandwidth of the response filter, which must be estimated in order to sufficiently use the energy properties of reflections [10].

The relatively low mobility of the UAV makes it possible to use the coherent properties of signals [11], that is, to accumulate signals reflected from objects. This can provide the necessary energy performance for long range detection and with acceptable performance.

Another important aspect is that in addition to the response in a certain frequency range, there will be extraneous noise in the received signal. Therefore, for reliable detection of the response, it is necessary to use techniques to reduce their influence. As such a technique, the article proposes to use sub-band processing, that is, analysis of the properties of received signals from the standpoint of dividing the frequency band into sub-bands [12-14]. For this, the article has developed a special mathematical apparatus and methods of its application.

2. Mathematical foundations of sub-band signal analysis

Let further the symbol $x(t), t \in (0, T)$ means some signal of finite duration, the Fourier transform of which is determined by the following relation [15]

$$X(\omega) = \int_0^T x(t) \exp(-j\omega t)\, dt. \quad (1)$$

The domain of definition of the transformant (Fourier spectrum) is the entire numerical axis and it is assumed that the conditions [16] of the existence of the frequency representation of the original signal in the domain of originals are fulfilled (inverse Fourier transform)

$$x(t) = \int_{-\infty}^{\infty} X(\omega) \exp(j\omega t)\, d\omega/2\pi, \quad (2)$$

and the validity of Parseval's equality

$$\|x\|^2 = \int_0^T x^2(t)dt = \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega/2\pi. \quad (3)$$

If the spectrum definition area is divided into symmetric sub-bands of the form

$$\Omega_r = [-\Omega_{2r}, -\Omega_{1r}) \cup [\Omega_{1r}, \Omega_{2r}),$$
$$\Omega_{11} = 0, \Omega_{2,(r-1)} = \Omega_{1r}, r = 2, 3, \quad (4)$$

where the index r denotes the number of the frequency interval, then the Parseval equality can be given the following form

$$\|x\|^2 = \sum_{r=1}^{\infty} P_r(x), \quad (5)$$

where

$$P_r(x) = \int_{z\in\Omega_r} |X(z)|^2 dz/2\pi. \quad (6)$$

It seems natural to call characteristics of the form (6) the sub-band parts of the signal energy. It is clear that the definition of these characteristics refers to the techniques of sub-band analysis [12].

It is important that the feasibility of such calculations is available directly in the signal definition area (originals area). This conclusion can be easily substantiated by substituting representation (1) into definition (6). As a result of simple transformations, we obtain the quadratic form

$$P_r(x) = \int_0^T \int_0^T A_r(t_1 - t_2) x(t_1) x(t_2) dt_1 dt_2, \quad (7)$$

whose core is determined by the sub-band integral

$$A_r(t) = \int_{z\in\Omega_r} \exp(-jzt) dz/2\pi. \quad (8)$$

After integration, from this we have

$$A_r(t) = (\sin(\Omega_{2r}t) - \sin(\Omega_{1r}t))/\pi t,$$
$$A_r(0) = (\Omega_{2r} - \Omega_{1r})/\pi. \quad (9)$$

Kernels of the form (8) will be called sub-band kernels. They serve as the basis for the mathematical apparatus of sub-band analysis developed in the article.

In addition to calculating the sub-band parts of the signal energies, it is of interest to separate their sub-band additive components

$$x(t) = y_r(t) + \varepsilon_r(t), t \in (0, T), \quad (10)$$

which are uniquely determined based on the following requirement for the spectrum of the sought-for component

$$Y_r(\omega) = \int_0^T y_r(t) \exp(-j\omega t) dt. \quad (11)$$

Namely

$$Y_r(\omega) = X(\omega), \omega \in \Omega_r, \quad (12)$$

$$Y_r(\omega) \equiv 0, \omega \notin \Omega_r. \quad (13)$$

It is clear that requirements (12) and (13) with a finite duration of the signal components cannot be met, but a functional can be introduced that determines the sub-band measure of the error of their fulfillment in the space of signals of the original duration

$$S_r(y) = P_r(x - y) + \|x\|^2 - P_r(y). \quad (14)$$

It can be seen that the first term is the measure of the error in meeting requirement (12), while the other two, taking into account equality (3), determine the measure of the error of the deviation from zero of the squared modulus of the spectrum of the sought component (requirement (13)). Therefore, the natural principle is the variational condition for minimizing the error measure in the space of signals of the original duration with limited energy

$$S_r(y_r) = \min S_r(y), y(t) \in L_2(T). \quad (15)$$

Omitting details, we present the solution of the variational problem (15)

$$y_r(t) = \int_0^t A_r(t - \tau) x(\tau) d\tau. \quad (16)$$

Thus, in this case, sub-band cores are of decisive importance.

Note one more important feature of the components of the initial signals obtained on the basis of representation (16). If representation (8) is substituted into (16), then after obvious transformations, taking into account definition (1), it is easy to obtain the following relation

$$y_r(t) = \int_{\omega \in \Omega_r} X(\omega) \exp(j\omega t)\, d\omega / 2\pi. \quad (17)$$

It shows that the component of the form (16) is completely determined by the segment of the spectrum of the original signal in the original sub-band. This is a very important property that cannot be achieved with any other filter.

In addition, unlike other filtering methods, the additivity property $\sum_{r=1}^{\infty} y_r(t) = x(t)$ holds. It follows directly from the definition of sub-band kernels.

3. Computational aspects of sub-band analysis

The implementation of calculating integrals (8) and (16) using computer technology involves the use of quadrature formulas. From many points of view, it seems acceptable to carry out equidistant discretization of the integration regions (signal definition regions) and use the quadrature formula of rectangles, so that analogs of (8) and (16) are

$$P_r(\vec{x}) \cong \vec{x}' A_r \vec{x}, \quad (18)$$

$$\vec{y}_r = A_r \vec{x}, \quad (19)$$

where the top stroke means transpose:

$$\vec{x} = (x_1, \cdots, x_N)'; \; x_k = x(k\Delta t); \; k = 1, \cdots, N;$$

$$\vec{y}_r = (y_{1r}, \cdots, y_{Nr})'; \; y_{kr} = y_r(k\Delta t); \; k = 1, \cdots, N;$$

$$A_r = \{a_{ik}^r\};$$

$$a_{ik}^{r}=\frac{\sin(V_{2r}(i-k))-\sin(V_{1r}(i-k))}{\pi(i-k)},\ a_{ii}^{r}=\frac{V_{2r}-V_{1r}}{\pi}; \tag{20}$$

$$V_{mr}=\Delta t\Omega_{mr};m=1,2; \tag{21}$$

where $\Delta t=T/N$ - sampling step.

It seems that the use of the same symbols to denote discrete analogs of sub-band cores (sub-band matrices) and signals does not lead to distortion of the essence.

The discretization step must be small enough in the sense of fulfilling the inequalities

$$\varepsilon_G=1-\int_0^T\int_0^T A_G(t_1-t_2)x(t_1)x(t_2)dt_1dt_2/\|x\|^2\ll 1,$$

where G - corner frequency interval satisfying the condition,

$$G\Delta t=(-\pi,\pi). \tag{22}$$

The product on the left side of (22) is usually called the area of normalized circular frequencies, which determines the period of changes in the spectrum of the sample vector (keeping the notation):

$$X(\omega)=\sum_{k=1}^{N}x_k\exp(j\omega k)\,;\ -\pi\le\omega\le\pi. \tag{23}$$

It is clear that with a limited range of definition of the spectrum of the sampled signal, only a finite number of sub-bands are formed, which completely cover the area (22). Under conditions of uncertainty regarding the properties of the MBPLA structural elements reacting to sounding, it seems natural to divide this area as follows (a zero sub-band is introduced)

$$V_{01}=0;\ V_{02}=\Delta V/2\,;\ \Delta V=V_{2r}-V_{1r}=const;$$
$$r=1,\cdots,R; \tag{24}$$

$$\Delta V(2R+1)=2\pi. \tag{25}$$

It is easy to show that then, in accordance with (20), the sub-band matrices can be represented in the following form

$$A_r=C_rA_0C_r+S_rA_0S_r, \tag{26}$$

where

$$A_0=\{a_{ik}^{0}\};$$
$$a_{ik}^{0}=\frac{\sin(\Delta V(i-k)/2)}{\pi(i-k)};\ a_{ik}^{0}=\frac{\Delta V}{2\pi};\ i=1,\cdots,N; \tag{27}$$

$$\begin{aligned}C_r&=\mathrm{diag}(\cos(\omega_r),\cdots,\cos(\omega_rN))\,;\\ S_r&=\mathrm{diag}(\sin(\omega_r),\cdots,\sin(\omega_rN))\,;\end{aligned} \tag{28}$$

$$\omega_r=r\Delta V;r=0,\cdots,R. \tag{29}$$

Due to the symmetry of the matrices A_r and their positive definiteness (follows from the positivity of the sub-band parts of the energies of signals of finite duration), they are matrices of a simple structure and their eigenvalues are positive [17]. Computational experiments, however, show that only defined by the following relation (the square bracket means the integer part of the number)

$$J_0=[N\Delta V/2\pi]+2, \tag{30}$$

some of the eigenvalues of the zero sub-band matrix are significantly different from zero. Therefore, the relation

$$A_0=Q_0L_0Q'_{0'}, \tag{31}$$

where L_0 – diagonal matrix of nonzero eigenvalues,

$$L_0 = \mathrm{diag}\left(\lambda_1^0, \cdots, \lambda_{J_0}^0\right), \tag{32}$$

Q_0 – matrix of eigenvectors corresponding to nonzero eigenvalues,

$$A_0 Q_0 = Q_0 L_0. \tag{33}$$

Based on the definition of the zero sub-band matrix, one can prove the inequality for the eigenvalues [18]

$$1 \geq \lambda_1^0 \geq \cdots \geq \lambda_{J_0}^0 > 0. \tag{34}$$

In turn, using representation (26), one can prove the validity of the following relations for the eigenvalues of sub-band matrices of other sub-bands.

$$\lambda_{2k-1}^r = \lambda_{2k}^r = \lambda_k^0; k = 1, \cdots, J_0. \tag{35}$$

For matrices of eigenvectors corresponding to eigenvalues with odd and even indices, the following representations are valid [19]

$$\begin{gathered} Q_r^c = C_r Q_0; \\ Q_r^s = S_r Q_0. \end{gathered} \tag{36}$$

Note that it is expedient to compose the general matrix of eigenvalues bearing in mind the order determined by the indices in (35). Then the corresponding eigenvectors in the common matrix will also be interspersed in the same order.

It is easy to show that the substitution of (26) in (18) and (19) gives the corresponding representations in terms of the projections of the original vector onto the eigenvectors of the sub-band matrices

$$P_r(\vec{x}) = \sum_{k=1}^{J_0} \lambda_k^0 \left(\alpha_{kr}^2 + \beta_{kr}^2\right); \tag{37}$$

$$\vec{y}_r = \vec{y}_{cr} + \vec{y}_{sr}; \tag{38}$$

$$\begin{gathered} \vec{y}_{cr} = C_r Q_0 L_0 \vec{\alpha}_r; \\ \vec{y}_{sr} = C_r Q_0 L_0 \vec{\beta}_r; \end{gathered} \tag{39}$$

$$\begin{gathered} \vec{\alpha}_r = \left(\alpha_{1r}, \cdots, \alpha_{J_0 r}\right)' = Q'_0 C_r \vec{x}; \\ \vec{\beta}_r = \left(\beta_{1r}, \cdots, \beta_{J_0 r}\right)' = Q'_0 S_r \vec{x}. \end{gathered} \tag{40}$$

Obviously, these relations show the possibility of efficient parallelization of the computations of these sub-band characteristics.

It is of interest to estimate the required dimension of the used sub-band matrices. For this, as a measure of the difference between the spectra of the original and the vectors obtained on the basis of relation (19), we use the concept of the sub-band part of the energy of their difference, which after obvious transformations takes the following form

$$P_r(\vec{x} - \vec{y}_r) \sum_{k=1}^{J_0} \lambda_k^0 \left(1 - \lambda_k^0\right)^2 \left(\alpha_{kr}^2 + \beta_{kr}^2\right) = (1 - \lambda_{mean}^0)^2 P_r. \tag{41}$$

Here we mean that, in view of the positiveness of the factors in the representation of the terms, there is always some mean value of any of them, which makes it possible to express the sum in this form (mean value theorem). In this case, the multiplier in front of the sub-band part of the energy can be interpreted as a relative error. In view of inequality (34), its value will be less than one.

In order for vector (19) to sufficiently reflect the properties of the segment of the Fourier transform of the initial data, it is necessary to ensure that the right-hand side of (41) is close to zero. It is clear that this is achieved when the maximum projections onto the eigenvectors correspond to the largest eigenvalues, and the eigenvalues themselves are close to unity. Computational experiments show that the number of eigenvalues sufficiently close to unity is determined by the relation

$$I_0 = J_0 - 3. \tag{42}$$

Since the positivity of the right-hand side of (42) must be ensured, bearing in mind (30), we obtain the requirement for the number of samples

$$N \geq 4\pi/\Delta V = 2\nu_d/\Delta\nu, \quad (43)$$

where on the right we mean the sampling rate

$$\nu_d = 1/\Delta t, \quad (44)$$

and the width of the original sub-bands in the frequency domain

$$\Delta\nu = \Delta V/2\pi. \quad (45)$$

It is easy to show that, in this case, the duration of the processed continuous realization must satisfy the inequality

$$T = N\Delta t \geq 2/\Delta\nu. \quad (46)$$

4. Decisive procedure for sub-band response detection

It is assumed that after a certain delay in relation to sounding by short pulses at the output of the receiver, the vector of signal samples $\vec{x}$ of dimension N is recorded. It is necessary to make a decision regarding the validity of the following initial hypothesis [20].

H_9: the components of the vector $\vec{x}$ are generated in the absence of responses from the MBPLA.

$$\vec{x} \equiv \vec{u} = (u_1, \cdots, u_N)^T, \quad (47)$$

where $u_i, i = 1, \cdots, N$ – samples of some centered noise with uncorrelated samples (E is the mean symbol),

$$E[u_i u_{i+\tau}] = \sigma_u^2, \tau = 0; \quad E[u_i u_{i+\tau}] = 0, \tau \neq 0. \quad (48)$$

In the simplest case, the opposite hypothesis is formulated as follows.

$H_1 = \overline{H}_0$: the components of the vector $\vec{x}$ are recorded in the presence of a response/ $\vec{w}_r = (w_{1r}, \cdots, w_{Nr})'$ from MBPLA in some previously unknown sub-band from a previously formed set of them, that is

$$x_i = w_{ir} + u_i, i = 1, \cdots, N. \quad (49)$$

The sub-band test of the validity of the initial hypothesis should be performed by detecting a possible response in each of the sub-bands.

It is proposed to use components of vectors of the form (40) as features. Bearing in mind (47), when the initial hypothesis is fulfilled,

$$\vec{\alpha}_r^x = \vec{\alpha}_r^u = Q'_0 C_r \vec{u}; \quad \vec{\beta}_r^x = \vec{\beta}_r^u = Q'_0 S_r \vec{u}. \quad (50)$$

Comparison of these relations with definitions (28) of the matrices included in them shows that quadrature processing is implemented, in which the role of the filter is played by the matrix of eigenvectors of the zero sub-band matrix.

The centrality and the property of uncorrelatedness (48) of the noise, as well as the equality to unity of the Euclidean norms of the eigenvectors of the zero sub-band matrix, make it possible to obtain the following relations for the numerical characteristics of the components of the vectors (50)

$$E[\vec{\alpha}_r^u] = E[\vec{\beta}_r^u] = 0; m = 1, \cdots, J_0; \quad (51)$$

$$s_{mr}^{\alpha 2} = E[(\alpha_{mr}^2)^2] = \sigma_u^2/2 + \sigma_u^2 \sum_{k=1}^N \cos(2\omega_r k) q_{km}^2/2\,; \quad (52)$$

$$s_{mr}^{\beta 2} = E[(\beta_{mr}^2)^2] = \sigma_u^2/2 - \sigma_u^2 \sum_{k=1}^N \cos(2\omega_r k) q_{km}^2/2. \quad (53)$$

These relations imply the obvious equalities

$$E[(\vec{\alpha}_{mr}^{u})^2] + E\left[\left(\vec{\beta}_{mr}^{u}\right)^2\right] = \sigma_u^2. \tag{54}$$

It is clear that among the eigenvectors of the zero matrix there is one on which the minimum variance of the projection is achieved. This circumstance and relation (54) speaks in favor of using separately the responses of the quadrature components as a feature space for detecting the responses of the quadrature components.

Under the conditions of the opposite hypothesis (49), the quadrature components of the projections onto the eigenvectors are determined by the following relations

$$\vec{\alpha}_r^x = \vec{\alpha}_r^w + \vec{\alpha}_r^u = Q'_0 C_r(\vec{w} + \vec{u}); \tag{55}$$

$$\vec{\beta}_r^x = \vec{\beta}_r^w + \vec{\beta}_r^u = Q'_0 S_r(\vec{w} + \vec{u}). \tag{56}$$

It is also easy, taking into account (51), to obtain relations for the mathematical expectations of these vectors

$$\vec{e}_r^{\alpha} = \left(e_{1r}^{\alpha}, \cdots, e_{J_0 r}^{\alpha}\right)' = E[\vec{\alpha}_r^x] = E[\vec{\alpha}_r^w]; \tag{57}$$

$$\vec{e}_r^{\beta} = \left(e_{1r}^{\beta}, \cdots, e_{J_0 r}^{\beta}\right)' = E\left[\vec{\beta}_r^x\right] = E\left[\vec{\beta}_r^w\right]. \tag{58}$$

Assuming that the responses are stationary during some time, a large number of soundings of values in the sense of preserving the values of the components of vectors (57) and (58), in order to reduce the variances of the components of the second terms in (55) and (56), it is advisable to average the resulting projections, obtaining estimates of the vectors mathematical expectations

$$\hat{\vec{e}}_r^{\alpha} = \sum_{n=1}^{k} \vec{\alpha}_r^x(n)/K, \tag{59}$$

$$\hat{\vec{e}}_r^{\beta} = \sum_{n=1}^{k} \vec{\beta}_r^x(n)/K. \tag{60}$$

Here the argument in parentheses means the act of probing.

It is important that if the terms in (59) and (60) are uncorrelated from sounding to sounding, then the variances of their noise components compared with (52) and (53) will decrease by a factor of K

$$\begin{aligned} d_{mr}^{\alpha 2} &= s_{mr}^{\alpha 2}/K; \\ d_{mr}^{\beta 2} &= s_{mr}^{\beta 2}/K. \end{aligned} \tag{61}$$

When checking the validity of the initial hypothesis, we use the following decision rule: the hypothesis of the absence of a response in the tested sub-band is rejected if at least one of the following inequalities is satisfied

$$|\hat{e}_{mr}^{\alpha}| \geq h_{mr}^{\alpha}, m = 1, \cdots, J_0; \tag{62}$$

$$\left|\hat{e}_{mr}^{\beta}\right| \geq h_{mr}^{\beta}, m = 1, \cdots, J_0. \tag{63}$$

where h_{mr}^{α} и $h_{mr}^{\beta}, m = 1, \cdots, J_0$ – some thresholds that are determined in the learning process with a deliberate lack of responses.

The learning process is implemented on the basis of α given probability α of errors of the first kind (false alarms). It determines the required number M of repetitions of averaging procedures of the form (59) and (60) (here the square bracket is the integer part of the number) in the deliberate absence of responses

$$M = [1/\alpha] + 1. \tag{64}$$

In the absence of responses, the thresholds are determined from the condition

$$h_{mr}^{\alpha} = \max\left|\sum_{k=1}^{N} \cos(\omega_r k) q_{km}(k) \bar{u}_k^i\right|, \quad (65)$$

where $i = 1, \cdots, M$.

$$h_{mr}^{\beta} = \max\left|\sum_{k=1}^{N} \sin(\omega_r k) q_{km}(k) \bar{u}_k^i(i)\right|, \quad (66)$$

where $i = 1, \cdots, M$.

Here the symbol $\bar{u}_k^i$ means the component of the noise vector obtained with the *i* averaging (see (59) and (60))

$$\bar{u}_k^i = \sum_{n=1}^{K} u_k^i(n)/K. \quad (67)$$

Here, again, the symbol n in brackets denotes the probing number in the formation of sums of the form (59) and (60). Thus, averaging is reduced to averaging the noise vectors.

It is of interest to estimate the probability of errors of the second kind (missing a target) [20]. To do this, assume that the noise has a Gaussian distribution. Then the ratios for the probabilities of errors of the second kind in the detection of responses have the form

$$p_{mr}^{\alpha} = F((h_{mr}^{\alpha} + |e_{mr}^{\alpha}|)/d_{mr}^{\alpha} + F\,(h_{mr}^{\alpha} - |e_{mr}^{\alpha}|)/d_{mr}^{\alpha}) - 1; \quad (68)$$

$$p_{mr}^{\beta} = F\left(\left(h_{mr}^{\beta} + \left|e_{mr}^{\beta}\right|\right)/d_{mr}^{\beta} + F\left(h_{mr}^{\beta} - \left|e_{mr}^{\beta}\right|\right)/d_{mr}^{\beta}\right) - 1. \quad (69)$$

Here we mean the integral of the probability for the Gaussian distribution [20]

$$F(c) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{c} \exp(-x^2/2)dx. \quad (70)$$

These relations show that in order to achieve the probability of correct detection of the order of 0.98 with the probability of a type I error of the order of 0.0001, at least one of the following inequalities must be satisfied

$$|e_{mr}^{\alpha}| > 2d_{rm}^{\alpha}. \quad (71)$$

5. Conclusion

Within the framework of this article, the urgent problem of radar detection of small-sized UAVs to ensure the safety of flights of aircraft at low altitudes is considered. The conditions of reaction to sounding of only individual parts of structures of small-sized UAVs of unknown dimensions are considered. Under these conditions, the responses to sounding are formed on the basis of resonances in a previously unknown frequency band. Therefore, it is advisable to use sounding with broadband signals, for example, multifrequency signals in the form of a chirp with digital shaping.

Sub-band processing of the received signals, which is realized on the basis of dividing the domain of spectral definitions into sub-bands, is of paramount importance, this is how you can adapt to the frequency response band from small objects and filter noise.

The article describes the elements of the mathematical apparatus of sub-band signal analysis in the form of sub-band matrices and their eigenvalues and vectors. The concept of sub-band parts of signal energies is introduced and an optimal solution to the problem of filtering responses in given sub-bands is obtained in the sense of minimizing the sub-band measure of the square error of approximations.

A procedure for processing received signals when making decisions about the presence of a response in a given sub-band is developed and an estimate of the probabilities of erroneous decisions is given for a given probability of errors of the first kind.

Acknowledgements

The article was prepared with the financial support of a grant from the President (project NSH-2686.2020.8 «Models, methods and means of receiving and processing information about space objects in a wide spectral range of electromagnetic waves».

This study was supported by the Russian Federation State Task № 0555-2021-0005.

References

[1] Makarenko S I, Timoshenko A V and Vasilchenko A S 2020 Counter Unmanned Aerial Vehicles. Part 1. Unmanned aerial vehicle as an object of detection and destruction *Systems of Control, Communication and Security* **1** 109-46
[2] Zhukov A O, Valyaev I N, Kovalenko V P, Turlov Z N, Bondareva M K and Kartsan I N 2020 Synthesis of an optimal algorithm for processing random signals during phase direction finding *Journal of Physics: Conf. Ser.* **1679(3)** 032021
[3] Dembickij N L, Pankratov V A, Timoshenko A V, Zhukov A O, Kartsan I N, Chefonov N S and Chefonov V S 2020 Distributed control network for complex radio engineering systems on continuum computers *Journal of Physics: Conf. Ser.* **1515(4)** 042076
[4] Kobzareva Yu B 1969 *Modern radar (analysis, evaluation and system design)* (Moscow: Soviet radio) 699
[5] James D Taylor 2000 *Ultra-wideband radar technology* ed (Florida, Boca Raton: CRC Press LLC) 500
[6] Kartsan I N, Zhukov A O, Timoshenko A V, Perlov A Y and Ryabchenko D V 2020 Precising the evaluation of radio information system reliability characteristics based on the empirical failure distribution *IOP Conf. Ser.: Mater. Sci. Eng.* **919(5)** 052002
[7] Zhukov A O, Valyaev I N, Kovalenko V P, Turlov Z N, Novikov A S, Kartsan I N and Malanina Y N 2020 Narrow-band correlation processing of signals in a phase direction-finding *Journal of Physics: Conf. Ser.* **1515(2)** 022044
[8] Zhukov A O, Valyaev I N, Kovalenko V P, Turlov Z N, Chebotarev A S, Kartsan I N and Shumakova N A 2020 Adaptation of receiving channels to the spectrum of the received signal *Journal of Physics: Conf. Ser.* **1515(3)** 032070
[9] Murav'iov V V, Korenevsky S A, Kostykevich K A and Stanul A A 2013 The experementional studies of the generator of wideband signals were conducted by the method of direct digital synthesis *Devices and methods of measurements* **2(7)** 28-34
[10] Koshelev V I, Sarychev V T, Shipilov S E and Yakubov V P 2001 Estimation of radar objects information features at ultrawideband sounding *Journal of Radio Electronics* **5**
[11] David K Barton 2013 *Radar Equations for Modern Radar* (Boston, London: Artech house)
[12] Zhilyakov E G, Belov S P, Oleinik I I, Babarinov S L, and Trubitsyna D I 2020 Generalized sub band analysis and signal synthesis *Bulletin of Electrical Engineering and Informatics* **1(9)** 964-72
[13] Zhukov A O, Valyaev I N, Kovalenko V P, Turlov Z N, Minin I V, Zaverzaev A A, Kartsan I N and Malanina Y N 2020 Method for phase direction finding of signals with asymmetric spectra *Journal of Physics: Conf. Ser.* **1515(2)** 022051
[14] Kartsan I N, Zhukov A O, Goloshchapova B A, Zalivin A N, Oleynik I I and Orishchuk S G 2020 Experimental studies of differences in the distribution parameters of multidimensional spatial-subband object vectors on video images during subband information processing *Journal of Physics: Conf. Ser.* **1679(5)** 052025
[15] Ronald L Allen and Duncan W Mills 2004 *Signal analysis. Time, frequency, scale, and structure* (Ieee Press. Wiley-interscience. A John Wiley & Sons, Inc., Publication) 500
[16] Khurgin YA I and Yakovlev V P 1971 *Finite functions in physics and technology* (Moscow: Nauka) 408
[17] Gantmakher F R 2010 *Matrix theory* (Moscow: FIZMATLIT) 560
[18] Horn R and Dzhonson CH 1989 *Matrix Analysis* (Moscow: Mir) 655
[19] Zhilyakov E G 2015 Optimal sub-band methods for analysis and synthesis of finite-duration signals *Automation and Remote Control* **76(4)** 589-602
[20] Tikhonov V I 1966 *Statistical radio engineering* (Moscow: Soviet radio) 678