MULTI-WAVE EFFECTS IN PXR GENERATED BY LOW ENERGY PARTICLES

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PXR - Parametric X-Ray Radiation Is produced by a charged particle moving through a crystal with a unifom velocity.

The Bragg diffraction condition is fulfilled for emitted photons

$$(\vec{k}+\vec{h})^2 = k^2 , \qquad (1)$$

$\vec{k}$ - the wave vector of emitted p!loton,

$\vec{h}$ - the crystal reciprocal lattice vector corresponding to, the Bragg planes.

Under the condition of multi-wave diffraction condition (1) is fulfilled for several different $\vec{h}$ simultaneously.

For thick perfect crystal where the Bragg diffraction of emitted quanta is **dynamical** the condition of Cherenkov resonance between the particles and the quanta can be fulfilled (ħ= c = 1)

$$1 - Vn_j(\vec{k},\omega)\cos\theta = 0$$

V is the velocity of particles,

$n_j(\vec{k},\omega)$ is the refractive indices of X-rays in crystal under the Bragg diffraction. Now there are several refractive indices, (for each energy branch).

The experiments on the observation of PXR under the condition of multilpe diffraction of emitted photons have been conducted in Tomsk synchrotron (V.P.Afanasenko et al Phys.Lett.A 141 (1989) 311, Sov.Phys. -JETP Lett. 51 (1990) 242).

The conditions of these experiments are the following:

GaAs crystal

E_e = 500MeV, L = 100μk

4-wave diffraction condition, (000), (220), ($\bar{1}\bar{5}\bar{3}$), ($\bar{1}\bar{5}3$);

ω_B =18,4keV; $\Delta\theta_1$=0,1 mrad; $\Delta\theta_2$=1,0 mrad; Ψ_S=2,5 mrad

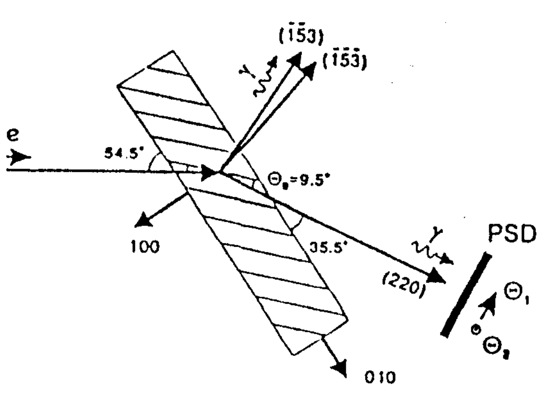
E_e = 900 MeV, 8-wave diffraction condition L = 400μk

8-wave diffraction condition - (000), (400), (022), (02$\bar{2}$), (202), (20$\bar{2}$), (040), (440) ,

ω_B =6,2keV; $\Delta\theta_1$=0,3 mrad; $\Delta\theta_2$=3,0 mrad; Ψ_S=7,0 mrad vspacelcm.

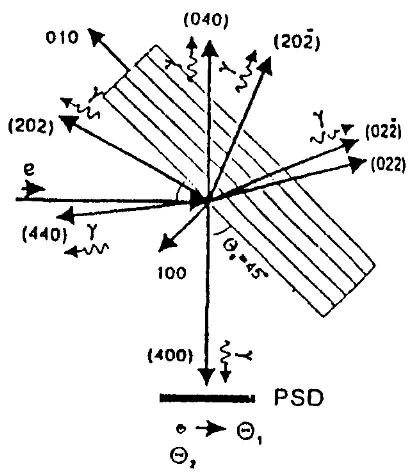
Schematic layouts of the experiments

4-wave experiment

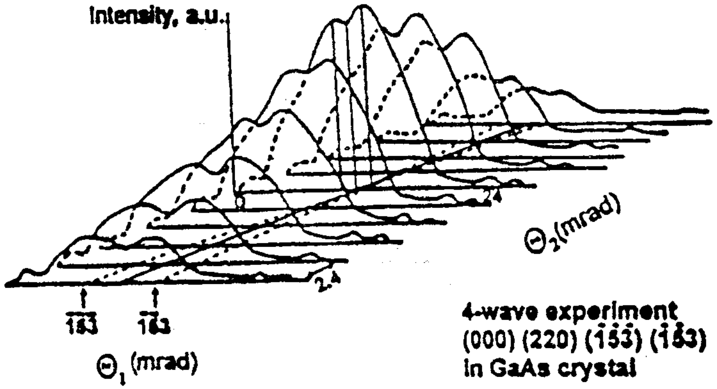


(a)

8-wave experiment



(b)



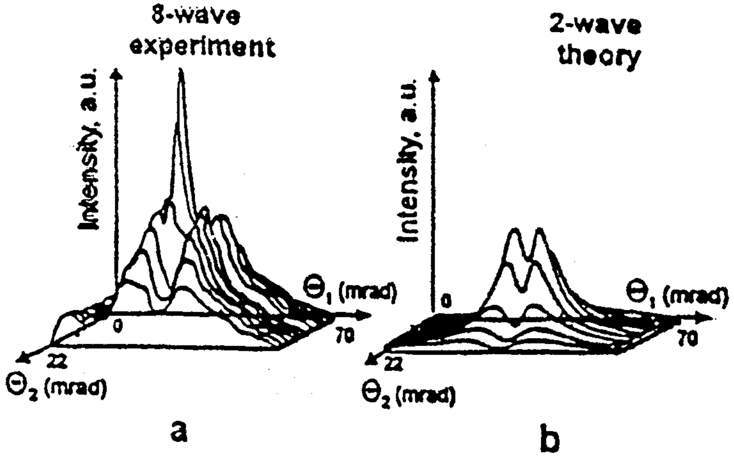


Fig. 2. (Left) The PXR angular distribution for (220) reflection in GaAs measured in Ref. [17] under 4-wave diffraction conditions.

Fig. 3. (Right) The PXR angular distribution for (400) reflection in GaAs. (a): the experimental distribution measured in Ref. [19] under 8-wave diffraction conditions, (b): the theoretical distribution calculated in 2-wave approximation.

Our algorithm is based on the numerical methods used in the dynamical theory of X-ray multiple diffraction (V.G.Kohn Phys.Stat.Sol.(a) 54 (1979) 375) and has been published in J.Phys.C: Condens.Matter 5 (1993) 7771.

The application of the dynamical diffraction theory is argued by the three factors:

1. the conditions of experiments presume the dynamical diffraction and the dominating role of the quasi-Cherenkov effect;
2. the analysis of interaction between different peaks of multiple Bragg diffraction can be made only in the frame of dynamical diffraction theory;
3. the dynamical diffraction theory is more general and permits easily to transit to the kinematic one.

The expression for a number of PXR quanta emitted by a particle in a Bragg reflection h can be written:

$$dN^{S}_{\vec{n},\omega} = \left(\frac{e\omega}{2\pi}\right)^{2}\left|\int_{t_0}^{t_1} E^{(+)S}_{-k}\left(\vec{r}(t),\omega\right)\cdot\vec{V}(t)\exp(i\omega t)dt\right|^{2}\frac{d\omega}{\omega}d\Omega\ .$$

$\vec{r}=\vec{V}t$, $E^{(+)S}_{-k}$ is the wavefield produced by the particle in crystal, s denotes σ- and π-polarizations.

Representing $E^{(+)S}_{-k}$ as a sum of transverse Bloch waves which travel in all Bragg diffraction directions $\vec{k}_m=\vec{k}+\vec{m}$ and correspond to 2N-dispersion branches

$$E^{(+)S}_{-\vec{k}}(\vec{r},\omega)=\vec{e}^{\,S}_{m}\sum_{m}D^{S}_{m}(\vec{r}),$$

$$D^{S}_{m}(\vec{r})=\sum_{j=2}^{2N}\lambda^{(j)}D^{S(j)}_{m}\exp\left(i\left[k_m-\omega(\varepsilon^{(j)}-\alpha_m)\vec{n}\right]\vec{r}\right)$$

N is the order of multiple Bragg diffraction

$$\alpha_m=\left[\left(\vec{k}_0+\vec{m}\right)^2-\omega^2\right]/\,2\omega^2\gamma_m\ ,$$

γ_m are the cosines of the angles between the X-ray wave vectors ane the internel normal to the crystal surface;

$\lambda^{(j)}$ are the coefficients of excitation of different dispersion branches;

$D^{S(j)}$ are the field amplitudes for each branches;

$\in^{(j)}$ determine the refraction of X-ray Bloch waves of different branches as a function of α_m.

The wave amplitudes $D^{S(j)}_{m}$ and the parameters $\in^{(j)}$ can be found as iegen vectors and iegen values of the dynamical diffraction equations which have the form

$$\sum_{S'=\sigma,\pi}\ \sum_{m'=1,N}G^{SS'}_{mm'}D^{S'(j)}_{m}=\in^{(j)}D^{S(j)}_{m}$$

where

$$G^{SS'}_{mm'}=\sigma_m\delta^{SS'}_{mm'}-\frac{1}{2\gamma_m}\left\{\chi_{mm'}\left(\vec{e}^{\,S}_{m}\cdot\vec{e}^{\,S'}_{m'}\right)+i\chi^{Q}_{mm'}\left[\left(\vec{k}_m\cdot\vec{k}_{m'}\right)\left(\vec{e}^{\,S}_{m}\cdot\vec{e}^{\,S'}_{m'}\right)+\left(\vec{e}^{\,S}_{m}\cdot\vec{k}_{m'}\right)\left(\vec{k}_m\cdot\vec{e}^{\,S'}_{m'}\right)\right]/\,\omega^2\right\}$$

$\chi_{mm'}$ and $\chi^{Q}_{mm'}$, are the dipole and quadrupole components of the expansion of crystal dielectric susceptibility $\chi(\vec{r},\omega)$ in a Fourier series over the reciprocal lattice vectors.

$\lambda^{(j)}$ are determined by the boundary conditions:

$$\sum_{j=1}^{2N} C_m^{s(j)} \lambda^{(j)} = \delta_{m0}\left(\delta^{s\sigma}\cos(o) + \delta^{s\pi}\sin(o)\right)$$

where $C_m^{s(j)} = D_m^{s(j)}$ for the Lane-case Bragg waves ($\gamma_m > 0$, $C_m^{s(j)} = D_m^{s(j)} \exp(-i\omega \in^{(j)} l)$ for the Bragg-case waves ($\gamma_m < 0$).

Substituting the expressions given above the number of quanta emitted by a particle in crystal

$$dN_{\vec{n},\omega}^{S} = \left(\frac{e\omega}{2\pi}\right)^2 \left| \sum_{s'=\sigma}^{\pi} \sum_{j=1}^{2N} \left(\vec{V} \cdot \vec{e}_h^{S'}\right) \lambda_S^{(j)}(\omega) \frac{D_h^{S'(j)}(\omega)}{Q_h^{(j)}(\omega)} \right| \frac{d\omega}{\omega}$$

Here

$$Q^{(j)}(\omega) = \omega\left(\gamma^{-2} + \Theta_1^2 + \Theta_2^2 + \vec{\Psi}_v^2 + \Psi_s^2\right)/2 + \omega\gamma_h(\in^{(j)} - \alpha_h)$$

where $\gamma = E/m$, $\vec{\Psi}_v$ is tire angular deviation of particles from the exact incidence direction of multiple Bragg diffraction.

Ψ_s is the mean square angle of multiple scattering of particle inside the crystal.

Θ_1 and Θ_2 determine the deviations of the emitted quanta from the exact direction of multiple Bragg diffraction.

The angular distribution measured can be found by integrating

$$dN_k = \sum_{s=\sigma,\pi} \int_{-\infty}^{\infty} dN_{\vec{k},\omega}^{s} \frac{d\omega}{\omega}$$

within narrow bands of the Bragg ancl Cherenkov peaks.

PXR has been calculated for tile 8-wave diffraction of photond produced by an electron with the energy

$E_e = 7.5$ MeV, L = 100μk

In this case $\hat{\Psi}_S^2$=7.8 mrad

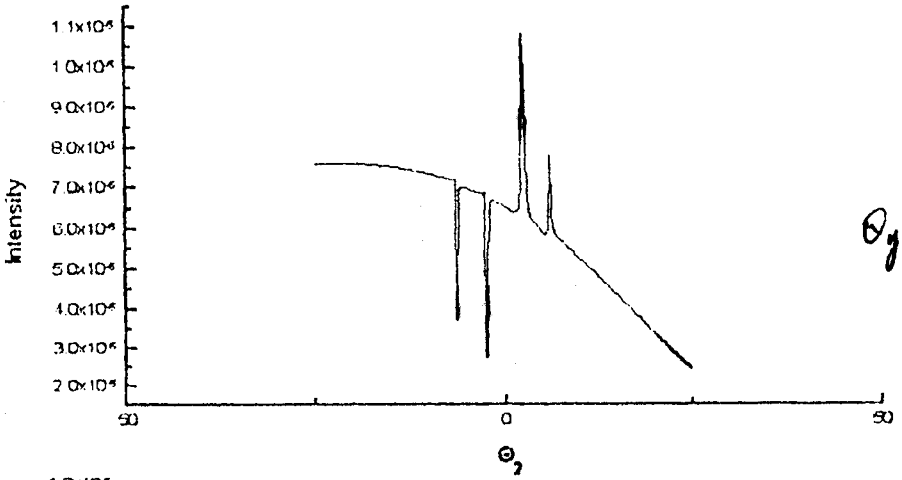
and the effective radiation angle is $1{,}28 \cdot 10^{-2}$ rad

he position of the maximum of two-wave PXR distribution is (θ_v = (38,5 - 55,5) mrad

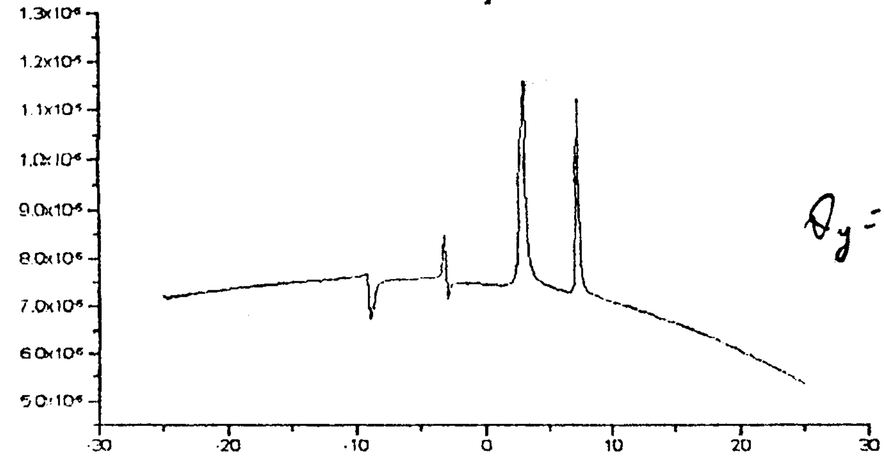
The influence of multiple scattering of electrons on PXR distribution has been taken into account by its averaging over initial divergence of transverse velocities of electrons in a beam

$$f(E,\vec{\theta},t) = \frac{1}{\sqrt{\pi \hat{\Psi}_S^2}} \exp - \frac{\left(\theta - \hat{\theta}_c\right)^2}{2\hat{\theta}_S^2}$$

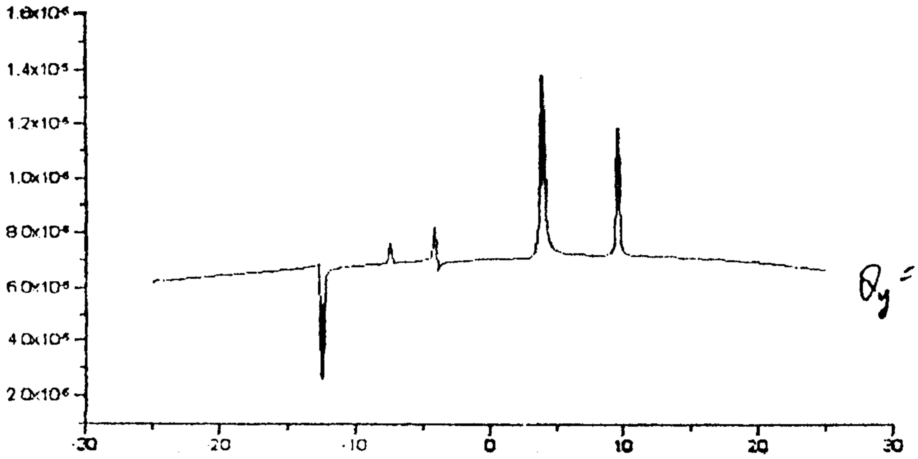
$\Psi_x=30$



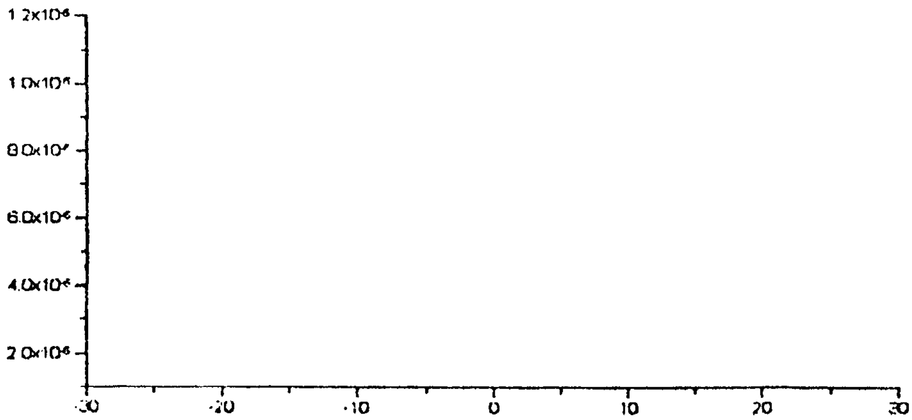
$\theta_y=-35$

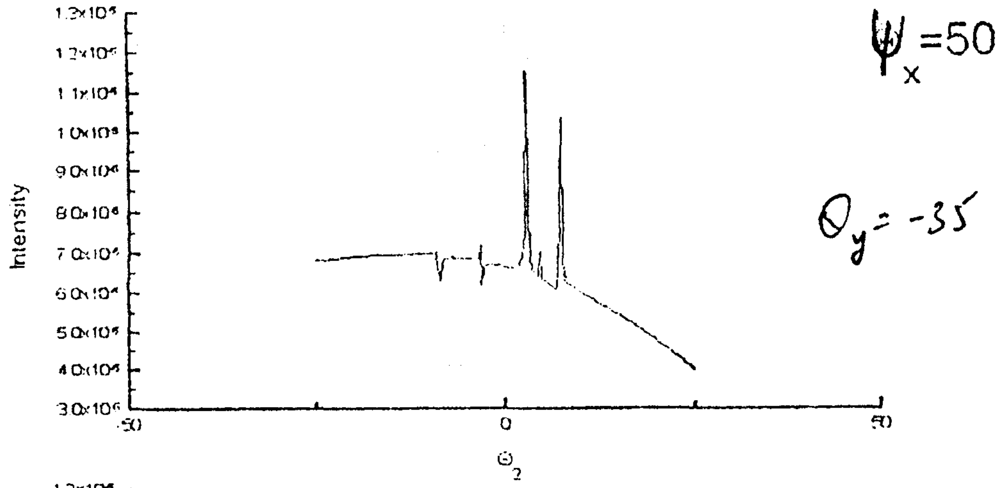


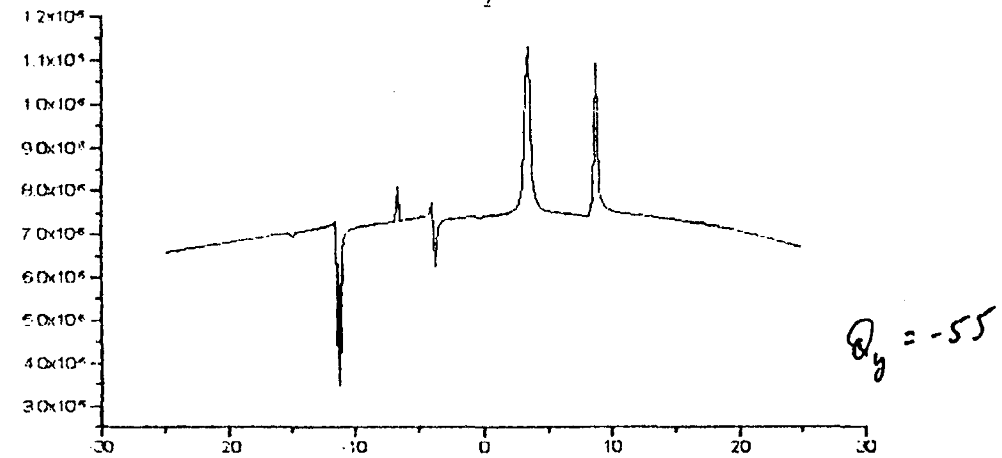
$\theta_y=-55$

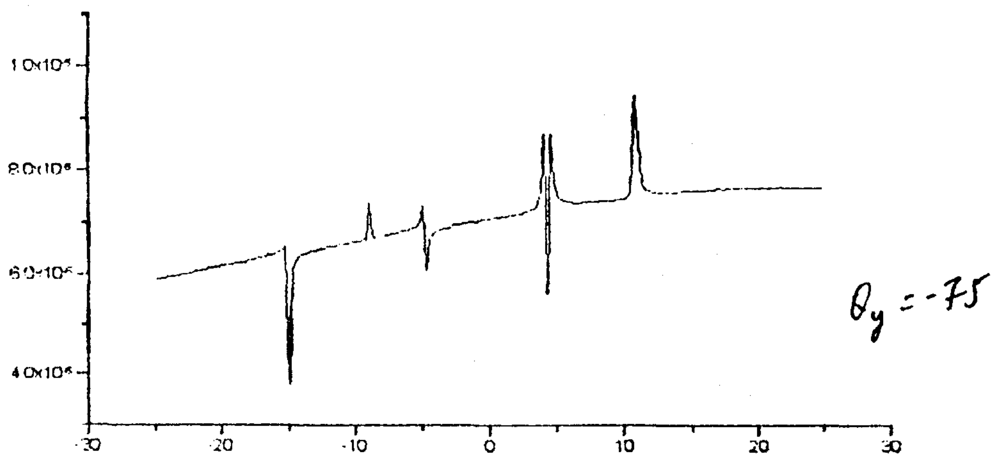


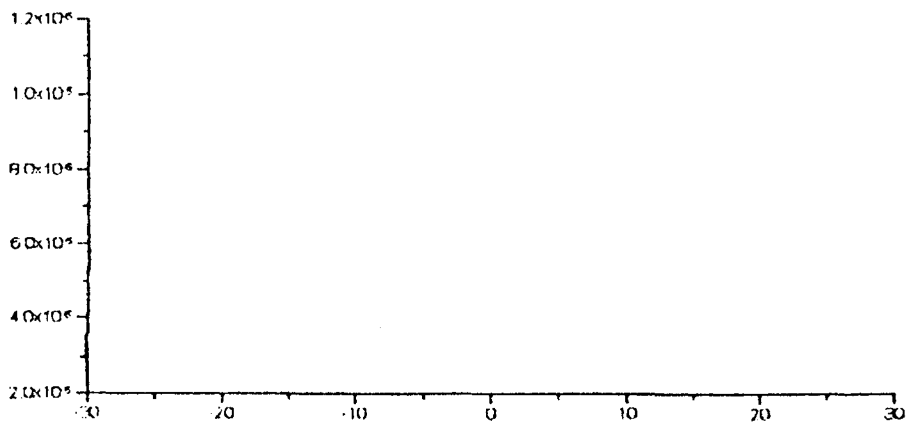
$\theta_y=-75$



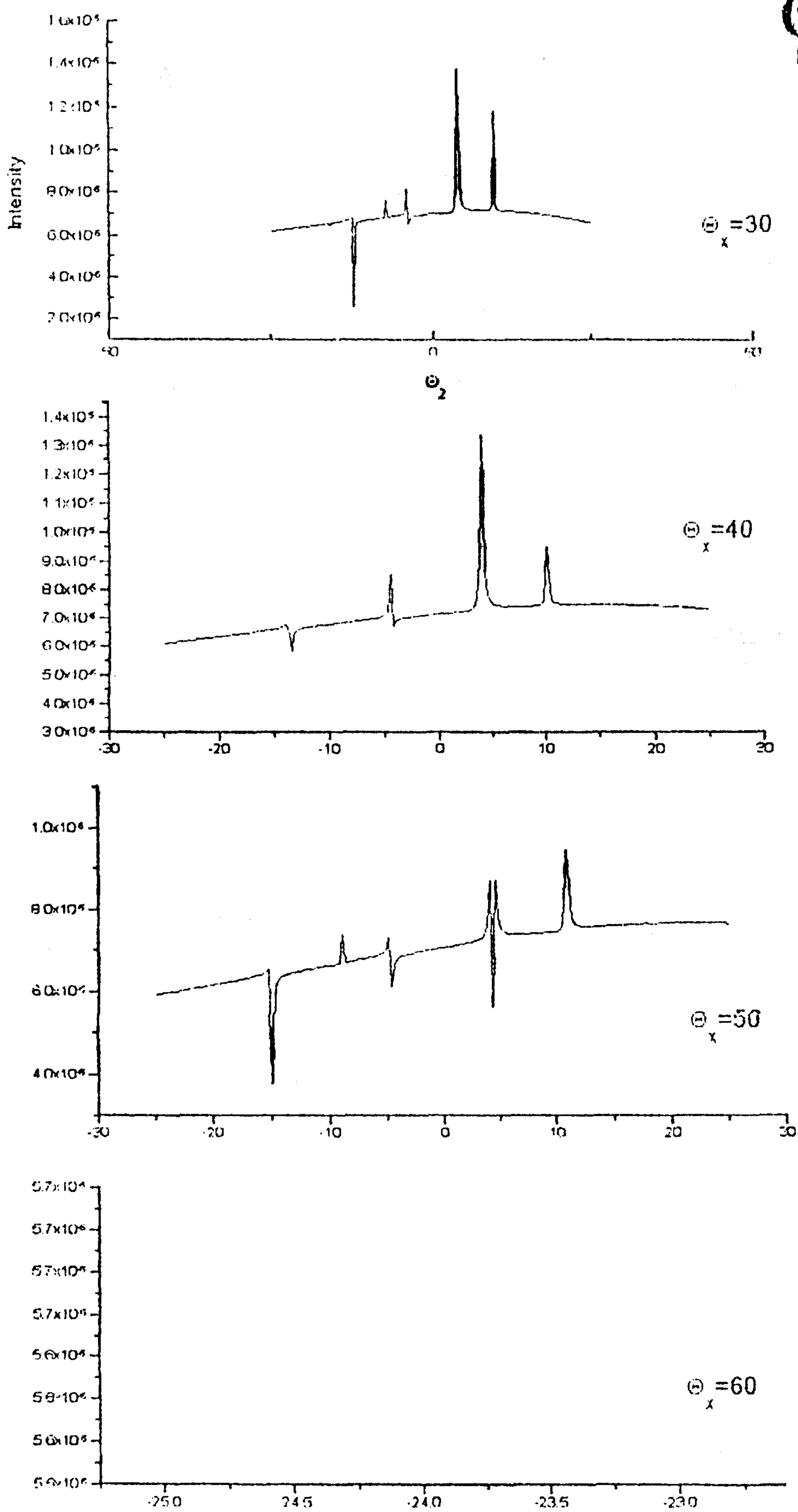

Ψx=50
Qy= -35
Intensity
Θ2
-50
0
50

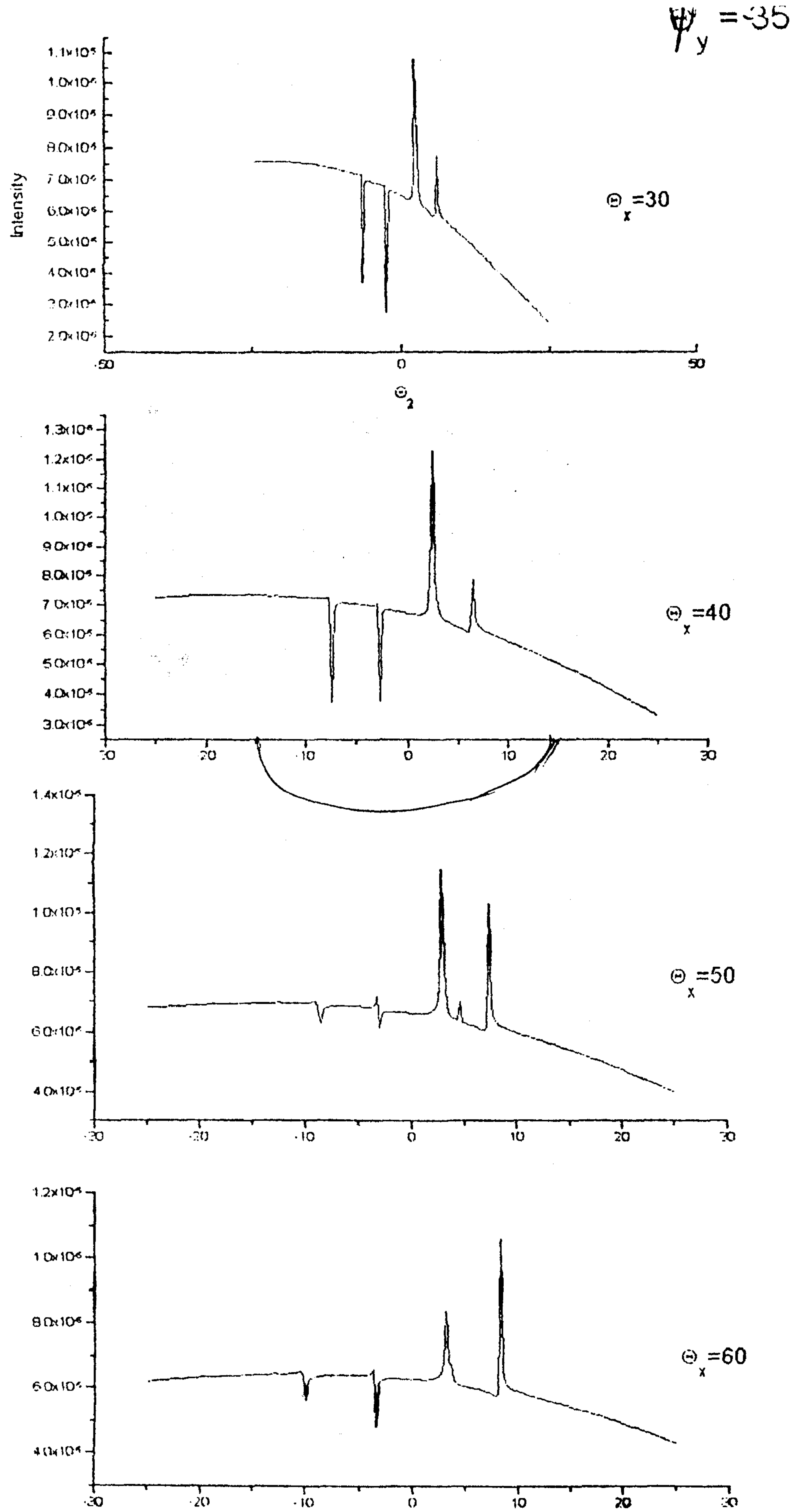

Qy = -55
-30
-20
-10
0
10
20
30

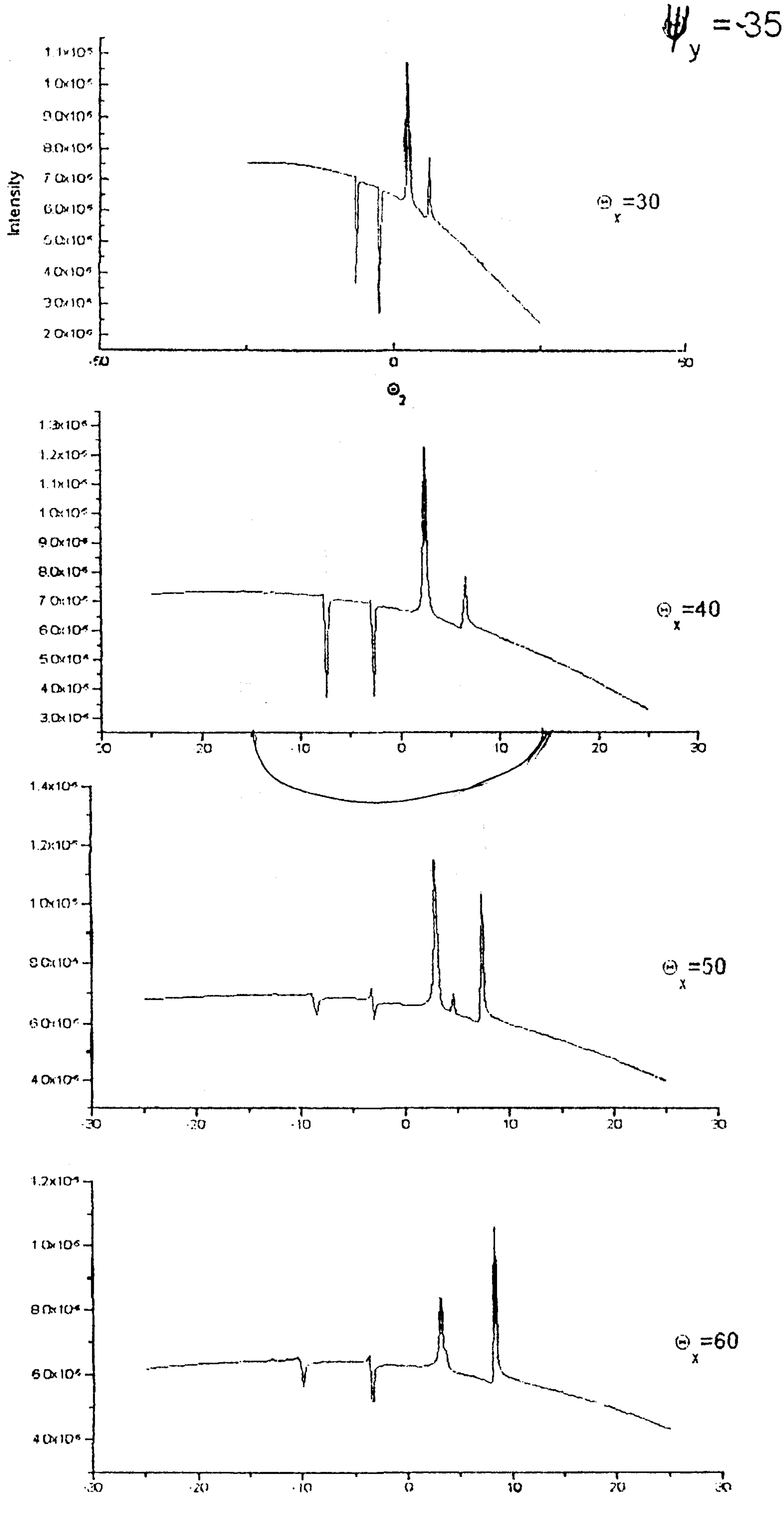

Qy = -75
-30
-20
-10
0
10
20
30


-30
-20
-10
0
10
20
30

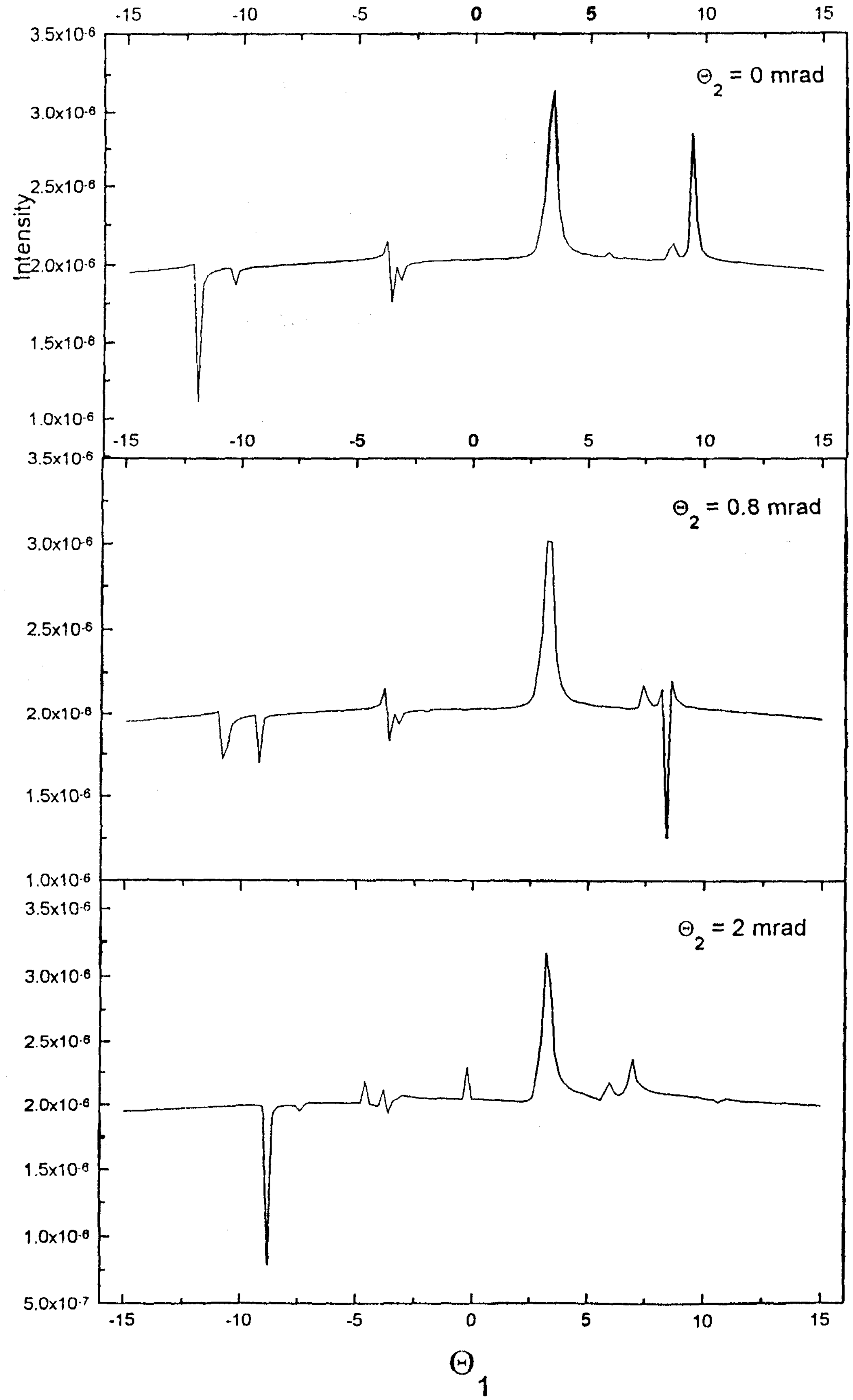
$\Phi_y=75$



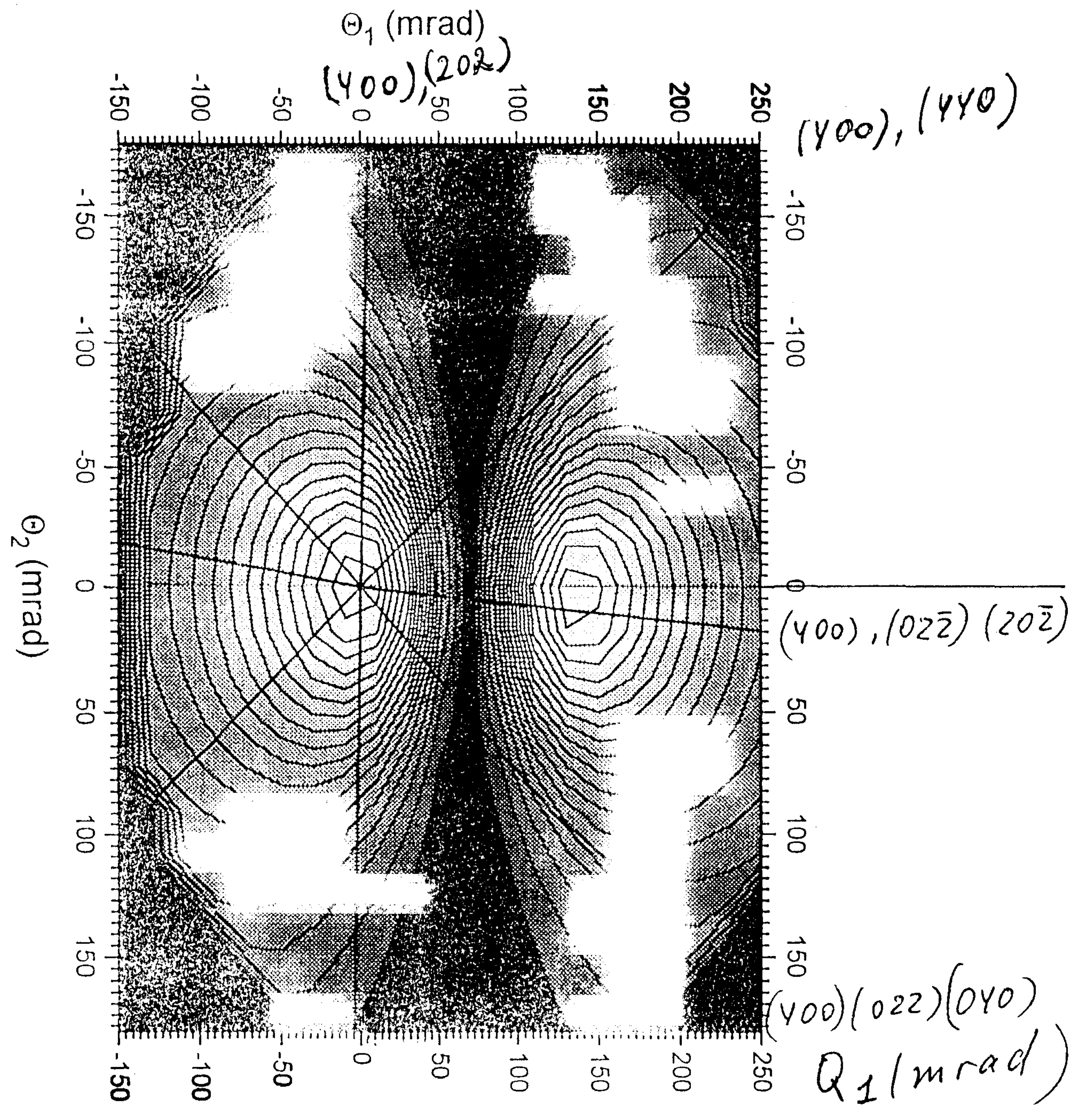

Ψy =-35
Intensity
Θx =30
Θ2
Θx =40
Θx =50
Θx =60



ψy =-35
Intensity
Θx=30
Θ2
Θx=40
Θx=50
Θx=60



Θ2 = 0 mrad
Θ2 = 0.8 mrad
Θ2 = 2 mrad
Intensity
Θ1


Θ1 (mrad)
Θ2 (mrad)
(400),(202)
(400),(440)
(400),(022̄) (202̄)
(400)(022)(040)
Q1(mrad)