On the Transition Radiation and Bremsstrahlung from a Relativistic Electron with a Nonequilibrium Field

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The problem of the transition radiation from an electron with a nonequilibrium self-field appearing in view of the sharp scattering of the electron has been considered. It has been shown that the state of the electron with the nonequilibrium field is manifested in the suppression of transition radiation and in the oscillatory dependence of its characteristics on the distance from a plate on which radiation occurs to the scattering point. The problem of the measurement of the characteristics of bremsstrahlung under the conditions when macroscopic transverse distances are responsible for the scattering process has been considered. It has been shown that the results of the measurement in this case significantly depend on the size of the detector and on its position with respect to the scattering point.

1. The scattering of an electron is accompanied by the rearrangement of the field around it, leading to the radiation of electromagnetic waves. Radiation from an ultrarelativistic particle is formed at lengths along the momentum of the particle that are much larger than the wavelength of the radiated wave. Such lengths are called the coherence lengths of the radiation process [1–3]. The field around the electron within these lengths is strongly different from a Coulomb field. Such a state of the electron with the nonequilibrium field is manifested in the next scattering events, leading to various effects of the suppression of the bremsstrahlung in the process of the motion of the electron in matter, in particular, the Landau–Pomeranchuk–Migdal effect [4, 5] and the effect of the suppression of radiation in a thin matter layer (TSF effect [6, 7]). Recent detailed experimental investigations of these effects at ultrahigh energies of particles accelerated at the SLAC [8] and CERN [9] confirm the main predictions of the theory. These investigations were carried out in X- and gamma-ray ranges of radiated photons.

In this work, we emphasize that the state of the electron with the nonequilibrium field can be manifested not only at ultrahigh energies of the electron, but also when the energy of the electron is 10–100 MeV in a millimeter range of radiated waves. The coherence length of the radiation process can be macroscopic in this case, so that it covers not only the target, but also detecting instruments. In this case, new manifestations of the state of the electron with the nonequilibrium field are possible in view of the presence of not only larger longitudinal sizes, but also transverse sizes responsible for the radiation process. We show that the characteristics of bremsstrahlung in the case under consideration can strongly depend both on the size of a detector and on its position with respect to the scattering point. The effects appearing in bremsstrahlung in this case are close to similar effects in transition radiation in the prewave zone [10–13]. We also show that the state of the electron with the nonequilibrium field is strongly manifested in the process of the subsequent radiation of transition radiation from such an electron. In this case, both the suppression of transition radiation and the oscillatory dependence of the characteristics of radiation on the distance between the plate on which transition radiation appears and the electron scattering point are possible. The causes of such effects are discussed.

2. If the velocity of the electron sharply changes at the time $t = 0$ from the initial direction $\mathbf{v}$ to the final direction $\mathbf{v}'$, the retarded solution for the scalar potential of the electromagnetic field at $t > 0$ has the form [14, 15]

$$\varphi(\mathbf{r}, t) = \theta(r - t)\varphi_{\mathbf{v}}(\mathbf{r}, t) + \theta(t - r)\varphi_{\mathbf{v}'}(\mathbf{r}, t), \quad (1)$$

where $\varphi_{\mathbf{v}}$ and $\varphi_{\mathbf{v}'}$ are the Coulomb potentials of the electrons uniformly moving along the z and z' axes, respectively (see Fig. 1). The speed of light c is taken to be unity.

The first term in Eq. (1) describes the field "escaping" from the particle in the process of scattering. It is

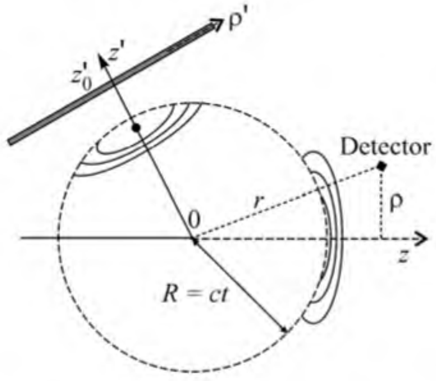

Fig. 1. Normal incidence of a scattered electron with a nonequilibrium field on a thin ideally conducting plate.

nonzero outside the sphere with the radius $r = t$ with the center at the scattering point.

The second term in Eq. (1) is the Coulomb field "grown" to this time around the electron moving with the velocity **v**' in the scattering direction. This field is nonzero inside the sphere $r = t$, i.e., in the spatial region at which the signal of the scattering of the electron arrives. The Fourier expansion of this term is the sum of the Fourier expansions of the Coulomb field of an electron moving in the direction **v**' and a package of free waves $\varphi^{f}_{v'}$ moving in the same direction:

$$\varphi_{v'} + \varphi^{f}_{v'} = \theta(t-r)\varphi_{v'}(\mathbf{r}, t)$$
$$= \frac{e}{2\pi^2}\mathrm{Re}\int\frac{d^3k}{k}\frac{e^{i\mathbf{kr}}[e^{-i\mathbf{kv}'t} - e^{-ikt}]}{k - \mathbf{kv}'}. \quad (2)$$

The expression for the vector potential of the field under consideration is similar to Eq. (1).

According to Eq. (2), the Fourier components making the main contribution to the Coulomb field of the electron are suppressed in the field around the electron for a long time $\Delta t \approx 2\gamma^2/\omega$. The electron in such a state is called the electron with the nonequilibrium self-field.

The electron with an energy of 10–100 MeV in the millimeter range of wavelengths in such state can cover macroscopic distances reaching several dozen meters. Let us show that the transition radiation of the electron with the nonequilibrium field can significantly differ from the transition radiation of a uniformly moving particle. To this end, we consider the backward transition radiation appearing on an ideally conducting plate located at a distance z'_0 from the scattering point in the direction of the final motion of the electron (see Fig. 1). The field incident on the plate can be represented in this case in the form of the Fourier expansion given by Eq. (2). Using the boundary condition that the tangential component (along ρ') of the total electric field on the surface of the plate is zero, we find that the tangential component of the harmonic of the electric field of transition radiation in the backward direction in the wave zone (i.e., for $-z' \gg 2\gamma^2/\omega$) has the form

$$\mathbf{E}^{f}_{\omega\perp}(\mathbf{r}) = \frac{2e}{v'}\frac{e^{i\omega r}}{r}\frac{\boldsymbol{\vartheta}}{\boldsymbol{\vartheta}^2 + 1/\gamma^2}\left\{e^{i\frac{z'_0\omega}{2\gamma^2}(1+\gamma^2\vartheta^2)} - v'\right\}. \quad (3)$$

Here, the z' axis is directed along the velocity **v**' and the angle ϑ is measured from the direction $-\mathbf{v}'$, r is the distance from the point of the intersection of the plate by the electron to the point at which the electric field is considered, $r \approx z'_0 - z' + \rho'^2/2(z'_0 - z')$, and γ is the Lorentz factor of the electron.

In this case, the spectral–angular density of the backward transition radiation is given by the expression

$$\frac{d\varepsilon}{d\omega do} = \frac{e^2}{\pi^2}\frac{\vartheta^2}{(\vartheta^2 + \gamma^{-2})^2}$$
$$\times 2\left\{1 - \cos\left[\frac{\omega z'_0}{2}(\gamma^{-2} + \vartheta^2)\right]\right\}. \quad (4)$$

Formula (4) differs from the corresponding expression for the spectral–angular density of the transition radiation of the electron with the equilibrium field in the interference term in the braces and in a factor of 2 in front of these braces. According to Eq. (4), under the condition $z'_0 \ll 2\gamma^2/\omega$, the backward transition radiation is suppressed as compared to the transition radiation of the particle always moving along the z' axis without scattering. This effect is caused by the long-term existence of the electron in the state with the nonequilibrium field in which the Fourier components with the wave vector **k** that make the main contribution to the Coulomb field of the electron are suppressed for a long time in the electromagnetic field around the electron.

According to Eq. (4), oscillations of the radiation intensity with an increase in z'_0 occur under the condition $z'_0 \geq 2\gamma^2/\omega$ with the period

$$\Lambda = 4\pi/\omega(\vartheta^2 + \gamma^{-2}), \quad (5)$$

which is of the order of the coherence length of the radiation process. These oscillations can be detected at the distances z'_0 satisfying the condition

$$z'_0 < 2\pi/\Delta\omega(\vartheta^2 + \gamma^{-2}), \quad (6)$$

where $\Delta\omega$ is the frequency resolution of the detector. At large distances $z'_0 \gg \Lambda$, in view of the nonzero resolution of the detector, the oscillations under consideration disappear and the spectral–angular density of radiation given by Eq. (4) is the sum of the contributions to transition radiation from the electron self-field reflected from the plate and the bremsstrahlung field in this direction. The contributions from these fields are identical; for this reason, a factor of 2 appears in front of the brace in Eq. (4). Note that this result is valid for the plate with sufficiently large transverse sizes covering the entire bremsstrahlung of the scattered electron. Thus, Eq. (4) shows that the possibility of the long-term existence of the electron in the state with the nonequilibrium field should be manifested in the process of the radiation of transition radiation from the electron.

3. For the considered energies of the electron in the millimeter range of wavelengths, not only the longitudinal lengths of the formation of radiation ($l_C \approx 2\gamma^2/\omega$), but also the characteristic transverse distances responsible for the radiation process ($l_T \approx \gamma/\omega$) are macroscopic. Let us demonstrate that under these conditions, the results of the measurement of the characteristics of bremsstrahlung appearing under the sharp deflection of the electron at a large angle significantly depend on the size of the used detector and on its position with respect to the scattering point. To this end, we consider the problem of the detection of the field escaping from the electron by the point detector. This field is a free wave packet, which moves in the direction of the initial velocity of the electron **v** and is continuously rearranged to the bremsstrahlung field [16]. The scalar potential of this field is determined by the first term in Eq. (1). Its Fourier expansion has the form

$$\varphi_{\mathbf{v}}^{\mathrm{f}} = \theta(r-t)\varphi_{\mathbf{v}}(\mathbf{r}, t) = \frac{e}{2\pi^2}\mathrm{Re}\int\frac{d^3k}{k}\frac{e^{i\mathbf{kr}-ikt}}{k-\mathbf{kv}}, \qquad (7)$$

where $k = \sqrt{k_z^2 + k_\perp^2}$ and $\mathbf{k}_z$ and $\mathbf{k}_\perp$ are the components of the vector **k** parallel and perpendicular to the velocity **v**, respectively. The Fourier expansion similar to Eq. (7) can also be obtained for the vector potential of the escaping field.

The point detector in the problem under consideration is a detector whose sizes are much smaller than the transverse length $\Delta\rho \approx \gamma/\omega$ within which the Fourier components with the frequency ω ($k = |\omega|$) in the wave packet given by Eq. (7) are located at $t = 0$. Such a detector records electromagnetic radiation with the frequency ω that falls into a small spatial region in which the detector is located (see Fig. 1). The spectral–angular density of this radiation can be calculated using the Fourier expansion of the scalar, φ^{f}, and vector, $\mathbf{A}^{\mathrm{f}}$, potentials of the escaping field.

For the characteristic radiation angles in the relativistic case $\vartheta \approx 1/\gamma$, we expand the expressions for φ^{f} and $\mathbf{A}^{\mathrm{f}}$ in the small parameter $k_\perp/k$ and retain terms up to the second order. In the framework of the used approximation, $|\mathbf{A}_\omega^{\mathrm{f}}| = |\varphi_\omega^{\mathrm{f}}|$ and the spectral–angular density of bremsstrahlung at the detector location point $\mathbf{r} = (\boldsymbol{\rho}, z)$ is given by the expression

$$\frac{d\varepsilon}{d\omega do} = \frac{r^2}{4\pi^2}|\mathbf{E}_{\omega\perp}(\mathbf{r})|^2 = \left(\frac{2e^2\xi}{\pi v}\right)^2\left|\int_0^\infty x^2dx\frac{J_1(x\xi\vartheta)}{x^2+\gamma^{-2}}\cos(\omega z x^2/2)\right|^2, \qquad (8)$$

where $\xi = \omega z$. This expression for the spectral–angular density of radiation is valid both in the wave ($z \gg l_C$) and prewave ($z \le l_C$) zones.

If $z \gg l_C$, the integral in Eq. (8) can be calculated using the stationary phase method. Then, this formula gives the known result of the theory of bremsstrahlung [17],

$$\frac{d\varepsilon}{d\omega do} = \frac{e^2}{\pi^2}\frac{\vartheta^2}{(\vartheta^2+\gamma^{-2})^2}. \qquad (9)$$

The situation with the emission of bremsstrahlung at small distances from the scattering point $z \ll l_C$ is different. In this spatial region, Eq. (8) gives the following spectral–angular distribution of radiation:

$$\frac{d\varepsilon}{d\omega do} = \frac{4e^2}{\pi^2}\frac{1}{\vartheta^2}\sin^2\left(\frac{\omega z\vartheta^2}{4}\right). \qquad (10)$$

According to this expression, radiation in the prewave zone is primarily concentrated in the region of the characteristic angles $\vartheta \approx 1/\sqrt{\omega z}$, which exceed the characteristic angles of radiation in the wave zone ($\vartheta \approx 1/\gamma$). This means that the angular distribution of bremsstrahlung detected by the point-like detector in the prewave zone ($z \le l_C$) is wider than the distribution recorded by this detector in the wave zone ($z \gg l_C$). Moreover, as follows from Eq. (10), the spectrum of bremsstrahlung at small distances from the scattering point depends on the frequency of radiated waves.

The above results correspond to the case where measurements are performed by the point-like detectors at various distances from the scattering point. However, measurements can be carried out by a detector much larger than the characteristic transverse lengths of the radiation process $\Delta\rho \approx \gamma/\omega$. In contrast to the point-like detector, such a detector records all waves with the frequency ω radiated in the direction of the wave vector **k** by the scattered electron. In order to calculate the spectral–angular distribution of bremsstrahlung detected by a long detector with a large area, it is necessary to integrate Eq. (8) over the entire considered area and to represent the result as the integral over the directions of the wave vectors of radiated waves. In this case, the integrand is the desired distribution. In this case, this distribution is given by Eq. (9), which is independent of the distance from the detecting plane to the scattering point. Therefore, the angular distribution of radiated waves

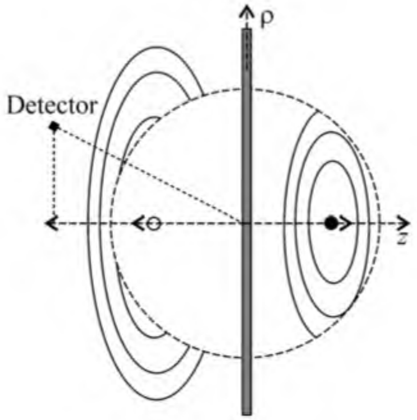

Fig. 2. Total field in the ultrarelativistic case after the intersection of the thin ideally conducting plate by the electron.

with the given vector **k** is the same in both the wave and prewave zones. In other words, the readings of a detector larger than the effective transverse region responsible for the formation of radiation $\Delta\rho \approx \gamma/\omega$ will be independent of the distance from the detector to the scattering point. They are also independent of the frequency of detected photons and coincide exactly with the readings of the point-like detector at large distances from the scattering point ($z \gg l_C$). The pointlike detector at small distances from the scattering point ($z < l_C$) will detect a wider distribution of radiation, according to Eq. (10).

Similar effects attributed to large transverse radiation lengths occur for backward transition radiation when the electron intersects the thin metallic plate [10–13]. This is explained by the fact that the structure of the fields appearing in this case is similar to that appearing in the instantaneous scattering of the electron at a large angle. Indeed, using the boundary condition for the field on the surface of the thin metal plate intersected by the electron, the transition radiation field can be represented in the form of the Fourier expansion

$$\varphi^{f}(\mathbf{r}, t) = -\frac{e}{2\pi^2 v}\int_{-\infty}^{+\infty} d\omega e^{-i\omega t} \times \int \frac{d^2 k_\perp}{k_\perp^2 + \omega^2/(v^2\gamma^2)} e^{i(\omega z\sqrt{1 - k_\perp^2/\omega^2} + \mathbf{k}_\perp \rho)}. \quad (11)$$

Analytically calculating integral (11), we arrive at the following explicit expression for the potential of the total field after the intersection of the plate by the electron:

$$\varphi(\mathbf{r}, t) = [\varphi_{\mathbf{v}}(\mathbf{r}, t) - \varphi_{-\mathbf{v}}(\mathbf{r}, t)] \times [\vartheta(r - t)\vartheta(-z) + \vartheta(t - r)\vartheta(z)], \quad (12)$$

where $\varphi_{\mathbf{v}}$ and $\varphi_{-\mathbf{v}}$ are the Coulomb potentials of the electron and its image in the plate, respectively. The pattern of the equipotential field surfaces determined by Eq. (12) in the ultrarelativistic case is shown in Fig. 2.

As is seen, the structure of the field to the left of the plate after the intersection of the plate by the ultrarelativistic electron (this field forms backward transition radiation) is similar, but not identical to the structure of the field escaping from the scattered electron. In this case, the field around the electron passed through the plate is similar in structure to the nonequilibrium field around the scattered electron. Such a structural similarity of the fields explains the existence of similar effects in bremsstrahlung and transition radiation in the considered cases.

REFERENCES

1. M. L. Ter-Mikaelyan, *High-Energy Electromagnetic Processes in Condensed Media* (AN ArmSSR, Yerevan, 1969; Wiley, New York, 1972).
2. V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Course of Theoretical Physics*, Vol. 4: *Quantum Electrodynamics* (Nauka, Moscow, 1989; Pergamon, Oxford, 1982).
3. A. I. Akhiezer and N. F. Shul'ga, *High Energy Electrodynamics in Matter* (Nauka, Moscow, 1993; CRC, Boca Raton, 1996).
4. L. D. Landau and I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR **92**, 73 (1953).
5. A. B. Migdal, Dokl. Akad. Nauk SSSR **94**, 49 (1954).
6. F. F. Ternovskii, Zh. Eksp. Teor. Fiz. **39**, 171 (1960) [Sov. Phys. JETP **12**, 123 (1960)].
7. N. F. Shul'ga and S. P. Fomin, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 126 (1978) [JETP Lett. **27**, 117 (1978)].
8. S. Klein, Rev. Mod. Phys. **71**, 1501 (1998).
9. H. D. Thomsen et al., Phys. Rev. D **81**, 052003 (2010).
10. V. A. Verzilov, Phys. Lett. A **273**, 135 (2000).
11. N. F. Shul'ga and S. N. Dobrovol'skii, Zh. Eksp. Teor. Fiz. **117**, 668 (2000 [J. Exp. Theor. Phys. **90**, 579 (2000)]).
12. M. Castellano, V. Verzilov, L. Catani, et al. Phys. Rev. E **67**, 015501 (2003).
13. A. P. Potylitsyn, M. I. Ryazanov, M. I. Strikhanov, and A. A. Tishchenko, *Diffraction Radiation from Relativistic Particles* (Tomsk. Politekhn. Univ., Tomsk, 2008; Springer, Berlin–Heidelberg, 2010).
14. E. L. Feinberg, Zh. Eksp. Teor. Fiz. **50**, 202 (1966) [Sov. Phys. JETP **23**, 132 (1966)].
15. A. I. Akhiezer and N. F. Shul'ga, Usp. Fiz. Nauk **151**, 385 (1987) [Sov. Phys. Usp. **30**, 197 (1987)].
16. N. F. Shul'ga, V. V. Syshchenko, and S. N. Shul'ga, Phys. Lett. A **374**, 331 (2009).
17. B. M. Bolotovskii and A. V. Serov, Usp. Fiz. Nauk **167**, 1107 (1997) [Phys. Usp. **40**, 1055 (1997)].

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