Peculiarities in the low energy range of the bremsstrahlung spectrum

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Abstract

Peculiarities in bremsstrahlung and polarization bremsstrahlung from fast electrons caused by the collective contribution of target's atoms to the formation of emitted photons are considered theoretically. It is shown that such collective contribution can result in the strong suppression or, in the contrary, in substantial growth of the emission yield relative to that in the case of fast electron interaction with a separate atom. Some new effects of this kind are analyzed in detail and the possibility to observe them experimentally is estimated.

Keywords: Bremsstrahlung; Polarization bremsstrahlung; Coherence length; Multiple scattering; Interference effect; Relativistic electron; Non-relativistic electron

1. Introduction

The collision of a fast charged particle with an atom is attended by the process of a photon emission. This emission known as bremsstrahlung (**B**) was considered until 1970s as being due to the acceleration of the incident particle in the screened Coulomb field of the atom. More recently it was shown (Buimistrov, 1972; Amusia et al., 1976; Zon, 1977) that the scattering of the projectile's Coulomb field by atomic electrons gives rise to an additional to **B** contribution known as so-called polarization bremsstrahlung (**PB**).

The very important property of emission process lies in the fact that the emitted photon is formed at a distance of the order of so-called coherence length l_{coh} (Ter-Mikaelian, 1972) along the emitting particle trajectory ($l_{\mathrm{coh}} \sim V/\omega$ for non-relativistic and $l_{\mathrm{coh}} \sim 2\gamma^2/\omega$ for relativistic particles, ω is the energy of emitted photon, γ is the Lorenz–factor of the emitting particle, V is its velocity). Different atoms make approximately independent contributions to the formation of the emission yield if the mean interatomic distance in a target $\bar{l}$ exceeds l_{coh}, otherwise these contributions are strongly correlated.

Such correlations well known in physics of ordinary **B** can occur even though a high energy particle moves through an amorphous dense medium. The classical electrodynamic effects: Landau–Pomeranchuk–Migdal (LPM) effect consisting in the suppression of **B** yield within the range of small energies of emitting photons due to the influence of multiple scattering of the emitting electron (Landau–Pomeranchuk, 1953) and

Ter-Mikaelian (TM) effect of **B** yield suppression due to the change of the phase velocity of emitted photon because of the polarization of target electrons (Ter-Mikaelian, 1954) are good cases in point. A great variety of another examples of the discussed correlations arises in **B** from relativistic particles crossing an aligned crystal (see for example Ter-Mikaelian, 1972; Akhieser and Shul'ga, 1996). Thus, the important role of the collective contribution of medium atoms to the formation of the ordinary **B** yield is recognized clearly.

As for analogous collective effects in **PB** processes the present state of the studies in the field is more complicated. It should be noted that the most part of theoretical and experimental studies of **PB** cross-section, i.e. the photon energy and angular distribution, has been concentrated on the process of the collision of a fast incident particle with a free atom (Tsytovich and Oiringel, 1993; Semaan and Quarles, 1981; Verkhovtseva et al., 1983; Korol and Solov'yov, 1997). Nevertheless, some experiments were devoted to measurements of **PB** from solid targets. Relativistic (Blazhevich et al., 1996, 1999) and non-relativistic electrons beams (Ambrose et al., 1996; Quarles and Portillo, 1999) were used in these experiments. The main result of non-relativistic experiments lies in the fact that no **PB** contribution to total emission yield has been observed. This unexpected result is sharply contradicted by the theory of **PB** on the free atom.

Analogous result consisting in the **PB** suppression in the overall spectrum of emitted photons has been obtained in the experiment (Blazhevich et al., 1996) where the relativistic electron **PB** from a thin diamond-like carbon foil was measured. The possible explanation of this results has been proposed (Nasonov et al., 2001a) on the base of the developed theory of **PB** from relativistic electrons crossing a fine grained medium. Not only the discussed suppression effect, but an existence of sharp peaks in **PB** spectrum has been observed in the experiment (Blazhevich et al., 1999) where **PB** spectrum from relativistic electrons crossing a thin *Al* foil was measured. It is significant that the experimental results (Blazhevich et al., 1999) are in a good agreement with theoretical predictions based on the developed theory of **PB** from relativistic electrons moving through a polycrystalline medium (Nasonov, 1998).

The role of **PB** is not reduced to an additional contribution to ordinary **B** yield only. This emission mechanism is substantial to the understanding of the nature of a variety of emission processes considered usually within the frame of macroscopic electrodynamics by the use of space-depending dielectric permeability $\varepsilon(\omega, \mathbf{r})$. Well-known examples of such processes are the parametric X-ray radiation (**PXR**) appearing due to the coherent Bragg diffraction of the Coulomb field of a fast charged particle crossing a crystalline target (Ter-Mikaelian, 1972; Garibian and Yang, 1971; Baryshevsky and Feranchuk, 1971) and the resonant transition radiation (**RTR**) arising from the constructive interference between transition radiation waves emitted by a fast electron crossing a periodically layered medium (Ter-Mikaelian, 1972). It is reasonable to assume that these and other emission mechanisms of this kind can be considered as the coherent part of **PB**. The deduction of **RTR** cross-section made on the base of microscopic approach accepted in **PB** theory has been presented (Platonov et al., 1990). **PXR** as the emission of atoms embedded at specific sites in the lattice of a crystal and excited by the Coulomb field of a fast incident particle was considered (Dialetis, 1978). The identity of **PXR** to the coherent part of **PB** from relativistic electrons crossing a crystalline target has been demonstrated (Lapko and Nasonov, 1990) and more recently (Nasonov and Safronov, 1993) where the strong suppression of incoherent part of **PB** from the crystal has been shown.

Since **PB** photons are emitted by non-relativistic atomic electrons **PB** spectrum is concentrated mainly in the range of small energies (X-ray range) for both non-relativistic and relativistic incident particles. It is in this range that the background from the side of ordinary **B** can be modified significantly due to coherent effects. Therefore, the study of the peculiarities in **B** spectral and angular distributions caused by the collective contribution of medium atoms is very important for revealing the relative contribution of ordinary **B** and **PB** to total emission yield. One of the aims of this paper consists in the study of normal **B** properties in circumstances where a new suppression effect caused by the saturation of the angle of emitting particle multiple scattering can be manifested. Characteristics of normal **B** produced by both non-relativistic electrons moving through an amorphous dense medium and relativistic electrons crossing an aligned crystal are studied in next two sections.

The occurrence of a coherent part in **PB** from relativistic electrons moving through different structurally targets is considered in Section 4 within the frame of perturbation theory. It should be noted in this connection that the use of relativistic electrons beams to search **PB** experimentally is very convenient because of the possibility to separate **PB** contribution by the measurement of an emission yield under the observation angle Θ large enough relative to characteristic emission angle of relativistic particle γ^{-1}.

PXR as a coherent part of **PB** from relativistic electrons crossing a perfect crystalline target is considered in the last section on the base of general dynamical diffraction theory (Pinsker, 1984). The possibility to observe dynamical diffraction effects in **PXR** experiment is discussed in this section.

2. Suppression of the bremsstrahlung from non-relativistic electrons due to multiple scattering of emitting particles

The emission of normal **B** photons in the single collision of a fast electron with an atom has been studied in detail (Tseng and Pratt, 1971; Kissel et al., 1983; Pratt and Feng, 1985). More complicated task is to describe the spectral and angular distributions of **B** photons emitted from a thick target, because of the necessity to take into account emitting particle energy losses and multiple scattering as well as the self-photoabsorption of emitted photons.

There are two specific suppression effects in **B** process (above outlined LPM and TM effects), but these effects are fundamentally relativistic in nature and cannot be realized for non-relativistic emitting electrons. We want to show here the new suppression effect caused by the correlations between elementary emission amplitudes corresponding to consecutive collisions of emitting non-relativistic electron with atoms in a dense amorphous medium.

Since our prime interest here is with the fundamental aspect of the discussed problem we will consider the spectrum of non-collimated ordinary **B** in the limit $\omega \ll \varepsilon$ (ε is the energy of an emitting electron) when a classical electrodynamics is legitimate. The spectral distribution of the number of emitted photons is given by well-known expression (Landau and Lifshitz, 1965)

$$\frac{\mathrm{d}N}{\mathrm{d}\omega} = \frac{2e^2}{3\pi\omega}\langle|\mathbf{W}(\omega)|^2\rangle, \tag{1a}$$

$$\mathbf{W}(\omega) = \int \mathrm{d}t\, \mathrm{e}^{\mathrm{i}\omega t}\dot{\mathbf{V}}(t), \tag{1b}$$

where $\mathbf{V}(t)$ is the velocity of emitting electron, $\dot{\mathbf{V}}(t) \equiv (\mathrm{d}/\mathrm{d}t)\mathbf{V}(t)$, brackets $\langle\rangle$ mean averaging over all possible trajectories of the electron in a target.

The acceleration $\dot{\mathbf{V}}(t)$ is different from zero during the small time intervals $\Delta t \approx R/V$ only corresponding to collisions of the electron with atoms (R is the characteristic size of an atom which is approximately equal to the screening radius in the Fermi–Thomas model of the atom; obviously, the interval Δt is much smaller than the average inter-collision time $\bar{\tau}$).

Let us consider the emission spectrum $\mathrm{d}N/\mathrm{d}\omega$ in the small frequency range $\omega\Delta t \ll 1$. As this takes place, the formula (1b) can be reduced to

$$\mathbf{W}(\omega) = \sum_n \mathrm{e}^{\mathrm{i}\omega t_n}\vec{\xi}_n, \tag{2}$$

where t_n is the point in time corresponding to nth collision of the emitting electron with an atom, $\vec{\xi}_n = \mathbf{V}_n - \mathbf{V}_{n-1}$, $\mathbf{V}_n$ and $\mathbf{V}_{n-1}$ are the vectors of the electron velocity after and before nth collision, summation in (2) is over all collisions.

Substituting the expression (2) to (1a) gives

$$\frac{\mathrm{d}N}{\mathrm{d}\omega} = \frac{2e^2}{3\pi\omega}\sum_n\sum_m\langle\cos\omega(t_n - t_m)\rangle\langle\vec{\xi}_n\vec{\xi}_m\rangle, \tag{3}$$

where averaging is performed over accidental quantities t_n and two-dimensional scattering angles $\mathbf{\Psi}_n \equiv (\Psi_n, \eta_n)$ describing the change of the electron velocity $\mathbf{V}$ in nth collision ($\mathbf{V}_n\mathbf{V}_{n-1} = V^2\cos^2\Psi_n$, η_n is the azimuth angle of the projection of the vector $\mathbf{V}_n$ on a plane normal to the vector $\mathbf{V}_{n-1}$, since atoms are located accidentally in the amorphous medium the angle η_n is uniformly distributed in the interval 0, 2π). Taking into account the statistical independence of different $\mathbf{\Psi}_n$ one can obtain the following formulae:

$$\begin{aligned}\langle\xi_k^2\rangle &= 4V^2\langle\sin^2(\Psi/2)\rangle, \langle\vec{\xi}_k\vec{\xi}_{k+l}\rangle \\ &= -4V^2\langle\sin^2(\Psi/2)\rangle\langle\cos\Psi\rangle^{l-1},\end{aligned} \tag{4}$$

where $\langle\sin^2(\Psi/2)\rangle = \sigma^{-1}\int\sin^2(\Psi/2)\,\mathrm{d}\sigma$, σ is the cross-section of an elastic scattering of the electron in a single collision.

Since $t_n = \sum_{l\leqslant n}\tau_l$, τ_l are the statistically independent accidental quantities—time intervals between consecutive collisions of the electron with different atoms, the formula $\langle\cos\omega(t_n - t_m)\rangle$ can be presented as

$$\langle\cos\omega(t_n - t_m)\rangle = \tfrac{1}{2}\langle\mathrm{e}^{\mathrm{i}\omega\tau}\rangle^{|n-m|} + \tfrac{1}{2}\langle\mathrm{e}^{-\mathrm{i}\omega\tau}\rangle^{|n-m|}. \tag{5}$$

Using the distribution function $f(\tau) = \bar{\tau}^{-1}\exp(-\tau/\bar{\tau})$ (Rytov, 1976) ($\bar{\tau}^{-1} = \sigma n_0 V$, n_0 is the density of atoms in the target) one can obtain from (5) the following result:

$$\langle\cos\omega(t_n - t_m)\rangle = \mathrm{Re}\left(\frac{1}{1 - \mathrm{i}\omega\bar{\tau}}\right)^{|n-m|}. \tag{6}$$

The final result following from the general formula (3) with account of the expressions (4) and (6) is given by

$$\frac{\mathrm{d}N}{\mathrm{d}\omega} = \frac{T}{\bar{\tau}}\frac{\mathrm{d}N_{\mathrm{B-H}}}{\mathrm{d}\omega}F\left(\frac{\omega}{\omega_*}, \omega_* T\right), \tag{7a}$$

$$\frac{\mathrm{d}N_{\mathrm{B-H}}}{\mathrm{d}\omega} = \frac{8e^2V^2}{3\pi\omega}\langle\sin^2(\Psi/2)\rangle, \tag{7b}$$

$$\begin{aligned}F(x,y) = \frac{1}{1+x^2}\Bigg[x^2 + \frac{1}{y}\Bigg(&\frac{1-x^2}{1+x^2}(1 - \mathrm{e}^{-y}\cos yx) \\ &+ \frac{2x}{1+x^2}\mathrm{e}^{-y}\sin yx\Bigg)\Bigg],\end{aligned} \tag{7c}$$

where T is the full time of the electron motion in the target (obviously, the ratio $T/\bar{\tau}$ is the total number of collisions), $\mathrm{d}N_{\mathrm{B-H}}/\mathrm{d}\omega$ is the classical limit of the Bethe–Heitler formula, the very important quantity ω_* is determined by the formula

$$\omega_* = 2n_0V\int\sin^2(\Psi/2)\,\mathrm{d}\sigma = 16\pi Z^2e^4n_0R^3m\Phi(2mRV), \tag{8a}$$

$$\Phi(x) = \frac{1}{x^3}\left[\ln(1+x^2) - \frac{x^2}{1+x^2}\right], \tag{8b}$$

where Z is the number of electrons in an atom, m is the electron mass, the simplest approximation for the potential of the atom $\varphi = (Ze/r)\exp(-r/R)$ has been used when deriving the formula (8a).

Let us analyze the obtained results. In accordance with (7a) and (7b) a difference between the derived formula for $dN/d\omega$ and classical Bethe–Heitler result $dN_{B-H}/d\omega$ is described by the function $F(x, y)$. As evident from (7c), $F(x, y) \approx 1$ if $x \gg 1$ (or $\omega \gg \omega_*$) so that the spectrum of B from non-relativistic electrons moving through a dense amorphous medium coincides with Bethe–Heitler spectrum within the range of high enough energies of emitted photons $\omega \gg \omega_*$. The spectrum behaviour in the small frequency range $\omega \leqslant \omega_*$ depends strongly on the parameter $y = \omega_* T$. The function $F(x, y) \approx 1$ if $y < 1$ so that Bethe–Heitler result is correct in the overall emission spectrum for thin enough targets when $\omega_* T < 1$. The new effect in non-relativistic electron B consisting in the strong suppression of an emission yield in the photon energy range $\omega \leqslant \omega_*$ appears with the proviso that $\omega_* T \gg 1$. The function $F(\omega/\omega_*, \omega_* T)$ is presented in Fig. 1.

To explain the predicted suppression effect it should be stressed that this effect is caused by the correlations between consecutive collisions of the emitting electron with target's atoms described by non-diagonal terms in

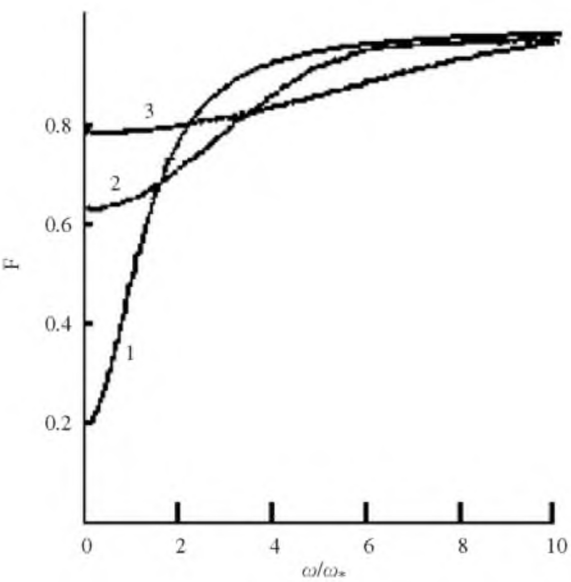

Fig. 1. Suppression of low energy bremsstrahlung from non-relativistic electrons in a dense medium. The function $F(\omega/\omega_*, \omega_* T)$ and the characteristic frequency ω_* are determined by Eqs. (7), (8). Curves 1, 2 and 3 have been calculated for the values of the parameter $\omega_* T = 5$, 1 and 0.5, respectively.

the general formula (3). Indeed, Bethe–Heitler result (7b) follows immediately from (3) if it is granted that $\langle \vec{\xi}_k \vec{\xi}_{k+l} \rangle = 0$. With allowance made for such correlations, target's atoms can make a collective contribution to the formation of B photons. Since this photon is formed at the segment of the emitting electron trajectory of the order of l_{coh} (Ter-Mikaelian, 1972) only the contribution of atoms located along such segments must be taken into account. It is clear (see the expression (6) as well) that the expected modification of B spectrum can be essential in the small frequency range $\omega \ll 1/\bar{\tau}$ (or $l_{coh} \gg V\bar{\tau}$) only where the emitting electron suffers many collisions at the distance l_{coh}. Separating the trajectory of this electron into above segments one can estimate the emission spectrum $dN/d\omega$ as the sum of independent contributions from these segments

$$\omega \frac{dN}{d\omega} \approx \frac{VT}{l_{coh}} \frac{dE_{coh}}{d\omega} = \frac{T}{\bar{\tau}} \frac{1}{M_{coh}} \frac{dE_{coh}}{d\omega}, \tag{9a}$$

$$\frac{dE_{coh}}{d\omega} \approx \frac{2e^2}{3\pi} \langle (\mathbf{V} - \mathbf{V}_{coh})^2 \rangle = \frac{8e^2 V^2}{3\pi} \langle \sin^2(\Psi_{coh}/2) \rangle, \tag{9b}$$

where Ψ_{coh} is the scattering angle achievable at the distance l_{coh} ($\mathbf{V}\mathbf{V}_{coh} = V^2 \cos \Psi_{coh}$), $M_{coh} = l_{coh}/V\bar{\tau} = 1/\omega\bar{\tau}$ is the number of collisions at the coherence length l_{coh}, the important quantity $(1/M_{coh})\,(dE_{coh}/d\omega)$ in (9a) describes an average yield per single collision.

Let us consider (9) in two limiting cases $\omega_* T \ll 1$ and $\omega_* T \gg 1$. With the constraint $\omega_* T = 2(T/\bar{\tau}) \langle \sin^2(\Psi/2) \rangle \ll 1$ a total scattering angle is small and in consequence the angle Ψ_{coh} is small as well. As this takes place $\langle \Psi_{coh}^2 \rangle = M_{coh} \langle \Psi^2 \rangle$ (Ψ is the angle of electron scattering in the single collision) and the quantity $(1/M_{coh})\,(dE_{coh}/d\omega)$ is reduced to Bethe–Heitler result (7b).

The condition $\omega_* T \gg 1$ implies that the total scattering angle is large. The angle Ψ_{coh} can be large as well in the range of small enough photon energies ω because $M_{coh} \sim 1/\omega$. In contrast with the dependence $M_{coh}(\omega)$ the function $dE_{coh}(\omega)/d\omega \sim \langle (\mathbf{V} - \mathbf{V}_{coh})^2 \rangle$ is bounded above and therefore the quantity $(1/M_{coh})\,(dE_{coh}/d\omega)$ decreases in value with decreasing ω. In such a manner the suppression of **B** from non-relativistic electrons is caused by the limitation on the possible change of the emitting electron velocity $\mathbf{V}(t)$ induced by multiple scattering.

Such an effect is impossible for relativistic electrons emitting B photons in an amorphous medium because the condition $\langle \Psi_{coh}^2 \rangle = M_{coh} \langle \Psi^2 \rangle$ is always fulfilled for the discussed emission process. On the other hand the coherent **B** from relativistic electrons crossing an aligned crystal can be suppressed by the analogous effect as will be shown in the next section.

Let us estimate the field of existence of the predicted suppression effect. Since the argument of function Φ

in (8) is usually much more than unit the characteristic frequency ω_* can be determined as

$$\omega_* \approx 4\pi\left(\frac{Z}{13t}\right)^2 \frac{n_0 \ln(2mRV)}{m^2 V^3}. \tag{10}$$

In line with this formula thin foils consisting of heavy metals are most suited for experimental search of the predicted suppression effect. For example: $\omega_* \approx 42\,\text{eV}$ and the parameter $\omega_* T \approx \omega_* L/V \approx 336 \gg 1$ for golden target with the thickness $L = 10^{-5}\,\text{cm}$ and incident electron beam with particle's energy $\varepsilon = 1\,\text{keV}$ ($\omega_* \simeq 118\,\text{eV}$ if $\varepsilon = 500\,\text{eV}$ under the same other conditions.) Based on these estimations the experimental observation of the predicted effect appears quite possible.

3. Peculiarities in the coherent bremsstrahlung from relativistic electrons crossing a thin aligned crystal

As indicated above, the strong modification of bremsstrahlung spectrum can be realized not only in the emission from non-relativistic electrons but in coherent bremsstrahlung from relativistic electrons as well. This effect is caused by the peculiarity of an emitting electron scattering in an average potential of atomic strings in a crystal. This peculiarity consists in the conservation of the transverse component of the momentum of incident electron $p_\perp = p \sin\Psi \simeq p\Psi$ (Ψ is the orientation angle between the full momentum $\mathbf{p}$ and string's axis) (Akhieser and Shul'ga, 1996). Vector $\mathbf{p}_\perp$ is rotated around the string's axis because of scattering and therefore the maximum possible scattering angle $\eta_{\max}$ is bounded above by the value 2Ψ. Because of this, the scattering angle η_{coh} achievable at the distance of the order of coherence length l_{coh} and the energy distribution of emitted photons $\text{d}E_{\text{coh}}/\text{d}\omega$ (see formulae 9) are bounded above as well. As a result a photon yield must be suppressed in the range of small enough photon energies ω where the coherence length $l_{\text{coh}} \sim 1/\omega$ increases beyond all bounds with decreasing of ω.

Let us consider this effect in greater detail. Characteristics of coherent bremsstrahlung depend strongly in the correlations between consecutive collisions of the emitting electron with different atomic strings. If the orientation angle Ψ exceeds the critical angle of axial channelling Ψ_{ch} and the transverse component of the electron's momentum $\mathbf{p}_\perp$ is oriented close to directions of planar channelling a constructive interference between emission amplitudes on different strings can be realized. As this takes place strong maxima appear in the spectrum of coherent bremsstrahlung (Ter-Mikaelian, 1972). The mentioned constructive interference is destroyed in the range of small orientation angles $\Psi \sim \Psi_{\text{ch}}$ due to the essential growth of coherent azimuthal scattering of emitted particles by atomic strings (Akhieser and Shul'ga, 1996). In the case being considered the spectrum of coherent bremsstrahlung in the range of high enough photon energies ω where the coherence length l_{coh} is less than the average path of emitting electron between consecutive collisions of this electron with atomic strings $\bar{l}_\perp/\Psi$ ($\bar{l}_\perp = 1/\sqrt{n_0 d}$, n_0 is the density of atoms, d is the distance between neighbouring atoms in the string) is determined by the electron interaction with a single string. More complicated situation is realized in small photon energy range where $l_{\text{coh}} \gg \bar{l}_\perp/\Psi$ and the emission yield is formed in the process of multiple scattering of emitted electrons on atomic strings. It is precisely this photon energy range that is of prime interest for our studies.

To describe the spectral-angular distribution of emitted photons let us introduce two-dimensional angular variables $\mathbf{\Theta}$ and $\mathbf{\Psi}_t \equiv \mathbf{\Psi}(\mathbf{t})$ determining the unit vector $\mathbf{n}$ to the direction of emitted photon propagation and the velocity of emitting electron $\mathbf{V}(t)$ in accordance with formulae

$$\mathbf{n} = \mathbf{e}(1 - \tfrac{1}{2}\Theta^2) + \mathbf{\Theta}, \quad \mathbf{e\Theta} = 0, \tag{11a}$$

$$\mathbf{V}(t) = \mathbf{e}(1 - \tfrac{1}{2}\gamma^{-2} - \tfrac{1}{2}\Psi_t^2) + \mathbf{\Psi}_t, \quad \mathbf{e\Psi}_t = 0, \tag{11b}$$

where $\mathbf{e}$ is the string's axis, it is suggested that the electron moves in a crystal at small angles Ψ_t relative to $\mathbf{e}$.

Taking into account transition radiation and bremsstrahlung emission mechanisms and using generally accepted methods of classical electrodynamics (Landau and Lifshitz, 1965) (we consider an emission process in the small photon energy range $\omega \ll \varepsilon = m\gamma$), one can obtain the following expression for the total emission amplitude $\mathbf{A_n}$ in a very convenient form for further analysis (Nasonov, 2002):

$$\begin{aligned}\mathbf{A_n} = \frac{e}{\pi}\Bigg[&(\mathbf{\Psi}_\text{i} - \mathbf{\Theta})\left(\frac{1}{\gamma^{-2} + (\mathbf{\Psi}_\text{i} - \mathbf{\Theta})^2} - \frac{1}{\gamma_*^{-2} + (\mathbf{\Psi}_\text{i} - \mathbf{\Theta})^2}\right) \\ &- (\mathbf{\Psi}_\text{f} - \mathbf{\Theta})\left(\frac{1}{\gamma^{-2} + (\mathbf{\Psi}_\text{f} - \mathbf{\Theta})^2} - \frac{1}{\gamma_*^{-2} + (\mathbf{\Psi}_\text{f} - \mathbf{\Theta})^2}\right) \\ &\times \exp\left(\frac{\text{i}\omega}{2}\int_0^L \text{d}t(\gamma_*^{-2} + (\mathbf{\Psi}_t - \mathbf{\Theta})^2)\right) \\ &- \int_0^L \text{d}t \exp\left(\frac{\text{i}\omega}{2}\int_0^t \text{d}\tau(\gamma_*^{-2} + (\mathbf{\Psi}_\tau - \mathbf{\Theta})^2)\right) \\ &\times \frac{\text{d}}{\text{d}t}\frac{\mathbf{\Psi}_t - \mathbf{\Theta}}{\gamma_*^{-2} + (\mathbf{\Psi}_t - \mathbf{\Theta})^2}\Bigg],\end{aligned} \tag{12}$$

where $\mathbf{\Psi}_\text{i}$ and $\mathbf{\Psi}_\text{f}$ are initial and final values of $\mathbf{\Psi}_t$, respectively, L is the thickness of the target, $\gamma_*^{-2} = \gamma^{-2} + \omega_0^2/\omega^2$, ω_0 is the plasma frequency of the target.

As it is easy to see, the first and second terms in (12) describe transition radiation from *in* and *out*-surfaces of the target. These terms vanish without account of the target's dielectric susceptibility $\chi(\omega) = -\omega_0^2/\omega^2$. The last term in (12) corresponds to bremsstrahlung contribution. This term vanishes for the particle moving with a constant velocity.

Since the emitting particle velocity $\mathbf{V}(t)$ can be changed essentially during small time intervals only corresponding to collisions of this electron with atomic strings, one can represent the total emission amplitude (12) as a sum of elementary amplitudes describing the emission an different strings, much as it has been done in the previous section. In the small frequency range

$$\frac{\omega R}{2\gamma_*^2 \Psi} \ll 1, \tag{13}$$

where the coherence length l_{coh} exceeds essentially the effective path of emitting electron in a string's potential R/Ψ and in a consequence the emission appearing in kth collision is determined by the scattering angle $\mathbf{\Psi}_{k+1} - \mathbf{\Psi}_k$ only ($\mathbf{\Psi}_{k+1}$ and $\mathbf{\Psi}_k$ are the values of $\mathbf{\Psi}(t)$ after and before kth collision) the total emission amplitude (12) is reduced to

$$\begin{aligned}\mathbf{A_n} = \frac{e}{\pi}\Bigg[&(\mathbf{\Psi}_{\mathrm{i}} - \mathbf{\Theta})\left(\frac{1}{\gamma^{-2} + (\mathbf{\Psi}_{\mathrm{i}} - \mathbf{\Theta})^2} - \frac{1}{\gamma_*^{-2} + (\mathbf{\Psi}_{\mathrm{i}} - \mathbf{\Theta})^2}\right)\\ &- (\mathbf{\Psi}_{\mathrm{f}} - \mathbf{\Theta})\left(\frac{1}{\gamma^{-2} + (\mathbf{\Psi}_{\mathrm{f}} - \mathbf{\Theta})^2} - \frac{1}{\gamma_*^{-2} + (\mathbf{\Psi}_{\mathrm{f}} - \mathbf{\Theta})^2}\right)\\ &\times \exp\left(\frac{\mathrm{i}\omega}{2}\sum_k (\gamma_*^{-2} + (\mathbf{\Psi}_k - \mathbf{\Theta})^2)\tau_k\right)\\ &- \sum_k \left(\frac{\mathbf{\Psi}_k - \mathbf{\Theta}}{\gamma_*^{-2} + (\mathbf{\Psi}_k - \mathbf{\Theta})^2} - \frac{\mathbf{\Psi}_{k-1} - \mathbf{\Theta}}{\gamma_*^{-2} + (\mathbf{\Psi}_{k-1} - \mathbf{\Theta})^2}\right)\\ &\times \exp\left(\frac{\mathrm{i}\omega}{2}\sum_{j\leqslant k} (\gamma_*^{-2} + (\mathbf{\Psi}_j - \mathbf{\Theta})^2)\tau_j\right)\Bigg],\end{aligned} \tag{14}$$

where τ_k is the interval between $k-1$th and kth collisions.

Following from (14) expression for the spectral-angular distribution of emitted photons

$$\omega\frac{\mathrm{d}^3 N}{\mathrm{d}\omega\,\mathrm{d}^2\Theta} = \langle|\mathbf{A_n}|^2\rangle = \omega\frac{\mathrm{d}^3 N^{\mathrm{cb}}}{\mathrm{d}\omega\,\mathrm{d}^2\Theta} + \omega\frac{\mathrm{d}^3 N^{\mathrm{tr}}}{\mathrm{d}\omega\,\mathrm{d}^2\Theta} + \omega\frac{\mathrm{d}^3 N^{\mathrm{int}}}{\mathrm{d}\omega\,\mathrm{d}^2\Theta}, \tag{15}$$

takes into account among contributions of coherent bremsstrahlung (first term in (15) and transition radiation (second term) an interference between these emission mechanisms). Here, as in the previous section, the brackets mean averaging over emitting electron trajectories described by accidental variables $\mathbf{\Psi}_k$ and τ_k.

Let us consider coherent bremsstrahlung contribution using the general expressions (14) and (15). The most general formula for the spectral-angular distribution of emitted photons has the form

$$\begin{aligned}\omega&\frac{\mathrm{d}^3 N^{\mathrm{cb}}}{\mathrm{d}\omega\,\mathrm{d}^2\Theta}\\ &= \frac{e^2}{\pi^2}\sum_k \Bigg\langle \left(\frac{\mathbf{\Psi}_k - \mathbf{\Theta}}{\gamma_*^{-2} + (\mathbf{\Psi}_k - \mathbf{\Theta})^2} - \frac{\mathbf{\Psi}_{k-1} - \mathbf{\Theta}}{\gamma_*^{-2} + (\mathbf{\Psi}_{k-1} - \mathbf{\Theta})^2}\right)^2\\ &+2\,\mathrm{Re}\sum_{m\geqslant 1}\left(\frac{\mathbf{\Psi}_k - \mathbf{\Theta}}{\gamma_*^{-2} + (\mathbf{\Psi}_k - \mathbf{\Theta})^2} - \frac{\mathbf{\Psi}_{k-1} - \mathbf{\Theta}}{\gamma_*^{-2} + (\mathbf{\Psi}_{k-1} - \mathbf{\Theta})^2}\right)\\ &\times\left(\frac{\mathbf{\Psi}_{k+m} - \mathbf{\Theta}}{\gamma_*^{-2} + (\mathbf{\Psi}_{k+m} - \mathbf{\Theta})^2} - \frac{\mathbf{\Psi}_{k+m-1} - \mathbf{\Theta}}{\gamma_*^{-2} + (\mathbf{\Psi}_{k+m-1} - \mathbf{\Theta})^2}\right)\\ &\times\exp\left(\frac{\mathrm{i}\omega}{2}\sum_{p=1}^{m}(\gamma_*^{-2} + (\mathbf{\Psi}_{k+p} - \mathbf{\Theta})^2)\tau_{k+p}\right)\Bigg\rangle.\end{aligned} \tag{16}$$

Averaging of the expression (16) is very complicated task in the general case, therefore we restrict our consideration to the case of small enough scattering angles when the condition of a dipole approximation in the theory of relativistic particle emission

$$\gamma^2\langle(\Delta\Psi_{\mathrm{coh}})^2\rangle \ll 1, \tag{17}$$

is fulfilled, $\Delta\mathbf{\Psi}_{\mathrm{coh}}$ is the scattering angle achievable at the distance of the order of coherence length $l_{\mathrm{coh}}{\sim}2\gamma^2/\omega$.

Performing necessary expansions by the use of (17) we can reduce the general formula (16) to more simple one

$$\begin{aligned}\omega\frac{\mathrm{d}^3 N^{\mathrm{cb}}}{\mathrm{d}\omega\,\mathrm{d}^2\Theta} = \frac{e^2}{\pi^2}\frac{1}{(\gamma_*^{-2} + \tilde{\Theta}^2)^2}\sum_k \Bigg\langle &(\mathbf{\Psi}_k - \mathbf{\Psi}_{k-1})^2\\ &- \frac{4\gamma_*^{-2}}{\gamma_*^{-2} + (\tilde{\Theta}^2)^2}(\tilde{\mathbf{\Theta}}, \mathbf{\Psi}_k - \mathbf{\Psi}_{k-1})^2\\ &+ 2\,\mathrm{Re}\sum_{m\geqslant 1}(\mathbf{\Psi}_k - \mathbf{\Psi}_{k-1}, \mathbf{\Psi}_{k+m} - \mathbf{\Psi}_{k+m-1})\\ &- \frac{4\gamma_*^{-2}}{(\gamma_*^{-2} + \tilde{\Theta}^2)^2}(\tilde{\mathbf{\Theta}}, \mathbf{\Psi}_k - \mathbf{\Psi}_{k-1})\\ &\times(\tilde{\mathbf{\Theta}}, \mathbf{\Psi}_{k+m} - \mathbf{\Psi}_{k+m-1})\\ &\times\exp\left(\frac{\mathrm{i}\omega}{2}(\gamma_*^{-2} + \tilde{\Theta}^2)\sum_{p=1}^{m}\tau_{k+p}\right)\Bigg\rangle,\end{aligned} \tag{18}$$

where $\tilde{\mathbf{\Theta}} = \mathbf{\Theta} - \mathbf{\Psi}_{\mathrm{i}}$.

Averaging over τ_k in (18) is performed by analogy with (5) and (6)

$$\left\langle \exp\left(\frac{\mathrm{i}\omega}{2}(\gamma_*^{-2} + \tilde{\Theta}^2)\sum_{j=1}^{k}\tau_j\right)\right\rangle = \left(1 - \frac{\mathrm{i}\omega}{2}\bar{\tau}(\gamma_*^{-2} + \tilde{\Theta}^2)\right)^{-k}, \tag{19}$$

where $\bar{\tau} = \bar{l}_\perp/\Psi$.

When averaging over $\mathbf{\Psi}_k$ in (18) one should take into account the unique property of electron coherent scattering by the atomic potential of atomic strings consisting in the conservation law: $\Psi_k^2 = \Psi_{k+1}^2$. It is well known that only azimuthal angle χ of the vector $\mathbf{\Psi}_t$ is changed due to electron collision with atomic string. Therefore,

$$\mathbf{\Psi}_k = \Psi(\mathbf{e}_x \cos(\chi_k) + \mathbf{e}_y \sin(\chi_k)), \quad \chi_k = \sum_{j \leqslant k} \Delta\chi_j, \qquad (20)$$

where $\Delta\chi_j$ is the change of azimuthal angle χ in jth collision, it is clear that all $\Delta\chi_j$ are independent accidental quantities. To average the expression (18) over $\Delta\chi_j$ one should use the following formulae:

$$\langle\cos\chi_k\rangle = \langle\cos\Delta\chi\rangle^k, \quad \langle\sin\chi_k\rangle = 0,$$

$$\langle\cos\Delta\chi\rangle = \frac{2}{e_\perp}\int_0^\infty \mathrm{d}b\cos(\Delta\chi(b)),$$

$$\Delta\chi = \pi - 2b\int_{\rho_0}^\infty \frac{\mathrm{d}\rho}{\rho^2\sqrt{1-(b^2/\rho^2)+(\Psi_{\mathrm{ch}}^2/\Psi^2)f(\rho)}}, \qquad (21)$$

where $1-(b^2/\rho^2)+(\Psi_{\mathrm{ch}}^2/\Psi^2)f(\rho_0)=0$, b is the impact parameter the electron collision with a string, the string's potential is defined as $\varphi(\rho)=\varphi_0 f(\rho)$, $f(0)=1$, $\Psi_{\mathrm{ch}}^2 = 2e\varphi_0/m\gamma$.

The expression for $\langle\cos\Delta\chi\rangle$ in (21) shows clearly that only above-barrier electrons are taken into account of our approach. Ignoring the contribution from the side of channelling electrons to studied emission yield is justified by the small value of this contribution in the frequency range (13) under consideration. Indeed, the spectrum of channelling radiation is mainly concentrated in the range $\omega \geqslant 2\gamma^2\Psi_{\mathrm{ch}}/R$ (Akhieser and Shul'ga, 1996). The yield of channelling radiation photons with energies $\omega \ll 2\gamma^2\Psi_{\mathrm{ch}}/R$ is strongly suppressed due to periodicity of channelling particle trajectories, because $\mathbf{E}_\omega \sim \mathbf{W}_\omega \to 0$ if $\omega \to 0$ for the trajectories with $\langle\mathbf{W}(t)\rangle = 0$ ($\mathbf{E}_\omega$ and $\mathbf{W}_\omega$ are the Fourier-transforms of the emission electrical field $\mathbf{E}(\mathbf{r},t)$ and emitting electron acceleration $\mathbf{W}(t)$).

Using (20)–(21) one can perform averaging of the expression (18) and obtain the following result:

$$\omega\frac{\mathrm{d}^3N^{\mathrm{cb}}}{\mathrm{d}\omega\,\mathrm{d}\tilde{\Theta}\tilde{\Theta}} = \frac{4e^2\Psi^2}{\pi}(1-\langle\cos\Delta\chi\rangle)\frac{L}{\bar{\tau}} \times \frac{\gamma_*^{-4}+\tilde{\Theta}^4}{(\gamma_*^{-2}+\tilde{\Theta}^2)^4}\frac{\omega^2}{\omega^2+\omega_*^2(\tilde{\Theta})}, \qquad (22)$$

where the quantity $\omega_*(\tilde{\Theta})$ is defined by the formula

$$\omega_*(\tilde{\Theta}) = \frac{2}{\bar{\tau}}\frac{1-\langle\cos\Delta\chi\rangle}{\gamma_*^{-2}+\tilde{\Theta}^2} \equiv \omega\frac{1-\langle\cos\Delta\chi\rangle}{\bar{\tau}}l_{\mathrm{coh}}. \qquad (23)$$

The important formula (17) determining the field of applications of the result (22) can be rewritten as

$$\gamma^2\langle(\Delta\mathbf{\Psi})_{\mathrm{coh}}^2\rangle = 2\gamma^2\Psi^2(1-\langle\cos\Delta\chi\rangle^{l_{\mathrm{coh}}/\bar{\tau}}) \approx 2\gamma^2\Psi^2\left[1-\exp\left(-\frac{1-\langle\cos\Delta\chi\rangle}{\bar{\tau}}l_{\mathrm{coh}}\right)\right] \ll 1. \qquad (24)$$

Let us analyze the result (22). The distribution (22) coincides with the ordinary spectral-angular distribution of coherent bremsstrahlung (Akhieser and Shul'ga, 1996) in the range of high enough energies of emitted photons $\omega \gg \omega_*(\tilde{\Theta})$. On the other hand two effects of the emission suppression appear in the range of law energies ω. The well-known Ter-Mikaelian effect of dielectric suppression (Ter-Mikaelian, 1954) appears in the $\omega < \gamma\omega_0$. More unexpected suppression effect occurs in the frequency range $\omega < \omega_*(\tilde{\Theta})$, but this effect can be observed on condition that $\omega_*(\tilde{\Theta}) \gg \gamma\omega_0$ only. The nature of the discussed effect is analogous to that considered in the previous section. To show this one should estimate the total emission yield as the sum of independent contributions from segments of a fast electron trajectory of the order of l_{coh}. By analogy with (9) one can obtain from the general formula (14) the following estimation:

$$\omega\frac{\mathrm{d}^3N^{\mathrm{cb}}}{\mathrm{d}\omega\,\mathrm{d}^2\tilde{\Theta}} \simeq \frac{e^2}{\pi^2}\frac{L}{l_{\mathrm{coh}}}\left\langle\left(\frac{\Delta\mathbf{\Psi}_{\mathrm{coh}}-\tilde{\mathbf{\Theta}}}{\gamma_*^{-2}+(\Delta\mathbf{\Psi}-\tilde{\Theta})^2}+\frac{\tilde{\mathbf{\Theta}}}{\gamma_*^{-2}+\tilde{\Theta}^2}\right)^2\right\rangle \simeq \frac{e^2}{\pi^2}\frac{L}{l_{\mathrm{coh}}}\frac{\langle(\Delta\mathbf{\Psi}_{\mathrm{coh}})^2\rangle}{(\gamma_*^{-2}+\tilde{\Theta}^2)^2}. \qquad (25)$$

Substituting expressions (23) and (24) into (25) we obtain the formula

$$\omega\frac{\mathrm{d}^2N^{\mathrm{cb}}}{\mathrm{d}\omega\,\mathrm{d}\tilde{\Theta}\tilde{\Theta}} \simeq \frac{4e^2\Psi^2}{\pi}(1-\langle\cos\Delta\chi\rangle)\frac{L}{\bar{\tau}}\frac{1}{(\gamma_*^{-2}+\tilde{\Theta}^2)^2}\frac{\omega}{\omega_*(\tilde{\Theta})} \times\left[1-\exp\left(-\frac{\omega_*(\tilde{\Theta})}{\omega}\right)\right], \qquad (26)$$

near the exact result (22).

Obviously, the result (26) coincides with the exact result (22) in the range of high photon energies $\omega \gg \omega_*(\tilde{\Theta})$ with an accuracy of a factor $(\gamma_*^{-4}+\tilde{\Theta}^4)/(\gamma_*^{-2}+\tilde{\Theta}^2)^2 \simeq 1$. On the other hand suppression factors in (22) and (26) are very different in the range of law photon energies $\omega \ll \omega_*(\tilde{\Theta})$. Indeed, $\omega\,\mathrm{d}^3N^{\mathrm{cb}}/\mathrm{d}\omega\,\mathrm{d}^2\Theta \sim \omega^2/\omega_*^2(\tilde{\Theta})$ in accordance with the exact result (22), but the spectrum is proportional to $\omega/\omega_*(\tilde{\Theta})$ as is clear from (26). This strong discrepancy is due to an interference between contributions to emission yield from different segments of an emitting electron trajectory neglected when deriving the simple formula (26). The influence of this interference is diminished due to effective averaging over photon phases in the case of the measurement of the total spectrum. Integrating (22) over $\tilde{\mathbf{\Theta}}$ one can obtain the following result:

$$\omega\frac{\mathrm{d}N^{\mathrm{cb}}}{\mathrm{d}\omega} = \frac{2e^2}{\pi}\gamma_*^2\Psi^2(1-\langle\cos\Delta\chi\rangle)\frac{L}{\bar{\tau}}P\left(\frac{\omega}{\omega_s}\right), \qquad (27a)$$

$$P = \frac{\omega}{\omega_s}\left(1 - 2\frac{\omega^2}{\omega_s^2}\right)\arctan\left(\frac{\omega_s}{\omega}\right) + 2\frac{\omega^2}{\omega_s^2}\left(1 - \ln\sqrt{1 + \frac{\omega_s^2}{\omega^2}}\right), \qquad (27b)$$

where $\omega_s = (2/\bar{\tau})\gamma_*^2(1 - \langle\cos\Delta\chi\rangle) \equiv \omega_*(0)$.

Asymptotics of the function $P(\omega/\omega_s)$

$$P\left(\frac{\omega}{\omega_*}\right) = \begin{cases} \frac{\pi}{2}\frac{\omega}{\omega_s} & \text{if } \omega \ll \omega_s, \\ \frac{2}{3} & \text{if } \omega \gg \omega_s \end{cases} \qquad (28)$$

favours the view that the influence of the discussed interference is really suppressed in the total emission spectrum.

It should be noted that the studied effect of coherent bremsstrahlung suppression has been predicted in works (Laskin et al., 1985) where the peculiarities in the total emission spectrum were considered.

Let us consider now the contribution of transition radiation to total emission yield. This contribution concentrated in the range of low photon energies $\omega \leqslant \gamma\omega_0$ is of special interest because of the suppression of the contribution of coherent bremsstrahlung in the photon energy range considered. Starting from the general expression for the total emission amplitude (14) and using dipole approximation by analogy with calculations of coherent bremsstrahlung contribution one can obtain the following expression:

$$\omega\frac{d^2N^{tr}}{d\omega\, d\tilde{\Theta}\tilde{\Theta}} = \frac{4e^2}{\pi}\left(\frac{1}{\gamma^{-2} + \tilde{\Theta}^2} - \frac{1}{\gamma_*^{-2} + \tilde{\Theta}^2}\right)^2 \times\left[\tilde{\Theta}^2\left(1 - \cos\frac{\omega L}{2}(\gamma_*^{-2} + \tilde{\Theta}^2)\right) + \Psi^2(1 - \langle\cos\Delta\chi\rangle^{L/\bar{\tau}}) \times\left(1 - 2\tilde{\Theta}^2\left(\frac{1}{(\gamma^{-2} + \tilde{\Theta}^2)} + \frac{1}{\gamma_*^{-2} + (\tilde{\Theta})^2}\times\left(1 - \cos\frac{\omega L}{2}(\gamma_*^{-2} - \tilde{\Theta}^2)\right)\right)\right)\right], \qquad (29)$$

that is true for thin enough targets only when the condition $\omega L\Psi^2(1 - \langle\cos\Delta\chi\rangle^{L/\bar{\tau}}) \ll 1$ is fulfilled.

The first term in square brackets in (29) corresponds to ordinary transition radiation from relativistic electron moving with a constant velocity. The other describes an influence of multiple scattering. The changes in (29) caused by multiple scattering depend strongly on the orientation angle Ψ and the thickness L of the target.

To estimate the influence of azimuthal scattering of emitting electrons on the total emission yield it is necessary to take into account an interference between transition radiation and coherent bremsstrahlung. Spectral-angular distribution of the interference term following from the general expression (14) under condition identical to that for previous calculations can be presented in the form

$$\omega\frac{d^2N^{int}}{d\omega\, d\tilde{\Theta}\tilde{\Theta}} = \frac{4e^2}{\pi}\frac{\Psi^2}{\gamma_*^{-2} + \tilde{\Theta}^2}\left(\frac{1}{\gamma^{-2} + \tilde{\Theta}^2} - \frac{1}{\gamma_*^{-2} + \tilde{\Theta}^2}\right) \times\left[\frac{4\tilde{\Theta}^2}{\gamma_*^{-2} + \tilde{\Theta}^2}(F_1 - F_2) + \left(\frac{\gamma_*^{-4} + \tilde{\Theta}^4}{(\gamma_*^{-2} + \tilde{\Theta}^2)^2} - \frac{\tilde{\Theta}^2}{\gamma_*^{-2} + \tilde{\Theta}^2}\frac{\gamma_*^{-2} - \tilde{\Theta}^2}{\gamma_*^{-2} + \tilde{\Theta}^2}\right) \times(F_1 + F_2)\right], \qquad (30a)$$

$$F_1 = \frac{\omega_*^2(1 - \langle\cos\Delta\chi\rangle^{L/\bar{\tau}}\cos(\omega L/2)(\gamma_*^{-2} + \tilde{\Theta}^2)) + \omega\omega_*\langle\cos\Delta\chi\rangle^{L/\tau}\sin(\omega L/2)(\gamma_*^{-2} + \tilde{\Theta}^2)}{\omega^2 + \omega_*^2}, \qquad (30b)$$

$$F_2 = \frac{\omega_*^2(\cos(\omega L/2)(\gamma_*^{-2} + \tilde{\Theta}^2) - \langle\cos\Delta\chi\rangle^{L/\bar{\tau}}) - \omega\omega_*\langle\cos\Delta\chi\rangle\sin(\omega L/2)(\gamma_*^{-2} + \tilde{\Theta}^2)}{\omega^2\langle\cos\Delta\chi\rangle^2 + \omega_*^2}, \qquad (30c)$$

where the quantity $\omega_*(\tilde{\Theta})$ is defined by (23).

Formulae (22), (29) and (30) allow us to describe all properties of the dipole coherent emission from relativistic electrons penetrating a thin aligned crystal. In line with these formulae, the spectrum of emitted photons is determined in the main by the contribution of coherent bremsstrahlung in the range of high enough frequencies $\omega \gg \omega_*$, $\omega \gg \gamma\omega_0$. On the other hand transition radiation and the interference term (30) can contribute substantially to total emission yield in the range of small frequencies. In order to show this, let us consider the spectrum of strongly collimated emission ($\gamma^2\tilde{\Theta}^2 \ll 1$),where a sum of formulae (22), (29) and (30) can be reduced to

$$\omega\frac{d^2N}{d\omega\, d\tilde{\Theta}\tilde{\Theta}} = \frac{4e^2}{\pi}\Psi^2\left[(1 - \langle\cos\Delta\chi\rangle)\frac{L}{\bar{\tau}}\frac{\omega^2}{\omega^2 + \omega_S^2}\gamma_*^4 + (1 - \langle\cos\Delta\chi\rangle^{L/\bar{\tau}})(\gamma^2 - \gamma_*^2)^2 + (F_1 + F_2)\gamma_*^2(\gamma^2 - \gamma_*^2)\right]. \qquad (31)$$

The range in question is of interest because of the expected anomalous Ter-Mikaelian effect analogous to that studied theoretically and experimentally in the ordinary bremsstrahlung from relativistic electrons crossing a thin layer of amorphous medium (Arkatov et al., 1996). For simplicity, assume that the scattering angle $\Delta\chi$ achievable in the single collision of emitting electron with atomic string is small, so that $\langle\cos\Delta\chi\rangle \simeq 1-\frac{1}{2}\langle\Delta^2\chi\rangle$ (in accordance with the corresponding formula in (21) the condition $\Delta\chi \ll 1$ is fulfilled if $\Psi^2 \gg \Psi_{\rm ch}^2$). In the case under study formula (31) is reduced to more simple one

$$\omega\frac{{\rm d}^2N}{{\rm d}\omega\,{\rm d}\tilde{\Theta}\tilde{\Theta}} = \frac{4e^2\gamma^2}{\pi}\Psi^2 G(x,y,z), \tag{32a}$$

$$G = yz\frac{x^4}{(1+x^2)^2+z^2x^2} + (1-{\rm e}^{-yz})\frac{1}{(1+x^2)^2} + (1-{\rm e}^{-yz})\frac{z^2x^2(1+\cos y(x+x^{-1})) - zx(1+x^2)\sin y(x+x^{-1})}{(1+x^2)^2+z^2x^2}, \tag{32b}$$

where $x=\omega/\gamma\omega_0$, $y=\omega_0 L/2\gamma$, $z=\gamma\langle\Delta^2\chi\rangle/\omega_0\bar{\tau}$. The spectrum distribution by the universal function G depends strongly on the arguments x, y and z. As evident from (32) the contribution of coherent bremsstrahlung is suppressed in the range $x<1$ or $\omega<\gamma\omega_0$ due to the normal Ter-Mikaelian effect (Ter-Mikaelian, 1954). As a consequence the relative contribution of transition radiation and the interference term increases in this range of emitted photon energies. The shape of the spectrum is determined by the parameters y and z. The parameter z determines the region of the manifestation of above discussed coherent bremsstrahlung suppression due to azimuthal scattering of emitting electrons by atomic strings. Formally this effect manifests in the frequency range $\omega<\omega_S$ in response to (27), but in actuality it can be realized with the proviso that $\omega_S>\gamma\omega_0$ because of the Ter-Mikaelian effect of dielectric suppression. It is easy to see that the ratio $\omega_S/\gamma\omega_0$ is determined by the formula

$$\frac{\omega_S}{\gamma\omega_0} = z\frac{x^2}{1+x^2} \leqslant z, \tag{33}$$

so that the suppression of coherent bremsstrahlung due to multiple scattering manifests with the constant $z>1$ only. In the range $z\ll 1$ the spectrum is determined mainly by the relationship between transition radiation and coherent bremsstrahlung contributions. This relationship depends on the coefficient yz (obviously the coefficient $2yz=\langle\Delta^2\chi\rangle L/\bar{\tau}$ is equal to mean square of the total azimuth angle of multiple scattering). In the special case that $yz\ll 1$ contributions of coherent bremsstrahlung and transition radiation are comparable in the photon energy range $\omega<\gamma\omega_0$ and the shape of the spectrum takes the form characteristic for the spectrum under condition of anomalous Ter-Mikaelian effect manifestation in bremsstrahlung (Arkatov et al., 1996; Nasonov, 2001b). The spectral curve is presented in Fig. 2.

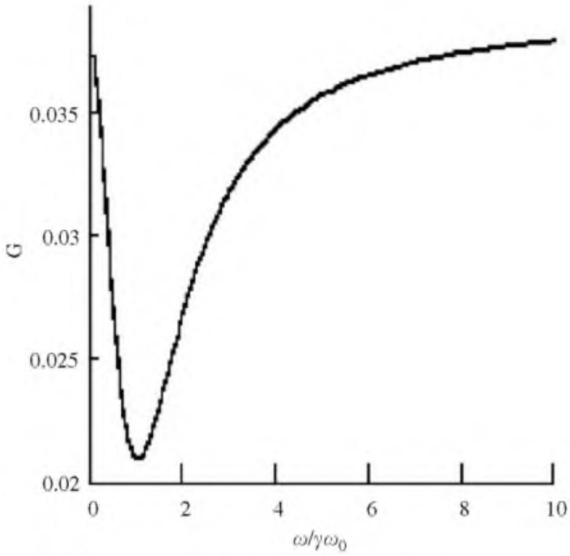

Fig. 2. Anomalous Ter-Mikaelian effect in coherent bremsstrahlung from relativistic electrons in an aligned crystal. The function $G(\omega/\gamma\omega_0, y, z)$ and the parameters y and z are determined by the Eq. (32). The presented curves has been calculated for the values of the parameters $y=0.1$, $z=0.4$.

In the case of thick enough target that $yz\gg 1$ the contribution of coherent bremsstrahlung dominates in the frequency range $\omega>\gamma\omega_0$. As a consequence, the shape of the spectrum comes close to that for coherent bremsstrahlung suppressed by the normal Ter-Mikaelian effect (see the spectral curve presented in Fig. 3).

The emission spectrum is changed substantially in the case $z\gg 1$. Suppression of coherent bremsstrahlung due to azimuth multiple scattering and increasing of the role of an interference between transition radiation and coherent bremsstrahlung are the new effects in the case in question.

The role of the interference term is demonstrated by Fig. 4, where the spectrum calculated for thin enough target ($y\ll 1$) is presented. Obtained result shows that the interference cannot only suppress completely the Ter-Mikaelian effect manifestation, but it gives rise to strong peak in the emission spectrum as well.

In the case $y>1$ (since $L/l_{\rm coh}=y(x+x^{-1})\geqslant 2y$, the interference term in (32b) oscillates in the case under consideration) the emission yield is suppressed due to azimuthal multiple scattering in a wide frequency range as it follows from Fig. 5.

Thus, obtained results show that the conventional theory of coherent bremsstrahlung from relativistic electrons crossing a thin aligned crystal is not sufficient for correct description of emission properties in the range of low photon energies where the contributions of coherent bremsstrahlung, transition radiation and interference between these emission mechanisms are comparable and must be taken into account.

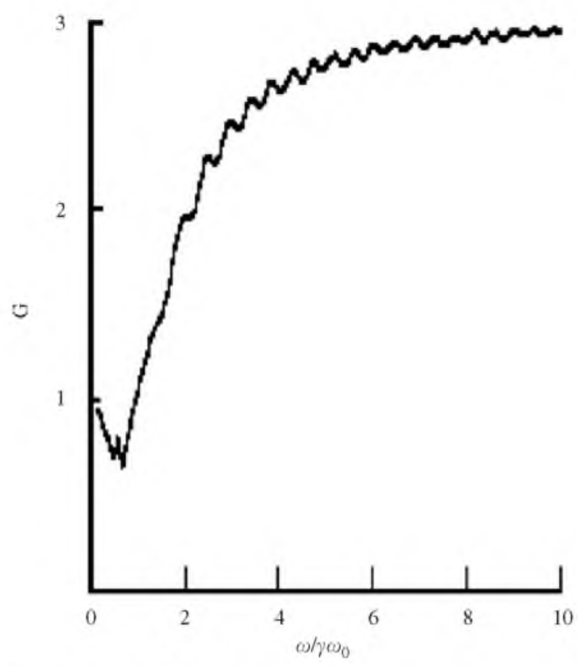

Fig. 3. Normal Ter-Mikaelian effect in coherent bremsstrahlung from relativistic electrons in an aligned crystal. The presented curves has been calculated for the values of the parameters $y = 12$, $z = 0.2$.

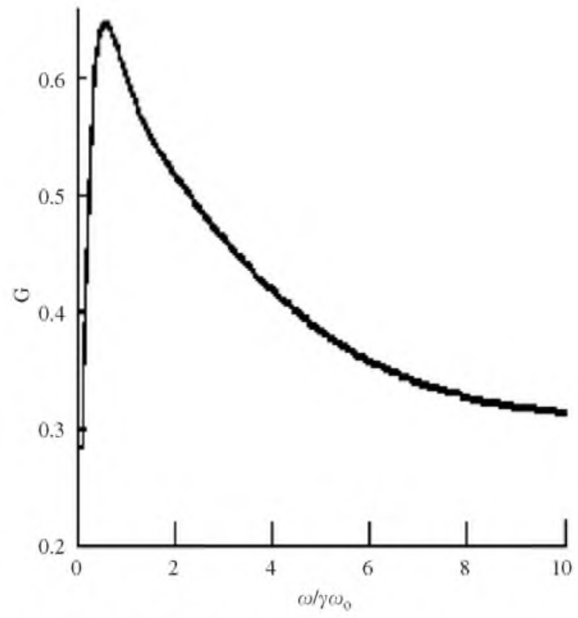

Fig. 4. The role of the interference between coherent bremsstrahlung and transition radiation from relativistic electrons in an aligned crystal. The presented curve has been calculated for the values of the parameters $y = 0.1$, $z = 4$.

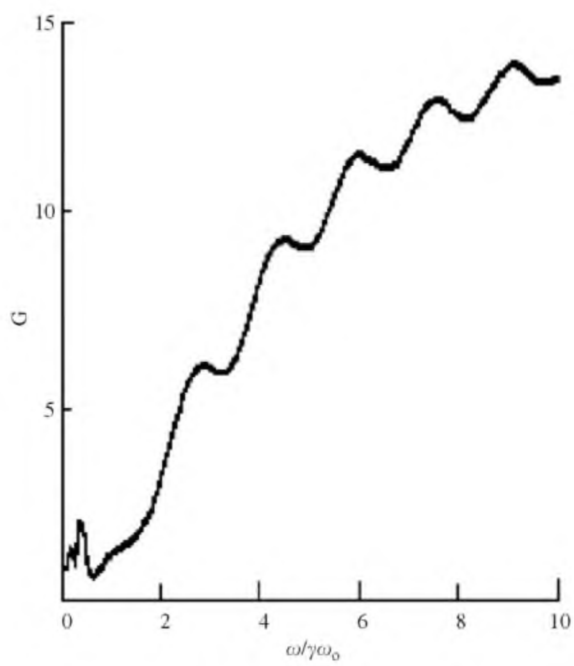

Fig. 5. Suppression of coherent bremsstrahlung from relativistic electrons due to azimuth multiple scattering. The presented curve has been calculated for the values of the parameters $y = 4$, $z = 4$.

4. Collective effects in polarization bremsstrahlung from relativistic electrons moving through dense media

Collective effects analogous to above demonstrated ones play an important role in the process of polarization bremsstrahlung from fast charged particles moving through a dense medium. In order to analyze such effects we use in this paper the simplest approach based on Maxwell equation

$$(k^2 - \omega^2)\mathbf{E}_{\omega\mathbf{k}} - \mathbf{k}(\mathbf{k}\mathbf{E}_{\omega\mathbf{k}}) = 4\pi i\omega \mathbf{J}_{e\omega\mathbf{k}} + 4\pi i\omega \mathbf{J}_{q\omega\mathbf{k}}, \quad (34)$$

where $\mathbf{E}_{\omega\mathbf{k}} = (2\pi)^{-1} \int dt\, d^3r\, \mathbf{E}(t,\mathbf{r}) e^{i\omega t - i\mathbf{k}\mathbf{r}}$ is the Fourier-transform of the electric component of electromagnetic field excited by a fast particle, $\mathbf{J}_{q\omega\mathbf{k}}$ and $\mathbf{J}_{e\omega\mathbf{k}}$ are the Fourier-transforms of current densities of the projectile and medium electrons, respectively. The classical formula for $\mathbf{J}_q$ is used in this work

$$\mathbf{J}_q = q\mathbf{V}_q(t)\delta(\mathbf{r} - \mathbf{r}_q(t)), \quad (35)$$

where q is the charge of the fast particle, $\mathbf{r}_q(t)$ is its trajectory $\mathbf{V}_q = (d/dt)\mathbf{r}_q$. To define the density of induced current of all medium electrons $\mathbf{J}_e$ we use the following simple formula:

$$\mathbf{J}_e = -\frac{e^2}{m}\mathbf{A}(t,\mathbf{r})\hat{n}, \quad \hat{n} = \sum_S \delta(\mathbf{r} - \mathbf{r}_S). \quad (36)$$

This expression averaged over electronic states in the medium is commonly used in the X-ray scattering theory (James, 1958; Pinsker, 1984). Here $\mathbf{A}(t,\mathbf{r})$ is the vector-potential of electromagnetic field. The final expressions for the emission cross-section will be averaged in the present work if the individual and collective contributions of medium electrons to the formation of the emission yield is to take into account. The formula (36) is valid in the frequency range $J \ll \omega \ll m$ (J is the mean ionization potential of an atom, m is the mass of an electron). The presented relations allows one to consider atomic electrons as being free during **PB** process and to neglect the Compton shift of the frequency of emitted photons.

Equation

$$(k^2-\omega^2)\mathbf{E}_{\omega\mathbf{k}} - \mathbf{k}(\mathbf{k}\mathbf{E}_{\omega\mathbf{k}}) + \int d^3k'\, G(\mathbf{k}'-\mathbf{k})\mathbf{E}_{\omega\mathbf{k}'} = \frac{i\omega q}{4\pi^3}\int dt\, \mathbf{V}_q e^{i\omega t - i\mathbf{k}\mathbf{r}_q}, \tag{37a}$$

$$G = \frac{e^2}{2\pi^2 m}\sum_S e^{i(\mathbf{k}'-\mathbf{k})\mathbf{r}_s}, \tag{37b}$$

following from (34) to (36) allows us to take into account not only **B** and **PB** contributions, but an interference between these emission mechanisms as well.

Eliminating from the function $G(\mathbf{k}'-\mathbf{k})$ the average components by means of averaging over medium electron coordinates $\mathbf{r}_s$

$$G(\mathbf{k}'-\mathbf{k}) \equiv \langle G(\mathbf{k}'-\mathbf{k})\rangle + \tilde{G}(\mathbf{k}'-\mathbf{k}), \tag{38}$$

$$\langle G(\mathbf{k}'-\mathbf{k})\rangle = \omega_0^2\delta(\mathbf{k}'-\mathbf{k}), \quad \omega_0^2 = \frac{4\pi e^2 n_e}{m}$$

let us rewrite the Eq. (37) in the form

$$(k^2-k_0^2)\mathbf{E}_{\omega\mathbf{k}} + \int d^3k'\, \tilde{G}(\mathbf{k}'-\mathbf{k})\left(\mathbf{E}_{\omega\mathbf{k}'} - \mathbf{k}\frac{\mathbf{k}\mathbf{E}_{\omega\mathbf{k}}}{k_0^2}\right) = \frac{i\omega q}{4\pi^3}\int dt\left(\mathbf{V}_q - \mathbf{k}\frac{\mathbf{k}\mathbf{V}_q}{k_0^2}\right)e^{i\omega t - i\mathbf{k}\mathbf{r}_q}, \tag{39}$$

where $k_0^2 = \omega^2(1-\omega_0^2/\omega) = \omega^2\varepsilon(\omega)$, $\varepsilon(\omega)$ is the ordinary average dielectric permeability in X-ray range, n_e is the density of medium electrons.

Determining the solution of Eq. (39) within the frame of perturbation theory and calculating the Fourier-integral $\mathbf{E}_\omega(r) = \int d^3k\, e^{i\mathbf{k}\mathbf{r}}\mathbf{E}_{\omega\mathbf{k}}$ by the stationary phase method one can obtain the following expression for the emission amplitude $\mathbf{A}_\mathbf{n}$

$$\int d^3k\, e^{i\mathbf{k}\mathbf{r}}\mathbf{E}_{\omega\mathbf{k}} \to \mathbf{A}_\mathbf{n}\frac{e^{ik_0 r}}{r}, \tag{40a}$$

$$\mathbf{A}_\mathbf{n} = \frac{i\omega q}{2\pi}\int dt\, e^{i\omega h t}\Bigg[(\mathbf{V}_q - \mathbf{n}(\mathbf{n}\mathbf{V}_q))e^{-k_0\mathbf{n}\mathbf{r}_q} - \int\frac{d^3k'}{k'^2-k_0^2} \times\tilde{G}(\mathbf{k}'-k_0\mathbf{n})\left(\mathbf{V}_q - \mathbf{k}'\frac{\mathbf{k}'\mathbf{V}_q}{k_0^2} - \mathbf{n}\left(\mathbf{n}\mathbf{V}_q - \mathbf{n}\mathbf{k}'\frac{\mathbf{k}'\mathbf{V}_q}{k_0^2}\right)\right)e^{-i\mathbf{k}'\mathbf{r}_q}\Bigg]. \tag{40b}$$

For further analysis one should integrate by parts in the first item in (40b) and express the acceleration of a fast electron $\mathbf{W}_q = (d/dt)\mathbf{V}_q$ in terms of the total potential induced by all atoms in the target. As before in this paper we shall restrict our consideration to the emission under condition of dipole approximation. The final expression for $\mathbf{A}_\mathbf{n}$ has the form

$$\mathbf{A}_\mathbf{n} = -iq\int d^3k\left[\frac{1}{k^2}Q(\mathbf{k})\mathbf{a}_\mathbf{k} + \frac{1}{k^2+2k_0\mathbf{n}\mathbf{k}}\tilde{G}(\mathbf{k})\mathbf{b}_\mathbf{k}\right] \times\delta\left[\omega(1-\sqrt{\varepsilon}\mathbf{n}\mathbf{V}) - \mathbf{k}\mathbf{V}\right], \tag{41a}$$

$$Q(\mathbf{k}) = \frac{eq}{2\pi^2 m\gamma}\sum_\alpha e^{i\mathbf{k}\mathbf{r}_\alpha}\left(Z - \sum_{\beta=1}^{Z} e^{i\mathbf{k}\mathbf{r}_{\alpha\beta}}\right), \tag{41b}$$

$$\mathbf{a}_\mathbf{k} = \frac{\mathbf{k}-\mathbf{V}\cdot\mathbf{k}\mathbf{V}}{1-\sqrt{\varepsilon}\mathbf{n}\mathbf{V}} - \frac{\mathbf{n}-\sqrt{\varepsilon}\mathbf{V}}{(1-\sqrt{\varepsilon}\mathbf{n}\mathbf{V})^2}(\mathbf{n}\mathbf{k}-\mathbf{n}\mathbf{V}\cdot\mathbf{k}\mathbf{V}), \tag{41c}$$

$$\tilde{G}(\mathbf{k}) = \frac{e^2}{2\pi^2 m}\sum_\alpha\sum_{\beta=1}^{Z} e^{i\mathbf{k}(\mathbf{r}_\alpha+\mathbf{r}_{\alpha\beta})} - \omega_0^2\delta(\mathbf{k}), \tag{41d}$$

$$\mathbf{b}_\mathbf{k} = \mathbf{V}\frac{\mathbf{k}\mathbf{V}}{1-\sqrt{\varepsilon}\mathbf{n}\mathbf{V}} - \mathbf{k}\frac{1}{\varepsilon} - \mathbf{n}\left(\mathbf{n}\mathbf{V}\frac{\mathbf{k}\mathbf{V}}{1-\sqrt{\varepsilon}\mathbf{n}\mathbf{V}} - \mathbf{n}\mathbf{k}\frac{1}{\varepsilon}\right), \tag{41e}$$

where $\mathbf{r}_\alpha$ is the coordinate of the nuclear of αth atom in the target, $\mathbf{r}_{\alpha\beta}$ is the coordinate of βth electron in αth atom, Z is the number of electrons in an atom, $\mathbf{n}$ is the unit vector to the direction of emitted photon propagation.

The first item in (41a) proportional to $Q(\mathbf{k})$ describes the contribution of **B** and the last one corresponds to the contribution of **PB** to total emission yield. Preparatory to analyzing spectral-angular properties of the emission, we will first point out that the solution (41) correctly described in non-relativistic limit ($V \ll 1$) the well-known stripping effect in the total **B** from electrons (Buimistrov and Trakhtenberg, 1977; Amusia et al., 1985; Avdonina and Pratt, 1999; Korol et al., 2002). Indeed, from (41) it follows that $\mathbf{a}_k \approx \mathbf{k} - \mathbf{n}(\mathbf{n}\mathbf{k})$ if $V \ll 1$ and therefore the expression in square brackets in (41) is reduced in the case of emitting electrons to simple formula

$$\frac{1}{k^2}Q(\mathbf{k})\mathbf{a_k}+\frac{1}{k^2+2k_0\mathbf{nk}}\tilde{G}(\mathbf{k})\mathbf{b_k} \simeq \frac{\mathbf{k}-\mathbf{n}(\mathbf{nk})}{k^2}\frac{Ze^2}{2\pi^2 m}\sum_\alpha e^{i\mathbf{kr}_\alpha}. \quad (42)$$

As one would expect, the interference between **B** and **PB** emission amplitudes suppress completely the contributions of **B** from the fast electron on medium electrons and **PB** from medium electrons. The formula (42) corresponds to the dipole **B** from the fast electron on atomic nuclei only.

Returning to the general formula (41) let us consider the spectral-angular distribution of emitted quanta

$$\omega\frac{dN}{d\omega\, d\Omega}=\langle|\mathbf{A_n}|^2\rangle=\sum_{j=1}^{3}\omega\frac{dN_j}{d\omega\, d\Omega}, \quad (43)$$

where three items correspond to **B**, **PB** and the contribution of the interference between these emission mechanisms. Brackets $\langle\rangle$ in (43) mean averaging over electronic states of an atom (for simplicity assume that wave functions of different atoms do not overlap) and over coordinates of atomic nuclei $\mathbf{r}_\alpha$.

In the case of relativistic emitting particle under consideration the contribution of **B** is concentrated in the narrow cone of the order of $\gamma^{-1}\ll 1$ around the electron's velocity **V**. Owing to this property, the emission yield in the range of large observation angles $\Theta\gg\gamma^{-1}$ is determined in the main by **PB** contribution. From (41) it follows that **PB** spectral-angular distribution has the form

$$\begin{aligned}\omega\frac{dN^{PB}}{d\omega\, d\Omega}&=e^2\int\frac{d^3k}{k^2+2k_0\mathbf{nk}}\frac{d^3k'}{k'^2+2k_0\mathbf{nk}'}\\&\times\langle\tilde{G}(\mathbf{k})\tilde{G}^*(\mathbf{k}')\rangle\mathbf{b_k}\mathbf{b_k}\delta\left[\omega\left(1-\sqrt{\varepsilon}\mathbf{nV}\right)-\mathbf{kV}\right]\\&\times\delta\left[\omega\left(1-\sqrt{\varepsilon}\mathbf{nV}\right)-\mathbf{k}'\mathbf{V}\right]. \quad (44)\end{aligned}$$

The dependence of **PB** properties on a target's atomic structure is described by the correlator $\langle\tilde{G}(\mathbf{k})\tilde{G}^*(\mathbf{k}')\rangle$. Inter-atomic correlations are liable to change **PB** spectrum as compared with **PB** on a single atom even in the simplest case of amorphous target. Using the simple statistical atom model with exponential screening and taking into account inter-atomic correlations caused by the finite size of an atom one can obtain the following expression (Nasonov and Safronov, 1992):

$$\begin{aligned}\langle\tilde{G}(\mathbf{k})\tilde{G}^*(\mathbf{k}')\rangle&=\frac{2e^4n_a}{\pi m^2}\left[Z\left(1-\frac{1}{(1+k^2R^2)^2}\right)\right.\\&+Z^2\frac{1}{(1+k^2R^2)^2}\left(1-\frac{4\pi}{k}n_a\right.\\&\left.\left.\times\int_0^\infty dr\, r\sin kr(1-W(r))\right)\right], \quad (45)\end{aligned}$$

where n_a is the density of atoms, R is the scattering radius in the Fermi–Thomas atom model, $W(r)$ is correlation function describing the probability to find two atoms positioned at r intervals (obviously, $W(r)\to 0$ is $r<l$, $\frac{1}{2}l$ is the characteristic ion radius of the atom and $W(r)\to 1$ if $r\gg l$).

The first item in (45) proportional to Z corresponds to individual contribution of medium electrons to the formation of **PB** yield. The second one proportional to Z^2 describes the coherent contribution of atomic electrons changed by inter-atomic correlations. It is precisely this contribution that determines in the main **PB** yield, therefore we concentrate our attention on the analysis of coherent **PB**, described by the formula

$$\begin{aligned}\omega\frac{d^3N^{PB}}{dt\, d\omega\, d\Omega}&=\frac{Z^2e^6n_a}{\pi^2m^2}\int\frac{d^3k}{(1+k^2R^2)^2}\\&\times\left[1-\frac{4\pi}{k}n_a\int_0^\infty dr\, r\sin kr(1-W(r))\right]\\&\times\frac{\left(\mathbf{k}-\sqrt{\varepsilon}\mathbf{V}k_0\right)^2-\left(\mathbf{nk}-\sqrt{\varepsilon}\mathbf{nV}k_0\right)^2}{(k^2+2k_0\mathbf{nk})^2}\\&\times\delta\left[\omega\left(1-\sqrt{\varepsilon}\mathbf{nV}\right)-\mathbf{kV}\right], \quad (46)\end{aligned}$$

that it follows from (44) and (45).

Let us consider spectral-angular distribution (46) in the best suited to the measurement of separate **PB** contribution range of observation angles $\Theta\gg\gamma^{-1}$. In order to estimate an influence of inter-atomic correlations on **PB** properties we choose the function $W(r)$ in the simplest form $W(r)=0$ if $r\leqslant l$ and $W(r)=1$ if $r>l$. On condition under consideration the formula (46) is reduced to more simple one

$$\begin{aligned}\omega\frac{d^3N^{PB}}{dt\, d\omega\, d\Omega}&=\frac{Z^2e^6n_a}{\pi m^2}\int_{\sin(\Theta/2)}^{\infty}\frac{d\tau}{(1+4\omega^2R^2\sin^2(\Theta/2)\tau^2)^2}\\&\times\left[1-\sigma f\left(2\omega l\sin\frac{\Theta}{2}\tau\right)\right]\\&\times\left[\frac{1+\cos^2\Theta}{\sqrt{(\tau^2-1)^2+\rho^2\cot^2(\Theta/2)}}-2\sin^2\left(\frac{\Theta}{2}\right)\right.\\&\times\left(1-\frac{\tau^2-1}{\sqrt{(\tau^2-1)^2+\rho^2\cot^2(\Theta/2)}}\right)\\&\left.-\frac{\rho^2\cos^2\Theta(\tau^2+\cos\Theta)}{2\sin^2(\Theta/2)((\tau^2-1)^2+\rho^2\cot^2(\Theta/2))^{3/2}}\right], \quad (47a)\end{aligned}$$

$$f(x)=\frac{3}{x^3}(\sin x-x\cos x), \quad (47b)$$

where $\sigma=(4\pi/3)l^3n_a$, $\rho^2=\gamma^{-2}+\omega_0^2/\omega^2$. The obtained result (47) describes an influence of two collective effects on **PB** properties. The first of them is well-known density effect or Fermi effect, consisting in the saturation of the dependence of **PB** yield on the emitting electron energy γm (the distribution (47) does not depend on γ in the range $\gamma > (\omega/\omega_0)$). This effect occurs because of the screening of relativistic electron's Coulomb field due to the polarization of medium electrons.

More unexpected effect consists in the suppression of **PB** yield due to inter-atomic correlations (Nasonov and Safronov, 1992). Since $f(2\omega l \sin(\Theta/2)\tau) \leqslant 1$ the discussed effect can occur in condensed medium only when $\sigma \approx 1$ and neighbouring atoms are arranged close to each other. Let us consider the nature of this suppression effect. **PB** appears due to the emission from atomic electrons excited by the Coulomb field of incident electron. It is very important that in the case of condensed medium under consideration the incident electron excites simultaneously many atoms arranged around its trajectory. Since induced currents of electrons being shared to atoms placed bilaterally along this trajectory are opposite in sign, the total current density of medium electrons is decreased (in other words, atoms placed around the trajectory of the fast electron screen each other). **PB** suppression caused by the discussed inter-atomic correlation is illustrated by the curves presented in Fig. 6. It should be noted that this effect is analogous to that known in the physics of free X-ray scattering is a dense amorphous medium (James, 1958).

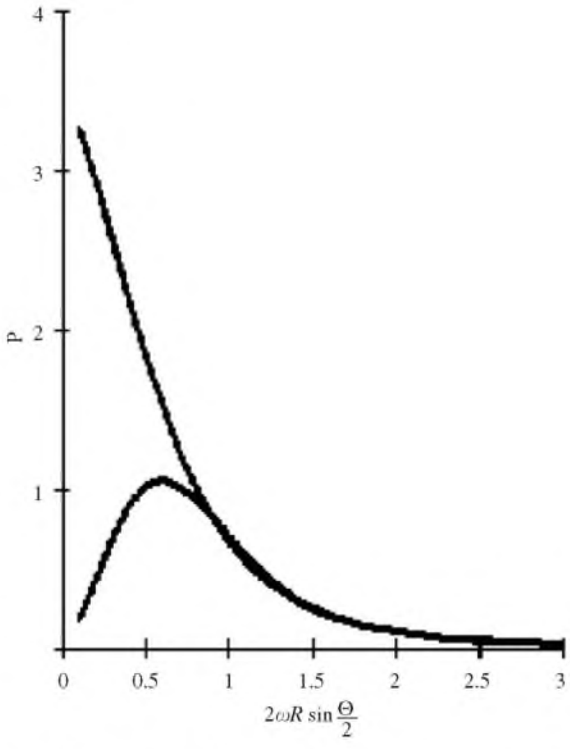

Fig. 6. Suppression of polarization bremsstrahlung from relativistic electrons in a dense medium. The function $P(2\omega R \sin(\Theta/2), \sigma, \Theta, \rho, l/R)$ determined by the Eq. (47) $P = (\pi M^2/Z^2 e^2 n_a)\cdot \omega(\mathrm{d}N^{\mathrm{PB}}/\mathrm{d}t\,\mathrm{d}\omega\,\mathrm{d}\Omega)$ is presented here. The curves 1 and 2 have been calculated for the same parameters $\Theta = \pi/2$, $\rho = 0.05$, $l/R = 5$ and different values of the parameter σ ($\sigma = 0$ and 1 for the curves 1 and 2, respectively).

Thus, even simplest correlations between positions of atoms in the target should determine substantial changes of **PB** spectrum. The role of inter-atomic correlations increases in the case of **PB** from relativistic electrons penetrating through a crystalline target. Coordinates of atoms in a crystalline lattice can be represented by the formula

$$\mathbf{r}_\alpha = \mathbf{r}_n + \mathbf{r}_{nl} + \mathbf{U}_{nl}, \tag{48}$$

where $\mathbf{r}_n$ is the coordinate of nth elementary cell, $\mathbf{r}_{nl}$ is the equilibrium position of lth atom in nth cell, $\mathbf{U}_{nl}$ is the accidental displacement of this atom due to the thermal excitation. Substituting (48) into (41a) one can obtain instead of (45) the following result:

$$\langle \tilde{G}(\mathbf{k})\tilde{G}^*(\mathbf{k}')\rangle = \frac{2e^4 n_a}{\pi m^2}\left[Z\left(1 - \frac{1}{(1+k^2R^2)^2}\right) + Z^2\frac{1-\mathrm{e}^{-k^2U_\mathrm{T}^2}}{(1+k^2R^2)^2} + (2\pi)^3 n_a Z^2 \times {\sum_{\mathbf{g}}}' \frac{|S(\mathbf{g})|^2 \mathrm{e}^{-g^2U_\mathrm{T}^2}}{(1+g^2R^2)^2}\delta(\mathbf{k}-\mathbf{g})\right]\delta(\mathbf{k}'-\mathbf{k}). \tag{49}$$

Two peculiarities of the expression (49) are noteworthy. The second term in (49) describing the individual contribution of atoms to **PB** yield predicts the strong suppression of this contribution (Nasonov and Safronov, 1993). Indeed, the effective momentum transfer $k_{\mathrm{eff}} \sim R^{-1}$ determined by the characteristic size of the distribution of atomic electron density is small enough, so that the factor $1 - \exp(-k^2U_\mathrm{T}^2) \sim 1 - \exp(-k_{\mathrm{eff}}^2 U_\mathrm{T}^2) \sim U_\mathrm{T}^2/R^2 \ll 1$ (mean square amplitude of thermal vibrations of atoms U_T is substantially less than R). The last term in (49) describes the coherent contribution of all crystal's atoms to the formation of **PB** yield. Let us consider in greater detail the coherent part of **PB**. Substituting (49) into (44) and determining the vectors **V** and **n** by the formulae

$$\mathbf{V} = \mathbf{e}_1 V, \quad \mathbf{n} = \mathbf{e}_2(1 - \tfrac{1}{2}\Theta^2) + \mathbf{\Theta}, \quad \mathbf{e}_2\mathbf{\Theta} = 0, \quad \mathbf{e}_1\mathbf{e}_2 = \cos\varphi \tag{50}$$

one can obtain the following result:

$$\omega\frac{\mathrm{d}^4 N_{\mathrm{coh}}^{\mathrm{PB}}}{\mathrm{d}t\,\mathrm{d}\omega\,\mathrm{d}^2\Theta} = {\sum_{\mathbf{g}}}' \frac{e^2\omega_0^4}{\pi g^2}\frac{|S(\mathbf{g})|^2\mathrm{e}^{-g^2U_\mathrm{T}^2}}{(1+g^2R^2)^2} \times \frac{\Theta_\perp^2 + (2\Theta' + \Theta_{\|})^2\cos^2\varphi}{[\gamma^{-2} + (\omega_0^2/\omega^2) + \Theta_\perp^2 + (2\Theta'+\Theta_{\|})]^2} \times\delta\left[\omega - \omega_\mathrm{B}\left(1 + (\Theta' + \Theta_{\|})\cot\frac{\varphi}{2}\right)\right], \tag{51}$$

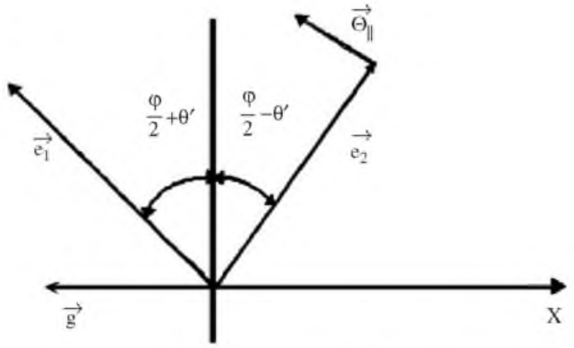

Fig. 7. Geometry of the parametric X-ray radiation process. The vector $\mathbf{e}_1$ and $\mathbf{e}_2$ are defined the Eq. (50). Here Θ' is the orientation angle describing the turning of the crystallographic plane G relative to fixed axes of the emitting electron beam ($\mathbf{e}_1$) and the X-ray detector ($\mathbf{e}_2$), φ is the fixed observation angle.

where Θ' is the orientation angle (see Fig. 7), $\mathbf{\Theta} = \mathbf{\Theta}_{\parallel} + \mathbf{\Theta}_{\perp}$, $\mathbf{\Theta}_{\parallel}\mathbf{\Theta}_{\perp} = 0$, $S(\mathbf{g})$ is the structure factor of an elementary cell, $\mathbf{g}$ is the reciprocal lattice vector, $\omega_0^2 = 4\pi^2 Ze^2 n_a/m^2$, $\omega_B = g/2\sin\varphi/2$ is the Bragg frequency in the vicinity of which the spectrum of coherent **PB** is concentrated, geometry of the considered emission process is presented in Fig. 7.

It is easy to see that the formula (51) completely coincides with that describing spectral-angular distribution of well-known parametric X-ray radiation (**PXR**) from relativistic electrons in a crystal (Ter-Mikaelian, 1972; Garibian and Yang, 1971; Baryshevsky and Feranchuk, 1971). Thus, **PXR** is the coherent part of **PB** in the crystal (Lapko and Nasonov, 1990).

Let us consider the last term in the general formula (45) caused by the interference between **B** and **PB** emission mechanisms. The contribution of this term depends on the sign of the emitting particle's charge q as it follows from the expression for the correlator $\langle Q(\mathbf{k})\tilde{G}^*(\mathbf{k}')\rangle$

$$\begin{aligned}
&\langle Q(\mathbf{k})\tilde{G}^*(\mathbf{k}')\rangle \\
&= \mathrm{sign}(q)\frac{2e^4 n_a}{\pi m^2}\left[-Z\left(1-\frac{1}{(1+k^2R^2)^2}\right)\right. \\
&\quad + Z^2\frac{k^2R^2(1-\mathrm{e}^{-k^2U_T^2})}{(1+k^2R^2)^2} \\
&\quad \left. + (2\pi)^3 n_a Z^2 {\sum_{\mathbf{g}}}' \frac{g^2R^2|S(\mathbf{g})|^2\mathrm{e}^{-g^2U_T^2}}{(1+g^2R^2)^2}\delta(\mathbf{k}-\mathbf{g})\right] \\
&\quad \times\delta(\mathbf{k}'-\mathbf{k}).
\end{aligned} \tag{52}$$

It should be noted that individual contributions of medium electrons to **PB** cross-section and to the interference term has different signs in accordance with (49) and (52). To account this fact one must take in mind that the discussed component of the interference term is proportional to the mean of the product of the variable current of incident particle appearing due to the scattering of this particle by medium electrons into the current of medium electrons induced by the particle. Obviously, these currents are facing away from each other. On the other hand the quantities of the same physical nature are averaged in the correlator $\tilde{G}(\mathbf{k})\tilde{G}^*(\mathbf{k}')$ (the same is true for $\langle Q(\mathbf{k})Q^*(\mathbf{k}')\rangle$) which determines the sign (+) in the corresponding component of this correlator.

In this paper we have restricted ourselves to the analysis of the coherent contribution of crystalline atoms to the formation of total emission yield. Using the general formulae (41), (43), correlators (49), (52) and the corresponding expression for $\langle Q(\mathbf{k})Q^*(\mathbf{k}')\rangle$ one can obtain the following formulae for the spectral-angular distribution of the coherent part of total emission:

$$\begin{aligned}
\omega\frac{\mathrm{d}^3N_{coh}^{tot}}{\mathrm{d}t\,\mathrm{d}\omega\,\mathrm{d}\Theta} &= \frac{e^2\omega_0^4}{2\pi}{\sum_{\mathbf{g}}}'\frac{|S(\mathbf{g})|^2\mathrm{e}^{-g^2U_T^2}}{(1+g^2R^2)^2} \\
&\times\left[\mathrm{sign}(q)\frac{R^2}{\gamma(1-\sqrt{\varepsilon}\mathbf{nV})}\right. \\
&\times\left(\mathbf{g}-\mathbf{V}(\mathbf{gV})-\frac{\mathbf{n}-\sqrt{\varepsilon}\mathbf{V}}{1-\sqrt{\varepsilon}\mathbf{nV}}(\mathbf{ng}-(\mathbf{nV})(\mathbf{gV}))\right) \\
&+\frac{1}{g^2+2\omega\sqrt{\varepsilon}\mathbf{ng}} \\
&\times\left.\left(\mathbf{V}\omega-\mathbf{g}\frac{1}{\varepsilon}-\mathbf{n}\left(\mathbf{nV}\omega-\mathbf{ng}\frac{1}{\varepsilon}\right)\right)\right]^2 \\
&\times\delta\left[\omega(1-\sqrt{\varepsilon}\mathbf{nV})-\mathbf{gV}\right],
\end{aligned} \tag{53}$$

pointing clearly to the interference between coherent parts of **B** and **PB**, or coherent bremsstrahlung and **PXR**.

The result (53) points to the strong difference between characteristics of the emission from electrons and positrons. For example, in the case of non-relativistic particles the expression in square brackets in (53) is reduced to more simple one

$$[\,]^2 \to \frac{g^2-(\mathbf{ng})^2}{g^4}[1-g^2R^2\,\mathrm{sign}(q)]^2. \tag{54}$$

In accordance with (53) and (54) the emission of non-relativistic electrons is reduced to coherent **B** on periodically arranged non-screened nuclei (stripping effect; see (42) as well), so that the total emission of electrons increases. On the other hand the discussed interference of **B** and **PB** channels suppress the total emission yield from positions.

The situation becomes more complicated in the case of relativistic emitting particles. Since the contribution of **B** from relativistic particles is concentrated within the range of small observation angles $\Theta \leqslant \gamma^{-1}$ near to the direction of emitting particle velocity **V** the interference between **B** and **PB** from relativistic particles may be substantial for small emission angles $\varphi \sim \gamma^{-1}$ only (see Fig. 7). On condition $\varphi \sim \gamma^{-1} \ll 1$ under consideration the coherent part of the total emission is described by the formula

$$\omega \frac{d^4 N_{coh}^{tot}}{dt\, d\omega\, d^2\Theta} = \sum_{\mathbf{g}}{}' \frac{e^2 \omega_0^4}{2\pi g^2} \frac{|S(\mathbf{g})|^2 e^{-g^2 U_T^2}}{(1+g^2R^2)^2} \times \Phi\delta\left[\omega - g\frac{\varphi + 2\Theta'}{\gamma_*^{-2} + \Theta_\perp^2 + (\varphi - \Theta_{\|})^2}\right], \quad (55a)$$

$$\begin{aligned}\Phi = \frac{1}{\Omega_1}\Bigg[&\frac{(\gamma_*^{-2} + \Theta_\perp^2 - (\varphi - \Theta_{\|})(2\Theta' + \Theta_{\|}))^2 + (\varphi + 2\Theta')^2\Theta_\perp^2}{\Omega_0^2} \\ &+ \frac{4g^4R^4}{\gamma^2}\frac{1}{\Omega_0^2}\left(1 - \frac{4\gamma_*^{-2}(\varphi - \Theta_{\|})^2}{\Omega_1^2}\right) \\ &- \mathrm{sign}(q)\frac{4g^2R^2}{\gamma}\frac{1}{\Omega_1}\frac{1}{\Omega_0} \\ &\times\Bigg(\gamma_*^{-2} + \Theta_\perp^2 + (\varphi - \Theta_{\|})(2\Theta' + \Theta_{\|}) \\ &-2\frac{\gamma_*^{-2}(\varphi - \Theta_{\|})(\varphi + 2\Theta')}{\Omega_1}\Bigg)\Bigg], \quad (55b)\end{aligned}$$

where $\Omega_0 = \gamma_*^{-2} + \Theta_\perp^2 + (2\Theta' + \Theta_{\|})^2$, $\Omega_1 = \gamma_*^{-2} + \Theta_\perp^2 + (\varphi + \Theta_{\|})^2$. The first item in square brackets in (55b) corresponds to **PXR** contribution, the second one describes the coherent bremsstrahlung contribution and the last item determines the influence of an interference between **PXR** and coherent **B**.

The interference is liable to change the emission properties very substantially. In order to show this let us consider the orientation dependence of strongly collimated emission ($\gamma\Theta_\perp \ll 1$, $\gamma\Theta_{\|} \ll 1$). Such dependence is determined by the function Φ which may be reduced to more simple one

$$\Phi/\gamma^2 = \left[\alpha\,\mathrm{sign}(q) - \frac{x - 1/\beta}{1 + x^2}\right]^2 \quad (56)$$

on condition $1 \gg \varphi \gg \gamma^{-1}$ under consideration. Here $\alpha = 2g^2R^2/\gamma^2\varphi^3$ is the coefficient proportional to coherent **B** amplitude, $\beta = \gamma\varphi \gg 1$, $x = 2\gamma\Theta'$. Orientation dependencies of the emission yield from electrons and positrons calculated by (56) are presented in Figs. 8 and 9, respectively. Characteristics of the total emission from electrons and positrons are very different in accordance with presented curves, although cross-sections of coherent **B** and **PXR** do not depend on the sign of emitting particle's charge. The interference between **PXR** and coherent **B** has been predicted in works (Nasonov and Safronov, 1993; Kleiner et al., 1994). This effect has beenobserved for the first time in work (Blazhevich et al., 1994).

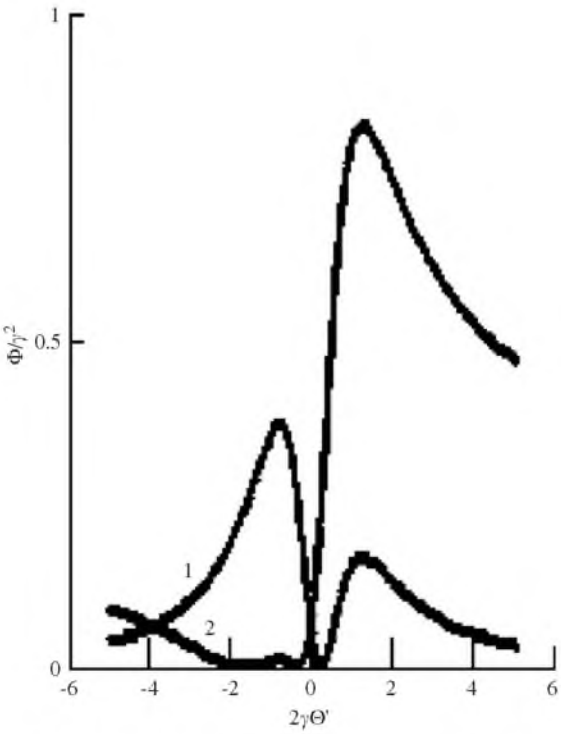

Fig. 8. Interference between coherent bremsstrahlung and parametric X-rays. Orientation dependence of the collimated emission yield from electrons. The function $\Phi(2\gamma\Theta', \alpha, \beta)/\gamma^2$ determined by the Eq. (56) is presented here. The curve 1 calculated for the values of parameters $\beta = 5$, $\alpha = 0$ describes **PXR** yield only. The curve 2 calculated for $\beta = 5$, $\alpha = 0.5$ takes into account both **PXR**.

Above indicated works were devoted to theoretical and experimental studies of the discussed interference effect for relativistic emitting particles. Recently the coherent X-rays from non-relativistic electrons crossing a crystalline target was considered theoretically with account of coherent **B** and **PXR** emission mechanisms (Feranchuk et al., 2000). It is very important that the developed theoretical model allows to explain experimental data (Korobochko et al., 1965; Reese et al., 1984).

Thus, as in the case of **B** process, characteristics of **PB** from fast charged particles moving a dense medium are changed very substantially as compared with **PB** on a single atom. It should be noted that **PB** properties depend strongly on the atomic structure of a target, suggesting that **PB** provide a useful method of the medium structure diagnostics. Such possibilities have been demonstrated in theoretical and experimental studies of **PB** from relativistic electrons moving through polycrystalline (Blazhevich et al., 1999) and small-grained targets (Blazhevich et al., 1996).

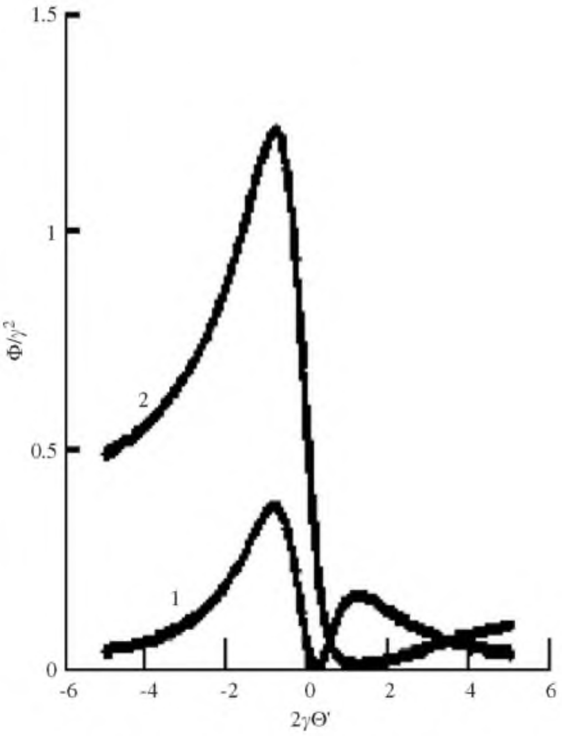

Fig. 9. The same but for positrons.

The performed above analysis was based on the perturbation theory. This approach, as a rule, is adequate to study of **PB** in amorphous media. On the other hand the scattered electromagnetic field appearing in the process of coherent **PB** or **PXR** in a crystalline target can be comparable with equilibrium electromagnetic field of an incident particle by analogy with the process of free X-ray scattering in a crystal (Pinsker, 1984). **PXR** in such conditions is studied in the next section of the paper.

5. Dynamical diffraction effects in PXR

Returning to the general equation (37) let us consider **PB** process beyond the frame of perturbation theory. Such analysis is of great interest for some reason. First of all, it is necessary to elucidate the physical reason of high efficiency of this simple perturbation approach used in the previous section and known as kinematical theory of **PXR** (this theory describes the most part of obtained experimental results). More general approach to the description of **PXR** is based on so-called dynamical diffraction theory widely used in the physics of free X-ray scattering in crystals. In the context of dynamical diffraction theory **PXR** is treated as quasi-Cherenkov emission mechanism realizing due to dynamical change of effective refractive index of a crystal (this index can exceed a unit and so the condition of Cherenkov radiation can be realized) (Baryshevsky and Feranchuk, 1983; Caticha, 1992). Contrary to that, **PXR** in the context of kinematic theory is treated as the result of scattering process (Ter-Mikaelian, 1972). The nature of **PXR** and its connection with discussed different pictures of this phenomenon is considered below.

In the case of relativistic emitting electrons under consideration the electromagnetic field associated with these electrons is practically transverse, so that $\mathbf{E}_{\omega\mathbf{k}} \approx \mathbf{E}^{\rm tr}_{\omega\mathbf{k}} = \sum_{\lambda=1}^{2} \mathbf{e}_{\lambda\mathbf{k}} E^{\rm tot}_{\lambda k}$, $\mathbf{e}_{\lambda\mathbf{k}}$ is the polarization vector, $\mathbf{k}\mathbf{e}_{\lambda\mathbf{k}} = 0$. Separating the total field $E^{\rm tot}_{\lambda\mathbf{k}}$ into average $E_{\lambda\mathbf{k}}$ and accidental $\tilde{E}_{\lambda\mathbf{k}}$ parts ($\langle E^{\rm tot}_{\lambda\mathbf{k}}\rangle = E_{\lambda\mathbf{k}}$) and substituting the sum of these quantities in (37), one can obtain the following system:

$$(k^2-\omega^2)E_{\lambda\mathbf{k}} + \sum_{\lambda'=1}^{2}\int \mathrm{d}^3k'\,\mathbf{e}_{\lambda\mathbf{k}}\mathbf{e}_{\lambda'\mathbf{k}'}(\langle G(\mathbf{k}'-\mathbf{k})\rangle E_{\lambda'\mathbf{k}'} + \langle \tilde{G}(\mathbf{k}'-\mathbf{k})\tilde{E}_{\lambda'\mathbf{k}'}\rangle) = \frac{\mathrm{i}\omega e}{2\pi^2}\mathbf{e}_{\lambda\mathbf{k}} - \mathbf{V}\delta(\omega-\mathbf{k}\mathbf{V}), \tag{57a}$$

$$(k^2-\omega^2)\tilde{E}_{\lambda\mathbf{k}} + \sum_{\lambda'=1}^{2}\int \mathrm{d}^3k'\,\mathbf{e}_{\lambda\mathbf{k}}\mathbf{e}_{\lambda'\mathbf{k}'}(\langle G(\mathbf{k}'-\mathbf{k})\rangle \tilde{E}_{\lambda'\mathbf{k}'} + \tilde{G}(\mathbf{k}'-\mathbf{k})E_{\lambda'\mathbf{k}'} + \tilde{G}(\mathbf{k}'-\mathbf{k})\tilde{E}_{\lambda'\mathbf{k}'} - \langle \tilde{G}(\mathbf{k}'-\mathbf{k})\tilde{E}_{\lambda'\mathbf{k}'}\rangle) = 0. \tag{57b}$$

Connection between field components $E_{\lambda\mathbf{k}}$ and $\tilde{E}_{\lambda\mathbf{k}}$ is responsible for the absorption of regular component $E_{\lambda\mathbf{k}}$, but this process is not very substantial in the case of perfect crystalline target under consideration and can be neglected in the first approximation. Dropping out the term in (57a) proportional to $\langle \tilde{G}(\mathbf{k}'-\mathbf{k})\tilde{E}_{\lambda\mathbf{k}}\rangle$ and using the formula

$$\langle G(\mathbf{k}'-\mathbf{k})\rangle = \omega_0^2\sum_{\mathbf{g}}\frac{S(\mathbf{g})\mathrm{e}^{-(1/2)g^2U_{\rm T}^2}}{1+g^2R^2}\delta(\mathbf{k}'-\mathbf{k}-\mathbf{g}) \equiv -\omega^2\sum_{\mathbf{g}}\chi_{-\mathbf{g}}\delta(\mathbf{k}'-\mathbf{k}-\mathbf{g}) \tag{58}$$

one can reduce (57a) to well-known system of dynamical diffraction equations

$$(k^2-k_0^2)E_{\lambda\mathbf{k}} - \omega^2\sum_{\mathbf{g}}{}'\chi_{-\mathbf{g}}\sum_{\lambda'=1}^{2}\mathbf{e}_{\lambda\mathbf{k}}\mathbf{e}_{\lambda'\mathbf{k}+\mathbf{g}}E_{\lambda'\mathbf{k}+\mathbf{g}} = \frac{\mathrm{i}\omega e}{2\pi^2}\mathbf{e}_{\lambda\mathbf{k}}\mathbf{V}\delta(\omega-\mathbf{k}\mathbf{V}), \tag{59}$$

describing **PXR** from relativistic electron moving through a crystalline target along a rectilinear trajectory. It should be noted that the presented derivation of Eq. (59) from (37) underlines the nature of **PXR** as the coherent part of **PB**.

Let us use Eq. (59) to consider the relationship between kinematical and dynamical pictures of **PXR**. Within the framework if two-wave approximation widely accepted in **PB** theory the system (59) is reduced to more simple one

$$(k^2 - k_0^2)E_{\lambda\mathbf{k}} - \omega^2\chi - \mathbf{g}\alpha_\lambda E_{\lambda'\mathbf{k}+\mathbf{g}} = \frac{\mathrm{i}\omega e}{2\pi^2}\,\mathbf{e}_{\lambda\mathbf{k}}\mathbf{V}\delta(\omega - \mathbf{kV}), \tag{60a}$$

$$((\mathbf{k}+\mathbf{g})^2 - k_0^2)E_{\lambda\mathbf{k}+\mathbf{g}} - \omega^2\chi_{\mathbf{g}}\alpha_\lambda E_{\lambda\mathbf{k}} = 0, \tag{60b}$$

where $\alpha_1 = 1$, $\alpha_2 = (\mathbf{k}, \mathbf{k}+\mathbf{g})/k|\mathbf{k}+\mathbf{g}|$.

In order to obtain the kinematical limit of **PXR** theory it is sufficient to eliminate the term $\omega^2\chi_{-\mathbf{g}}\alpha_\lambda E_{\lambda\mathbf{k}+\mathbf{g}}$ in (60a). As this takes place, the field $E_{\lambda\mathbf{k}}$ coincides with Coulomb field of fast electron moving through a homogeneous medium with dielectric permeability $\varepsilon(\omega) = 1 - \omega_0^2/\omega^2$. In this picture the field $E_{\lambda\mathbf{k}+\mathbf{g}}$ appears due to the Bragg scattering of mentioned Coulomb field by a system of parallel atomic planes, determined by the reciprocal lattice vector $\mathbf{g}$. It is easy to show that the spectral-angular distribution of **PXR** intensity following from (60) within the framework of kinematical approximation coincides with (51).

In accordance with (60) the main dynamical diffraction effect in **PXR** consists in the modification of the structure of the field $E_{\lambda\mathbf{k}}$ which can be presented in the form (Kubankin et al., 2003)

$$E_{\lambda\mathbf{k}} = \frac{\mathrm{i}\omega e}{2\pi^2}\frac{\mathbf{e}_{\lambda\mathbf{k}}\mathbf{V}}{k^2 - \omega^2\varepsilon_{\mathrm{eff}}(\omega,\mathbf{k})}\,\delta(\omega - \mathbf{kV}), \tag{61a}$$

$$\varepsilon_{\mathrm{eff}} = 1 + \chi_0 + \frac{\omega^2\chi_{\mathbf{g}}\chi_{-\mathbf{g}}\alpha_\lambda^2}{(\mathbf{k}+\mathbf{g})^2 - \omega^2(1+\chi_0)} \tag{61b}$$

coinciding formally with that describing an electromagnetic field excited by a fast particle moving in a homogeneous medium with the spatial dispersion. The presented expression for the effective dielectric permeability $\varepsilon_{\mathrm{eff}}$ shows clearly that the possible emission has the quasi-Cherenkov nature and this emission, known as the forward **PXR** (Baryshevsky, 1997) can occur due to dynamical changing of the refractive index only, since $\chi_0(\omega) = -\omega_0^2/\omega^2 < 0$ in X-ray range.

On the other hand the solution $E_{\lambda\mathbf{k}+\mathbf{g}}$ cannot be represented in the form, analogous to (61). In accordance with (60b) the expression for $E_{\lambda\mathbf{k}+\mathbf{g}}$ has the form

$$E_{\lambda\mathbf{k}+\mathbf{g}} = \frac{\mathrm{i}\omega e}{2\pi^2}\frac{\mathbf{e}_{\lambda\mathbf{k}}\mathbf{V}}{k^2 - \omega^2\varepsilon_{\mathrm{eff}}}\frac{\omega^2\chi_{\mathbf{g}}\alpha_\lambda}{k_{\mathbf{g}}^2 - k_0^2} \times\delta(\omega - \mathbf{k_g V} + \mathbf{gV}), \tag{62}$$

where we take into account that the momentum $\mathbf{k_g} \equiv \mathbf{k}+\mathbf{g}$ corresponds to the field $E_{\lambda\mathbf{k}+\mathbf{g}}$ propagating along the Bragg scattering direction. Obviously, the field $E_{\lambda\mathbf{k}+\mathbf{g}}$ appears due to the coherent Bragg scattering of the field $E_{\lambda\mathbf{k}}$ (the scattering process is indicated by the momentum transfer $-\mathbf{g}$ in the kinematical conservation low $\omega - \mathbf{k_g V} = -\mathbf{gV}$). It is easy to see that the emission field $E_{\lambda\mathbf{k}+\mathbf{g}}$ occurs independently on the character of the primary field $E_{\lambda\mathbf{k}}$. Particularly, the discussed emission occurs without account of the dynamical diffraction effects when $\varepsilon_{\mathrm{eff}} \to 1 + \chi_0$ and the field $E_{\lambda\mathbf{k}}$ is reduced to ordinary Coulomb field of a fast electron and **PXR** can be described within the framework of kinematical approach.

Let us consider an influence of dynamical diffraction effects on **PXR** properties in greater detail. Calculating the Fourier integral $E_\lambda^{\mathrm{Rad}} = \int \mathrm{d}^3k_g\, \mathrm{e}^{\mathrm{i}\mathbf{k}_g\mathbf{n}r} \times E_{\lambda\mathbf{k}+\mathbf{g}}$ by the stationary phase method one can obtain the following expression for the emission field in wave zone:

$$E_\lambda^{\mathrm{Rad}} = \frac{\mathrm{i}}{2}e\omega(\mathbf{e}_{\lambda\omega\mathbf{k}-\mathbf{g}}\mathbf{V})\frac{\omega\chi_{\mathbf{g}}\alpha_\lambda}{\sqrt{\Delta^2 + \omega^2\chi_{\mathbf{g}}\chi_{-\mathbf{g}}\alpha_\lambda^2(1 - \mathbf{ng}/\omega)}} \times\left[\mathrm{e}^{\mathrm{i}\xi_+ r}\delta\left(\omega\left(1 - \left(1 + \frac{1}{2}\chi_0\right)\mathbf{nV}\right) + \mathbf{gV} - \xi_+\mathbf{nV}\right) + \mathrm{e}^{\mathrm{i}\xi_- r}\delta\left(\omega\left(1 - \left(1 + \frac{1}{2}\chi_0\right)\mathbf{nV}\right) + \mathbf{gV} - \xi_-\mathbf{nV}\right)\right] \times\frac{\mathrm{e}^{\mathrm{i}k_0 r}}{r}, \tag{63}$$

where the phase addendum $\xi_\pm$ caused by dynamical diffraction effects is determined by the formula

$$2(1 - \mathbf{ng}/\omega)\xi_\pm = -\Delta \pm \sqrt{\Delta^2 + \omega^2\chi_{\mathbf{g}}\chi_{-\mathbf{g}}\alpha_\lambda^2(1 - \mathbf{ng}/\omega)}, \tag{64a}$$

$$\Delta = \frac{g^2}{2\omega} - \mathbf{ng}\left(1 + \frac{1}{2}\chi_0\right). \tag{64b}$$

The quantity Δ in (64b) is so-called resonance defect, describing the deviation of emitted wave from the exact Bragg resonance.

For the further analysis one should introduce the two-dimensional angular variable $\mathbf{\Theta}$ by the formula (50) and use the reciprocal lattice vector $-\mathbf{g}$ instead of $\mathbf{g}$ (see Fig. 7) since the scattered wave is described in the dynamical diffraction theory by the vector $\mathbf{k}_g$ in contrast with $\mathbf{k}$ in the kinematical theory. Since **PXR** spectrum is concentrated in the narrow vicinity to the Bragg frequency ω_{B} (see (51)) it is very convenient to use the resonance defect Δ instead of ω to analyze the obtained formulae (63) and (64). Indeed, one can see that only $\Delta(\omega)$ is the rapidly changed function on ω in these formulae (among other things the equality $\Delta(\omega) = 0$ is equivalent to another one $\omega = \omega_{\mathrm{B}}(1 + (\Theta' + \Theta_{||})\cot(\varphi/2)$ with an accuracy of γ^{-1}, so that this equality determines the emission line).

There are two branches of possible emitted waves in the dynamical diffraction picture as it follows from (63), but only one of them can be realized in fact. To show this let us analyze the expression for the spectral-angular distribution of the emission intensity

$$\omega\frac{d^4N_\lambda}{dt\,d\omega\,d^2\Theta}=\frac{e^2\omega^2}{4\pi}(\mathbf{e}_{\lambda\omega\mathbf{k}-\mathbf{g}}\mathbf{V})^2\frac{\omega^2|\chi_0|^2\alpha_\lambda^2}{\Delta^2+\omega^2\chi_\mathbf{g}\chi_{-\mathbf{g}}\alpha_\lambda^2\cos\varphi}\times\Big\{\delta\Big[\omega_\mathrm{B}(\gamma_*^{-2}+\Theta_\perp^2+(2\Theta'+\Theta_\parallel)^2)-\varDelta-\sqrt{\varDelta^2+\omega^2\chi_\mathbf{g}\chi_{-\mathbf{g}}\alpha_\lambda^2\cos\varphi}\Big]+\delta\Big[\omega_\mathrm{B}(\gamma_*^{-2}+\Theta_\perp^2+(2\Theta'+\Theta_\parallel)^2)-\varDelta+\sqrt{\varDelta^2+\omega^2\chi_\mathbf{g}\chi_{-\mathbf{g}}\alpha_\lambda^2\cos\varphi}\Big]\Big\},\quad(65)$$

following from (63) and (64).

It is easy to show that the argument of the first δ-function in (65) can be equal to zero. This argument is the dispersion equation for the branch corresponding to the phase addendum ξ_+. The discussed branch is generally identified as Cherenkov branch to underline the quasi-Cherenkov nature of **PXR** in the dynamical diffraction picture. But, as is evident from the foregoing, **PXR** propagation along the direction of Bragg scattering cannot be interpreted as Cherenkov radiation. In order to underline this fact let us consider the value of phase velocity V_ph of emitted **PXR** photons. Using (64) and the dispersion equation following from (65) one can obtain the formula

$$V_\mathrm{ph}=\frac{\omega}{k_g(\omega)}=\frac{\omega}{k_0+\xi_+}=\Big[1-\frac{1}{2}\frac{\omega_0^2}{\omega^2}+\frac{1}{2}\frac{\chi_\mathbf{g}\chi_{-\mathbf{g}}\alpha_\lambda^2}{\gamma^{-2}+(\omega_0^2/\omega^2)+\Theta_\perp^2+(2\Theta'+\Theta_\parallel)^2}\Big]^{-1}.\quad(66)$$

Obviously $V_\mathrm{ph}>1$, so that the discussed **PXR** wave cannot be generated by the Cherenkov mechanism.

The final expression for **PXR** spectral-angular distribution follows from (65) in the form

$$\omega\frac{d^4N}{dt\,d\omega\,d^2\Theta}=\frac{e^2\omega_0^4}{\pi g^2}\frac{|S(\mathbf{g})|^2e^{-g^2U_T^2}}{(1+g^2R^2)^2}\times\frac{\Theta_\perp^2}{(\gamma^{-2}+(\omega_0^2/\omega^2)+\Theta_\perp^2+(2\Theta'+\Theta_\parallel)^2)+\chi_\mathbf{g}\chi_{-\mathbf{g}}\cos\varphi}+\frac{(2\Theta'+\Theta_\parallel)^2\cos^2\varphi}{(\gamma^{-2}+(\omega_0^2/\omega^2)+\Theta_\perp^2+(2\Theta'+\Theta_\parallel)^2)+\chi_\mathbf{g}\chi_{-\mathbf{g}}\cos^3\varphi}\times\delta\Big[\omega-\omega_\mathrm{B}\Big(1+(\Theta'+\Theta_\parallel)\cot\frac{\varphi}{2}\Big)\Big].\quad(67)$$

Obviously, the dynamical distribution (67) differs from kinematical one (51) by the terms in the denominators in (67) proportional to $\chi_\mathbf{g}\chi_{-\mathbf{g}}=\omega_0^4|S(\mathbf{g})|^2\cdot e^{-g^2U_T^2}/\omega^4(1+g^2R^2)^2<\omega_0^4/\omega^4$.

In accordance with (67) dynamical diffraction effects can occur in the range of high enough emitting particle energies only, where the condition

$$\gamma\gg\gamma_0=\frac{\omega_\mathrm{B}}{\omega_0}=\frac{g}{2\omega_0\sin(\varphi/2)}\quad(68)$$

is fulfilled. But one further effect manifests in this range of electron energies. It is well-known density effect or Fermi-effect (brief mention has already been made of this effect in the section devoted to **PB** from amorphous medium). The influence of this effects is described by the term ω_0^2/ω^2 in the denominators in formulae (67) and (51). In order to elucidate relative contributions of discussed effects to the formation of **PXR** yield let us consider the dependence of the maximum of **PXR** angular density $(d^3N/dt\,d^2\Theta)_\mathrm{max}$ on the emitting electron energy. Reference to Eq. (67) shows that

$$\left(\frac{d^3N}{dt\,d^2\Theta}\right)_\mathrm{max}=\frac{e^2\omega_0^2\omega_\mathrm{B}}{2\pi g^2}\frac{|S(\mathbf{g})|^2e^{-g^2U_T^2}\cos^2\varphi}{(1+g^2R^2)^2}T(\gamma/\gamma_0,q),$$

$$T=\frac{\gamma^2/\gamma_0^2}{1+\gamma^2/\gamma_0^2+\sqrt{(1+\gamma^2/\gamma_0^2)^2+(\gamma^4/\gamma_0^4)q}},$$

$$q=\frac{|S(\mathbf{g})|^2e^{-g^2U_T^2}}{(1+g^2R^2)^2}\cos^3\varphi<1.\quad(69)$$

The function T allows us to estimate separate and simultaneous contributions of the density and dynamical effects. An influence of such contributions is illustrated by the curves $T(\gamma/\gamma_0)$ presented in Fig. 10. Here curve 1 corresponds to kinematical theory of **PXR** without regard for both discussed effects (this is accomplished by the transformations $(1+\gamma^2/\gamma_0^2\to 1)$ and $q\to 1$ in the denominator of the function $T(\gamma/\gamma_0)$). Curve 2 corresponds to dynamical theory of **PXR** but without considering the density effect (the transformation $(1+\gamma^2/\gamma_0^2\to 1)$ was used). This curve demonstrates the ordinary in dynamical diffraction theory saturation of the intensity of diffracted wave. Curve 3 corresponds to kinematical theory of **PXR** with regard to density effect (parameter $q=0$ in the function T). This curve shows the dominant role of the density effect in the saturation of **PXR** yield. This conclusion is confirmed by the last curve in Fig. 10 calculated with regard for both bounding effects.

Thus, the manifestation of the density effect is the physical reason of a weak influence of dynamical diffraction effects on **PXR** properties, as it has been shown for the first time (Nasonov and Safronov, 1993).

6. Conclusions

Low energy bremsstrahlung from fast charged particles moving in a dense medium can be substantially modified due to collective contribution of medium

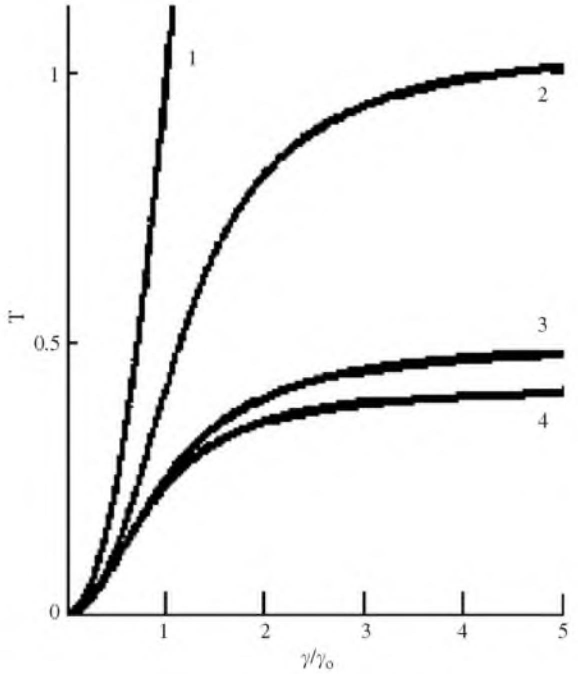

Fig. 10. Influence of both the density effect and dynamical diffraction effects on the parametric X-rays. The function $T(\gamma/\gamma_0, q)$ determined by the Eq. (69) is presented here. The curve 1 describes kinematical **PXR** without account of both the density effect and dynamical diffraction effects. The curve 2 corresponds to dynamical **PXR** but without account of the density effect. The curve 3 describes kinematical **PXR** but with account of the density effect. The curve 4 has been calculated with account of both discussed effects. The value of the parameter $q = 0.9$ was used when calculating of presented curves.

atoms to the formation of the emission yield. Some new examples of such modification of bremsstrahlung and an additional emission mechanism known as polarization bremsstrahlung have been discussed in the paper.

Multiple scattering of emitting particles can suppress not only bremsstrahlung from relativistic electrons (Landau–Pomeranchuk–Migdal effect), but low energy bremsstrahlung from non-relativistic electrons as well. In contrast with LPM effect the discussed suppression is caused by the limitation on the possible change of emitting electron velocity induced by multiple scattering.

The analogous suppression effect is possible in the process of coherent bremsstrahlung from relativistic electrons on a system of parallel atomic strings of a crystalline target. The emission properties are determined in the case in question not only by the contribution of coherent bremsstrahlung, but by the contribution of transition radiation and its interference with coherent bremsstrahlung as well. Depending on parameters determining the relative contribution of indicated emission mechanisms the total emission spectrum is subject to wide variations.

In the case of thin enough target when contributions of coherent bremsstrahlung and transition radiation are comparable the anomalous Ter-Mikaelian effect takes place. This effect analogous to that for amorphous target (Arkatov et al., 1996; Nasonov, 2001b) consists in the partial suppression of the total emission spectrum in the vicinity of $\omega = \gamma\omega_0$ (γ is the Lorentz-factor, ω_0 is the plasma frequency), where the spectrum has a dip hole.

In the case of thin target and small orientation angles Ψ of the emitting electron relative to the atomic string axis $\Psi \sim \Psi_{ch}$ (Ψ_{ch} is the critical channelling angle) when coherent bremsstrahlung is suppressed due to azimuthal multiple scattering of this electron on atomic strings, the interference between coherent bremsstrahlung and transition radiation can result in a strong peak in the spectrum instead of the hole.

Indicated and other peculiarities in the spectrum of the total coherent emission can be observed experimentally in X-ray range from tens to hundreds keV.

An influence of collective effects on polarization bremsstrahlung is found to be no less substantial than on bremsstrahlung. Even in the simplest case of a dense amorphous medium interatomic correlations in the target (caused by the finite size an atom) are responsible for the effect of polarization bremsstrahlung suppression. This effect is caused by the mutual screening of different atoms in the process of polarization bremsstrahlung.

The role of collective effects increases in the process of polarization bremsstrahlung from fast electrons penetrating a crystalline target. Periodical arrangement of atoms in a crystalline lattice results in two different components of the emission cross-section. First of them describing an individual contribution of target atoms to the emission yield is strongly suppressed. The second component describes the coherent contribution of all crystalline atoms to the emission yield. This part of polarization bremsstrahlung is found to be identical to well-known parametric X-ray radiation considered usually as the emission from a fast particle moving in a continuous medium with periodically changing macroscopic dielectric permeability $\varepsilon(\omega, \mathbf{r})$.

It is significant that not only the simplest kinematical model of parametric X-rays (perturbation theory) follows from the theory of polarization bremsstrahlung but the most general model based on dynamical diffraction approach as well. This circumstance allows to elucidate some unsolved questions in the physics of parametric X-rays. Specifically, as the performed analysis suggests, the known experimental result consisting in suppression of dynamical diffraction effects in parametric X-rays is caused by the well-known density effect.

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