THEORY OF DIELECTRIC FUNCTION AND STOPPING POWER IN INHOMOGENEOUS MANY-ELECTRON SYSTEMS

M. Kitagawa
Department of Electronics and Informaics, North Shore College
of Sony Institute, Atugi 243, Japan

We calculate the stopping power for planar channeled ions by use of the method of an inhomogeneous dielectric function. In an inverse inhomogeneous dielectric function, the density gradient effect appears. Such an effect contributes to the week dependence of the stopping power on impact parameter from the channel wall. We also derive not only the stopping power formula for the system that the electron density has an axial symmetry, but also the formula in which higher order terms of the inverse dielectric function are taken into account, using the method of decoupling.

1. Introduction

In 1964, the quantum mechanical analysis of the stopping power due to the dielectric func tion's method was performed by Lindhard and Winter under the system of free electron gas[1]. By introducing the local density approximation (LDA) to the above formula, various kind of theoretical calculations of the stopping power were performed[2]. LDA was phenomenologically proposed in order to analyze the inhomogeneous electronic systems from the statistical point of view[3]. The dielectric theory of inhomogeneous many-electron systems is a new and challenging field[4,5]. Lundqvist[6] and Lozovik et.al.[7] reviewed the framework of the general theory on dielectric function for inhomogeneous many-electron systems. The framework of the general theory of the dielectric function $\varepsilon^{6,7}$ is expressed as follows

$$\varepsilon(r_1, r_2, \omega) = \sigma(r_1 - r_2) - \int \prod (r_1, r_3, \omega) \upsilon_e(r_3 - r_2) dr_3, \tag{1}$$

where $\upsilon_e(r_3 - r_2) = e^2 / |r_3 - r_2|$ is Coulomb interaction between electrons. $\prod$ is a polarization operator and is also expressed by the following equation

$$\prod (r_1, r_3, \omega) = \frac{1}{\hbar} \sum_{j,k} p_j(r_1, r_3) p_k(r_3, r_1) \frac{\Theta(E_F - E_K) - \Theta(E_F - E_j)}{\omega + \hbar^{-1}(E_k - E_j) + i\eta} \tag{2}$$

In the above, p_j and Θ are tile j-th element of density matrix and Heaviside step function, respectively. E_F and E_j are Fermi energy and the eigen energy of the j-th electron. η denotes the positive infinitesimal.

In 1986, under the high frequency response, we derived the first order analytical formulae of dielectric and inverse dielectric functions, ε and ε^{-1}, in inhomogeneous systems of many electrons, and showed the theoretical interpretation of LDA[8]. ε and ε^{-1} in inhomogeneous system are given by

$$\begin{aligned} \varepsilon(r_1, r_2, \omega) &= \delta(r_1 - r_2) + \frac{1}{m\omega^2} div_{r_1}\left(n(r_1) grad_{r_1} \frac{e^2}{|r_1 - r_2|} \right) \\ &= \delta(r_1 - r_2)\left(1 - \frac{\omega_p^2(r_1)}{\omega^2}\right) - \frac{e^2(r_1 - r_2)}{m\omega^2 |r_1 - r_2|^3} grad_{r_1} n(r_1), \end{aligned} \tag{3}$$

$$\varepsilon^{-1}(r_1, r_2, \omega) \cong \frac{\omega^2}{\omega^2 - \omega_p^2(r_1)}\left(\delta(r_1 - r_2) - \frac{e^2}{m} \frac{1}{\omega^2 - \omega_p^2(r_2)} \frac{(r_1 - r_2)}{|r_1 - r_2|^3} grad_{r_1} n(r_1) \right). \tag{4}$$

Here, $n(r_1)$ is ale electron density, and r_1 and r_2 are position vectors of elrctron and projectile, respectively. In eqs.(3) and (4), ω_p is the plasma frequency which is defined due to the electron density n and mass m as follows

$$\omega_p^2(r_1) = \frac{4\pi e^2 n(r_1)}{m} \tag{5}$$

As a convenience, we express a complex variable ($= \omega + i\eta$) as ω in eqs.(3) and (4). Therefore, ε and ε^{-1} are complex and have real and imaginary parts.

In 1988, we also derived the general formula for stopping power for a system in which the electron distribution is possible to be treated to change one-dimensionally[9]. An arbitrary function of electron density is included in our formula. Our formula was successfully applied to analyze the stopping power at the surface[10]. In 1993, we also showed that eq.(3) is derived not only from the quantum mechanical standpoint of view, but also from the classical one[11].

In this work, using our general formula, we calculate the stopping power for planar channeled ions, and showed the week dependence of the stopping power on impact parameter from the channel wall[12], we also derive the stopping power formula for the system that the electron density has the axial symmetry, and derive the fundamental formula in which higher order contributions are taken into account by use of the method of decoupling.

2. Stopping Power for Planar Channeling

The definition of the stopping power due to electromagnetism is given by

$$S = -Z_1 e \,\mathrm{grad}_{r_1} \varnothing'(r_1, t_1)\Big|_{r_1 = r_2} \frac{V}{V} \quad (r_b = (b(Vt_1)), \tag{6}$$

where Z_1e, V and b are the charge, the velocity and the impact parameter of projectile, $\varnothing'$ is the dynamical polaization potential in which a bare Coulomb field υ_c is subtracted. $\varnothing'$ is represented as the following equation

$$\varnothing'(r_1,t_1) = \varnothing(r_1,t_1) - \upsilon_c(|r_{\perp 1} - b|, z_1 - Vt_1) \quad (\upsilon_c(r_1-r_2) = \frac{Z_1 e}{|r_1 - r_2|}). \tag{7}$$

In eq. (7), $\varnothing$ is the dynamical polarization potential which is expressed by use of ε^{-1} in r_1 -ω space as follows

$$\varnothing'(r_1,\omega) = \int dr_2 dr_3 \upsilon_c(|r_1 - r_3|)\varepsilon^{-1}(r_3, r_2, \omega)\rho^{ext}(r_2,\omega). \tag{8}$$

In the above, ρ^{ext} is the number density of projectile in r - ω space. If we take the classical straight path of projectile, ρ^{ext} becomes

$$\rho^{ext}(r_2,\omega) = \frac{1}{V}\delta(r_2 - b)\, e^{i\omega z_2/V}. \tag{9}$$

The formula for stopping power S(b) for the system, in which the electron density $n(x)$ is possible to be treated to change one-dimensionally and perpendicularly to the projection of an incident ion, is given as follows[9] (10)

$$S(b) = \frac{(Z_1 e)^2}{V^2}\left(\omega_p^2(b) \int_0^{q_{max}} \frac{dq_y}{\sqrt{q_y^2 + (\omega_p(b)/V)^2}} - \frac{1}{2}\int dx_3 \frac{\partial \omega_p^2(x_3)}{\partial x_3} \frac{b - x_3}{|b - x_3|} P \frac{1}{\omega_p^2(b) - \omega_p^2(x_3)} \right) \times$$

$$\left(\omega_p^2(b) \int_0^{q_{max}} \frac{dq_y}{\sqrt{q_y^2 + (\omega_p(b)/V)^2}} e^{-2|x_3 - b|\sqrt{q_y^2 + (\omega_p(b)/V)^2}} - \omega_p^2(x_3) \int_0^{q_{max}} \frac{dq_y}{\sqrt{q_y^2 + (\omega_p(x_3)/V)^2}} e^{-2|x_3 - b|\sqrt{q_y^2 + (\omega_p(x_3)/V)^2}} \right)$$

Here we take the propagating direction of the projectile as z-axis. b and x arc the impact parameter of the projectile from the channel wall and the position coordinate along x-axis, respectively. q and q_{max} are a wave number and a maximum one corresponding to the Fourier components over y-axis. P($\cdots$) indicates the principal value.

Fujii et.al. successfully applied our formula to analyze the position-dependent stopping power at, surfaces of $NaCl$-type crystals[10]. In planar channeling case, we use the following Moliére electron dendity

$$n(x_3)=\frac{Z_2 B}{2\sigma_c}\left(0{,}35e^{-B|x_3|}+2{,}2e^{-4B|x_3|}+2e^{-20B|x_3|}\right), \tag{11}$$

where $B=0{,}3/a_{TF}$ and $\sigma_c=1/(2LN_e)\cdot a_{TF}$, L and N_e are the Thomas-Fermi screening length, the half distance between planar plane and the density of lattice atom, respectively.

Using $S(b)$ given by eq.(10), we obtain the stopping power $S(X)$ for planar channeling given in eq.(12)

$$S(X)=\sum_{i=-n}^{i=n}\left(S(iL-X)+S(iL+X)\right). \tag{12}$$

In the above, X is the distance from the midpoint of the planar channel. We take n = 5 in performing numerical calculations.

In fig.l, for 3MeV He ions in the Au (111} channel, we show numerical results which is nomalized at the midpoint of the planar channel (X = 0). S,S_B and Sr denote the results for $S = S_B + S_N$, S_B only and Robinson's phenomenological analysis, respectively. We use S_R which is given as follows

$$S_R(X)=S_0+S_1\sigma(X), \qquad \sigma(X)=\frac{d}{dX}\sqrt{\frac{2}{V_p'(0)}\left(V_p(X)-V_p(0)\right)}, \tag{13}$$

where $V_p(X)$ denotes the planar continuum potential derived from Moliére model[13].

Deviations between S and S_B appear over the whole range of the planar channel, which mean that contributions of S_N to S are effective in our case. At the well defined channeling region, agreements between ours and Robinson's one are good. In such a region, S_N gives the week dependence of the stopping power on impact. parameter, which is caused by contributions due to distant collisions. Deviations appear as the projectile approches to the channel wall. If we take into account of the random stopping power, it is rather reasonable that the rapid increase appears in S than S_R at the neighborhood of the channel wall.

3. Stolming Power for Axial Case

In this chapter, we derive the formula pf stopping power for axial case in which the electron density of the many-electron system has the axial symmetry. In this case, taking into account that the functioh of electron density becomes $n(r_\perp)$, we can obtaih the following stopping power S(b) from eqs.(4) and (6)~(9)

$$S(b)=\frac{i(Z_1e)^2}{2\pi^2V^2}\int_{-\infty}^{\infty}d\omega\,\omega\int dq_\perp\frac{1}{q_\perp^2+(\omega/V)^2}\int dr_\perp e^{-iq_\perp(b-r_\perp)}\left(\varepsilon(r_\perp,b,\frac{\omega}{V},\omega)-\delta(r_\perp-b)\right)_{complex} \tag{14}$$

In the allove, b is the impact parameter from the string of lattice atoms. Performing tile following integration in eq. (14)

$$\int dq_{\perp} \frac{1}{q_{\perp}^2 + (\omega / V)^2} e^{-iq_{\perp}(b-r_{\perp})} = 2\pi \int_0^{q_{\perp\max}} dq_{\perp} \frac{q_{\perp}}{q_{\perp}^2 + (\omega / V)^2} J_0(q_{\perp}|b - r_{\perp}|), \tag{15}$$

we have

$$S(b) = \frac{i(Z_1 e)^2}{\pi V^2} \int_{-\infty}^{\infty} d\omega\, \omega \int_0^{q_{\perp\max}} dq_{\perp} \frac{q_{\perp}}{q_{\perp}^2 + (\omega / V)^2} \int dr_{\perp} J_0(q_{\perp}|b - r_{\perp}|) \left(\varepsilon(r_{\perp}, b, \frac{\omega}{V}, \omega) - \delta(r_{\perp} - b) \right) \tag{16}$$

where J_0 is the Bessel function of the zeroth order, and $q_{\perp\max}$ corresponds to the maximum momentum transfer in two dimentional $q_{\perp}$ space perpendicular to the direction of z-component.

From eq.(4) and the gradient in cylindrical coordinate ($r_{\perp}, \theta, z$)

$$grad\, n = \frac{\partial n}{\partial r_{\perp}} e_{r_{\perp}} + \frac{1}{r_{\perp}} \frac{\partial n}{\partial \theta} e_{\theta} + \frac{\partial n}{\partial z} e_z, \tag{17}$$

and taking into account of the axial symmetry of electron density n, we have the following result after performing Foulier transformation aver z-axis

$$\left(\varepsilon(r_{\perp}, b, \frac{\omega}{V}, \omega) - \delta(r_{\perp} - b) \right)_{complex} = \left(\frac{\omega_p^2(r_{\perp})}{\omega^2 - \omega_p^2(r_{\perp})} \right)_{complex} \delta(r_{\perp} - b) -$$

$$- \frac{e^2}{m} \left(\frac{\omega^2}{\omega^2 - \omega_p^2(r_{\perp})} \frac{1}{\omega^2 - \omega_p^2(b)} \right)_{complex} \frac{2(b\cos\theta - r_{\perp})}{|b - r_{\perp}|} \frac{|\omega|}{V} K_1\left(|b - r_{\perp}| \frac{|\omega|}{V} \right) \frac{\partial n(r_{\perp})}{\partial r_{\perp}} \tag{18}$$

where K_1 is the modified Bessel function of the second kind. In eq. (17), $e_{r_{\perp}}$, . . . are unit vectors of cylindrical coordinate.

In eq.(18), imaginary parts of $(\cdots\cdots)_{\text{complex}}$ are expressed as follows

$$\Im\left(\frac{\omega_p^2(b)}{\omega^2 - \omega_p^2(b)} \right)_{complex} = -\frac{\pi}{2} \omega_p(b) \left(\delta(\omega - \omega_p(b)) - \delta(\omega + \omega_p(b)) \right). \tag{19}$$

$$\Im\left(\frac{\omega^2}{\omega^2 - \omega_p^2(r_{\perp})} \frac{1}{\omega^2 - \omega_p^2(b)} \right)_{complex} = -\frac{\pi}{2} P\left(\frac{1}{\omega_p^2(r_{\perp}) - \omega_p^2(b)} \right)$$
$$\times \left(\omega_p(r_{\perp}) \left(\delta(\omega - \omega_p(r_{\perp})) - \delta(\omega + \omega_p(r_{\perp})) \right) - \omega_p(b) \left(\delta(\omega - \omega_p(b)) - \delta(\omega + \omega_p(b)) \right) \right) \tag{20}$$

Substituting eqs. (18)~(20) to eq. (16), we can obtain S(b). Taking into account that S(b) is dependent only on b, we express the stopping power as S. Therefore, we have

$$S = S_B + S_N \tag{21}$$

$$S_B = \frac{(Z_1 e)^2}{V^2} \omega_p^2(b) \log \frac{\sqrt{q_{\perp\max}^2 + (\omega_p(b) / V)^2}}{\omega_p(b) / V}, \tag{22}$$

$$S_N = -\frac{(Z_1 e)^2}{2\pi V^2}\int dr_\perp \frac{(b\cos\theta - r_\perp)}{|b-r_\perp|}\frac{\partial\omega_p^2(r_\perp)}{\partial r_\perp} P\left(\frac{1}{\omega_p^2(r_\perp)-\omega_p^2(b)}\right)$$

$$\times\left(\frac{\omega_p^3(r_\perp)}{V} F\left(|b-r_\perp|, \left(\frac{\omega_p(r_\perp)}{V}\right)^2, q_{\perp\max}\right) K_1\left(|b-r_\perp|\frac{\omega_p(r_\perp)}{V}\right)\right. \quad \mathbf{(23)}$$

$$\left. -\frac{\omega_p^3(b)}{V} F\left(|b-r_\perp|, \left(\frac{\omega_p(b)}{V}\right)^2, q_{\perp\max}\right) K_1\left(|b-r_\perp|\frac{\omega_p(b)}{V}\right)\right)$$

In eq. (23), F is defined by the following equation

$$F\left(|b-r_\perp|, \left(\frac{\omega_p}{V}\right)^2, q_{\perp\max}\right) = \int_0^{q_{\perp\max}} dq_\perp \frac{q_\perp}{q_\perp^2 + (\omega/V)^2} J_0(q_\perp |b-r_\perp|). \quad (24)$$

If we take as $q_{\perp\max}^2 + (\omega_p(b)/V)^2 = (2mV/\hbar)^2$ in eq. (22), we have

$$S_B = \frac{(Z_1 e)^2}{V^2}\omega_p^2(b)\log\frac{2mV^2}{\hbar\omega_p(b)}, \quad (25)$$

which is corresponding to the usual Bethe formula. Also if we take as $q_{\perp\max} \to \infty$ in eq. (24), we obtain

$$F\left(|b-r_\perp|, \left(\frac{\omega_p}{V}\right)^2, q_{\perp\max} = \infty\right) = K_0\left(|b-r_\perp|\frac{|\omega|}{V}\right). \quad \mathbf{(26)}$$

Then, eq.(23) reduces to the following equation

$$S_N = -\frac{(Z_1 e)^2}{2\pi V^2}\int dr_\perp \frac{(b\cos\theta - r_\perp)}{|b-r_\perp|}\frac{\partial\omega_\rho^2(r_\perp)}{\partial r_\perp} P\left(\frac{1}{\omega_\rho^2(r_\perp)-\omega_\rho^2(b)}\right)$$

$$\times\left(\frac{\omega_\rho^3(r_\perp)}{V} K_0\left(|b-r_\perp|\frac{\omega_\rho(r_\perp)}{V}\right) K_1\left(|b-r_\perp|\frac{\omega_\rho(r_\perp)}{V}\right) - \right)$$

$$-\frac{\omega_\rho^3(b)}{V} K_0\left(|b-r_\perp|\frac{\omega_\rho^3(b)}{V}\right) K_1\left(|b-r_\perp|\frac{\omega_\rho(b)}{V}\right) \quad (27)$$

In case of the high and nonrelativistic velocity region, we can approximate

$$K_0\left(|b-r_\perp|\frac{\omega_\rho(u)}{V}\right) \cong \log\frac{2V}{|b-r_\perp|\omega_\rho(u)} - \gamma = \log\frac{2V}{\chi|b-r_\perp|\omega_\rho(u)}, \quad (28)$$

$$K_1\left(|b-r_\perp|\frac{\omega_\rho(b)}{V}\right) \cong \frac{V}{|b-r_\perp|\omega_\rho(u)}, \quad (29)$$

where $\gamma = 0.5772$ is Euler constant, and $\chi = e^\gamma = 1.7812$. Therefore, we obtain the formula which corresponds to such a velocity region under the boundary condition that $\omega_\rho^2(\infty) = 0$

$$S_A = \frac{(Z_1 e)^2}{V^2}\left(\omega_p^2(b)\left(\log \frac{\chi^{\hbar\omega_p}(b)}{2V} + \xi \right) + \frac{1}{2}\left(\int_0^b dr_\perp - \int_b^\infty dr_\perp \right) \frac{\partial \omega_p^2(r_\perp)}{\partial r_\perp} \log\left(1 - \left(\frac{r_\perp}{b} \right)^2 \right) \right), \quad (30)$$

where $\xi = \pi^2 / 12 - 1/2 = 0,3225$

In fig.2 and fig.3, we show numerical results of the stopping power for 10 and 50MeV H ions as a function of b from the Au ⟨100⟩ string. Under the reasonable condition, S_A is more convenient than S_N because numerical computing times are shorter in S_A than in S_N. We confirm that the simplified formula S_A approaches to S_N as the incident energy of H becomes higher. In comparison between S_B and S_N, S_N has the weeker dependence on b than S_B. Such a dependence is the same effect with the one-dimensional case which is coming from distant collisions, as is already mentioned in chapter 2. It is an important point that as distances from the axial string become larger than a few $\mathring{A}$, S_N becomes more dominant, process than S_B. Recently, Fujii *et.al*, performed experiments to analyze string effects of the position-dependent stopping power[14]. In order to analyze such effects, it should be necessary to use the formula which is derived in this chapter.

4. Iteration of Higher Order Terms

Under the high freuancy approximation, Π given in eq.(2) is expanded by $1/\omega$ as follows

$$\Pi(r_1, r_3, \omega) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(\hbar\omega)^{n+1}} T_n(r_1, r_3) \quad (\omega \leftarrow \omega + i\eta), \quad (31)$$

$$T_n(r_1, r_3) = \sum_{j,k} (E_k - E_j)^n \rho_j(r_1, r_3)\rho_k(r_3, r_1)\left(\Theta(E_F - E_k) - \Theta(E_F - E_j)\right) \quad (32)$$

If thire is no total current in the system of many electrons, odd power parts of $1/\omega$ disappear. In such a case, we can take into account of even power parts only in Π. Then we have

$$\Pi(r_1, r_3, \omega) = \sum_{j,k} (E_k - E_j)\rho_j(r_1, r_3)\rho_k(r_3, r_1)\left(\Theta(E_F - E_k) - \Theta(E_F - E_j)\right)$$

$$\times \left(\sum_{n=1}^{\infty} \frac{(E_k - E_j)^{2(n-1)}}{(\hbar\omega)^{2n}} \right) \quad (33)$$

In eq. (33), performing that the term $(E_k - E_j)^2$ is replaced by the mean value $\langle (E_k - E_j)^2 \rangle$, we obtain

$$\sum_{n=1}^{\infty} \frac{\langle (E_k - E_j)^2 \rangle^{(n-1)}}{(\hbar\omega)^{2n}} = \frac{1}{\hbar^2\omega^2} \sum_{n=0}^{\infty} t^n = \frac{1}{\hbar^2\omega^2} \frac{1}{1-t} = \frac{1}{\hbar^2(\omega^2 - \omega_I^2)}, \quad (34)$$

where $\omega_I^2 = \langle (E_k - E_j)^2 \rangle / \hbar^2$, $t = \omega_I^2 / \omega^2$. Such a kind replacement corresponds to the decoupling usually used in Green's function. Therefore, we have Π

$$\Pi(r_1, r_3, \omega) = \frac{1}{\hbar^2(\omega^2 - \omega_I^2)} \sum_{j,k} (E_k - E_j)\rho_j(r_1, r_3)\rho_k(r_3, r_1)\left(\Theta(E_F - E_k) - \Theta(E_F - Ej)\right). \quad (35)$$

By use of the expression of eq.(35), we can use the same sum-rule by which we derive the basic equation of ε shown in eq.(3)[8,9]. Then, we have the following equations of ε and ε^{-1} in which higher order terms are taken into account through ω_I

$$\varepsilon(r_1,r_2,\omega)=\delta(r_1-r_2)+\frac{1}{m(\omega^2-\omega_l^2)}div_{r_1}\left(n(r_1)grad_{r_1}\frac{e^2}{|r_1-r_2|}\right)$$

$$=\delta(r_1-r_2)\left(1-\frac{\omega_p^2(r_1)}{\omega^2-\omega_l^2}\right)-\frac{e^2}{m}\frac{1}{(\omega^2-\omega_l^2)}\frac{(r_1-r_2)}{|r_1-r_2|^3}grad_{r_1}n(r_1), \tag{36}$$

$$\varepsilon^{-1}(r_1,r_2\omega)\simeq\frac{\omega^2-\omega_l^2}{\omega^2-\left(\omega_p^2(r_1)+\omega_l^2\right)}\left(\delta(r_1-r_2)-\frac{e^2}{m}\frac{1}{\omega^2-\left(\omega_p^2(r_2)+\omega_l^2\right)}\frac{(r_1-r_2)}{|r_1-r_2|^3}grad_{r_1}n(r_1)\right)$$

(37)

As is wellknown, elementaly excitation energies are expressed through poles about ω in ε^{-1}. Therefore, by taking into account of the iteration of higher order terms, excitation energies are given by $\hbar\sqrt{\omega_p^2(r_{1,2})+\omega_l^2}$ in eq.(37) although they are given by $\hbar\omega_p(r_1)$ in eq.(4). Effects of bound states of electrons are included in ω_l. In order to show the usefullness of eq.(37), we try to calculate $\hbar^2\omega_l^2$ for the case of the free electron gas system. In the free electron gas system, the mean value $\hbar^2\omega_l^2$ is defined and derived as follows

$$\hbar^2\omega_l^2=\frac{3}{4\pi q_F^3}\int_{q_j\le q_F}dq_j(E_k-E_j)^2$$
$$=\frac{3}{4\pi q_F^3}\frac{\hbar^4}{4m^2}\int_{q_j\le q_F}dq_j(k^4+4q_jk^3\cos\theta+4q_j^2k^2\cos^2\theta)$$
$$=\frac{\hbar^4}{m^2}\left(\frac{k^4}{4}+\beta_c^2k^2\right)\quad\left(\beta_c^2=\frac{q_F^2}{5}\right), \tag{38}$$

where $\hbar\beta_c/m$ is the phase velocity of plasmon based on the classical model (as is usually known[15], in case of quantum mechanics, we have $\beta_q^2=3q_F^3/5$. In eq.(38), the terms proportional to k^4 and k^2 correspond to the single electron excitation and the dispersion relation of plasmon, respectively. Therefore, in case of the free electron gas, we can obtain, from eqs.(37) and (38), the exactly same energy expression[15] of elementary excitation I_c.

$$I_e^2=\hbar^2\omega_p^2+\frac{\hbar^4\beta_c^2k^2}{m^2}+\frac{\hbar^4k^4}{4m^2}. \tag{39}$$

Then, from the mathematical standpoint of view, we show the usefullness of the decoupling given in eq.(34), and also confirm that through ω_l defined here, we can take into account of binding effects of electrons in inhomogeneous many-electron systems.

5. Concluding Remarks

In this work, using basic equations of ε and ε^{-1} given by eqs.(3) and (4)[8,9], we derive stopping power formula for planar channeling as one-dimensional case, which was successfully used for the case of surface[10]. Numerical calculations of the stopping power for planar channeling were performed for the case of 3McV He ions into the Au {111} channel. Comparisons between theoretical and experimental results were also performed, and we obtained qualitative agreements between them. The week dependence of the stopping power on impact parameter was typically caused by distant collisions. Such kind effects are not taken into account in the uasual Bethe-type theory.

In two-dimensional case, the stopping power formula in inhomogeneous system was constructed under the condition of the cylidrical symmetry of electron density. We performed numerical calculations for cases of 10 and 50MeV H ions in the Au, ⟨100⟩ string. As is same with one-dimensional case, we also cinfirm important contributions of distant collisions. Recently, Fujii et.al. performed experimrnts to analyze string effects of the position-dependent stopping power[14]. In order to analyze such effects, it should be necessary to use the formula discussed here.

Using tile decoupling method in Green's function, we took into account of the iteration of higher order contributions, and derived general formulae of the dielectric function and the inverse dielectric function. Applying such formulae to the case of the free electron gas, we obtained the exactly same dispersion relation of the free electron gas, in which terms of phase velocity of plasmon and single electron excitations appear. From the mathematical standpoint of view. we show the usefullness of the decoupling, and also confirm that through ω_I, we can generally take into account of binding effects of electrons in inhomogeneous many-electron systems.

REFERENCES

1. J. Lindhard and A. Winter, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. 34 (1964) no.4.
2. J.F. Ziegler, J.P. Biersack and U. Littmark, The Stopping and Range of Ions in Solids (Pergamon Press, New York, 1985).
3. J. Lindhard and M. Scharff, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. 27 (1953) no.15.
4. L. Hedin and S. Lundqvist, Solid State Phys. 23 (1969) 2.
5. A.L. Fetter and J.D. Walecka, Quantum Theory of Many-Particle Systems (MeGraw-Hill, New York, 1971).
6. S. Lundqvist and N.H. March, Theory of the Inhomogeneous Electron Gas (Plenum, New York, 1983).
7. Yu. E. Lozovik and A.V. Klyuchnik, The Dielectric Function of Condensed System, ed. L.V. Keldysh, D.A. Kitchnits and A.A. Maradudin (North-Holland, New York, 1989).
8. M. Kitagawa, Nucl. Instr. and Meth. BI3 (1980) 133.
9. M. Kitagawa, Nucl. Instr. and Meth. B33 (1988) 409.
10. Y. Fujii, K. Narumi, K. Kishine, K. Kimura and M. Mannami, Nucl. Instr. and Meth. B67 (1992) 82.
11. H. Nitta, S. Shindo and M. Kitagawa, Surf. Sci. 286 346 (1993).
12. M.T. Robinson, Phys. Rev. 179 (1969) 327.
13. D.V. Morgan, Channeling (John Wiley & Sons, New York, 1973).
14. Y. Fujii, Study on Ion Beam Interactions with Crystal Surfaces (Ph. D. Kyoto Universuty, 1993).
15. D. Pines, Elementary Excitations in Solids (W.A. Benjamin, New York, 1964).

Figure Captions

Fig.l Numerical calculations of the stopping power for planar channeling as a function of X for 3MeV He ions in the Au {111} channel.

Fig.2 Numerical calculations of the stopping power as a function of b for 10MeV H ions from the Au ⟨100⟩ string.

Fig.3 Numerical calculations of the stopping power as a function of b for 50 MeV H ions from the Au ⟨100⟩ string.

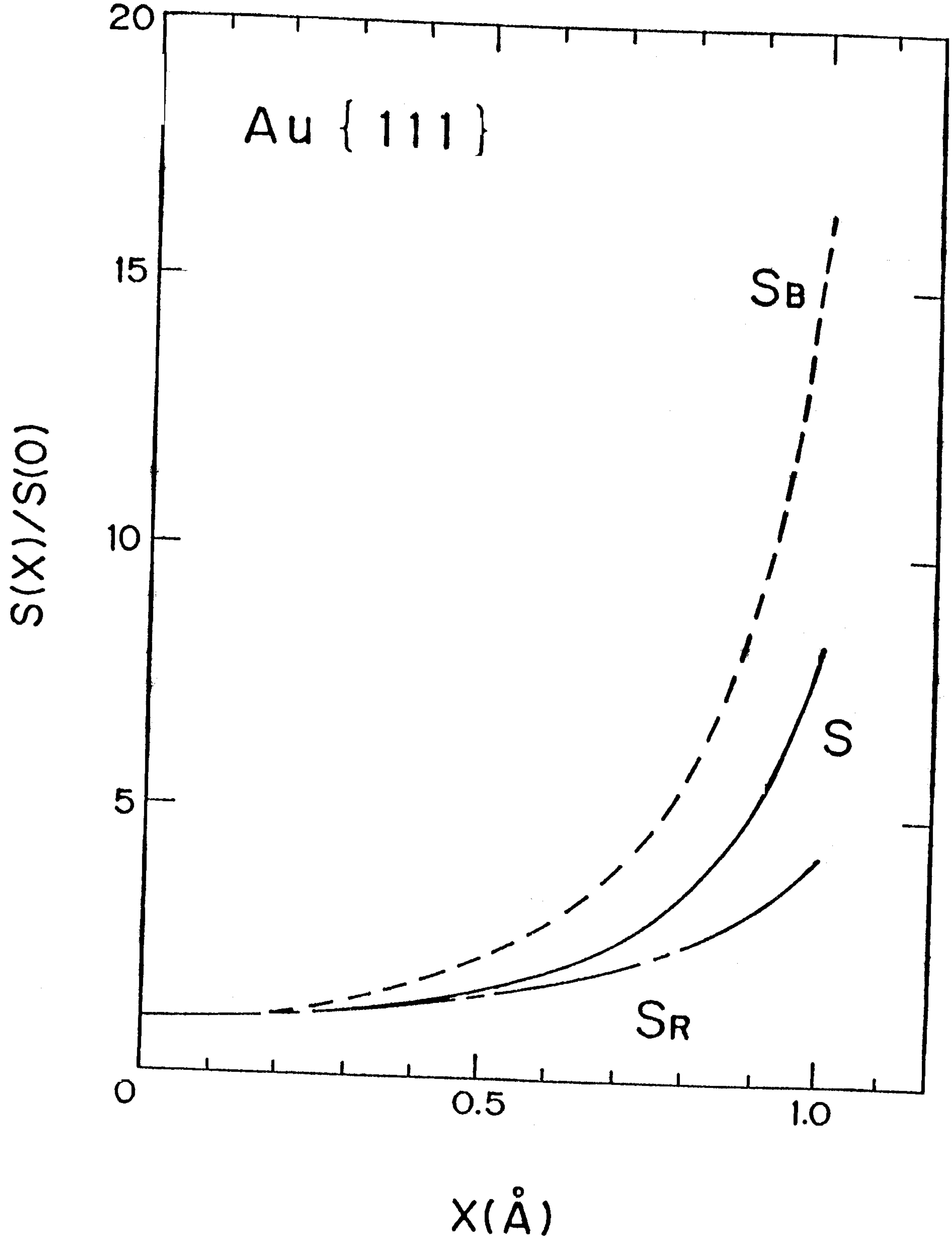

fig.1

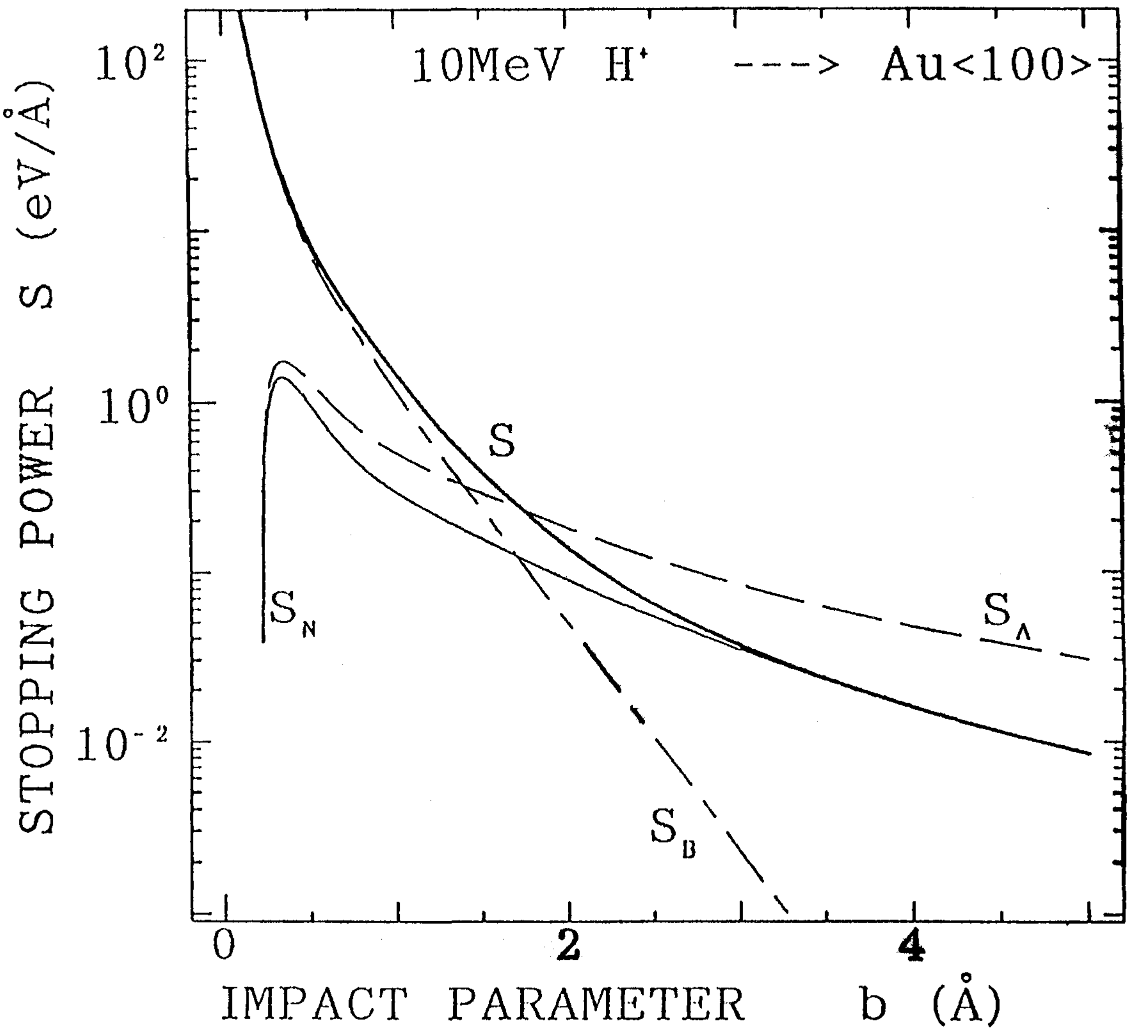
10MeV H+ ---> Au<100>
STOPPING POWER S (eV/Å)
IMPACT PARAMETER b (Å)
S
S_N
S_A
S_B
10^2
10^0
10^-2
0
2
4

fig.2

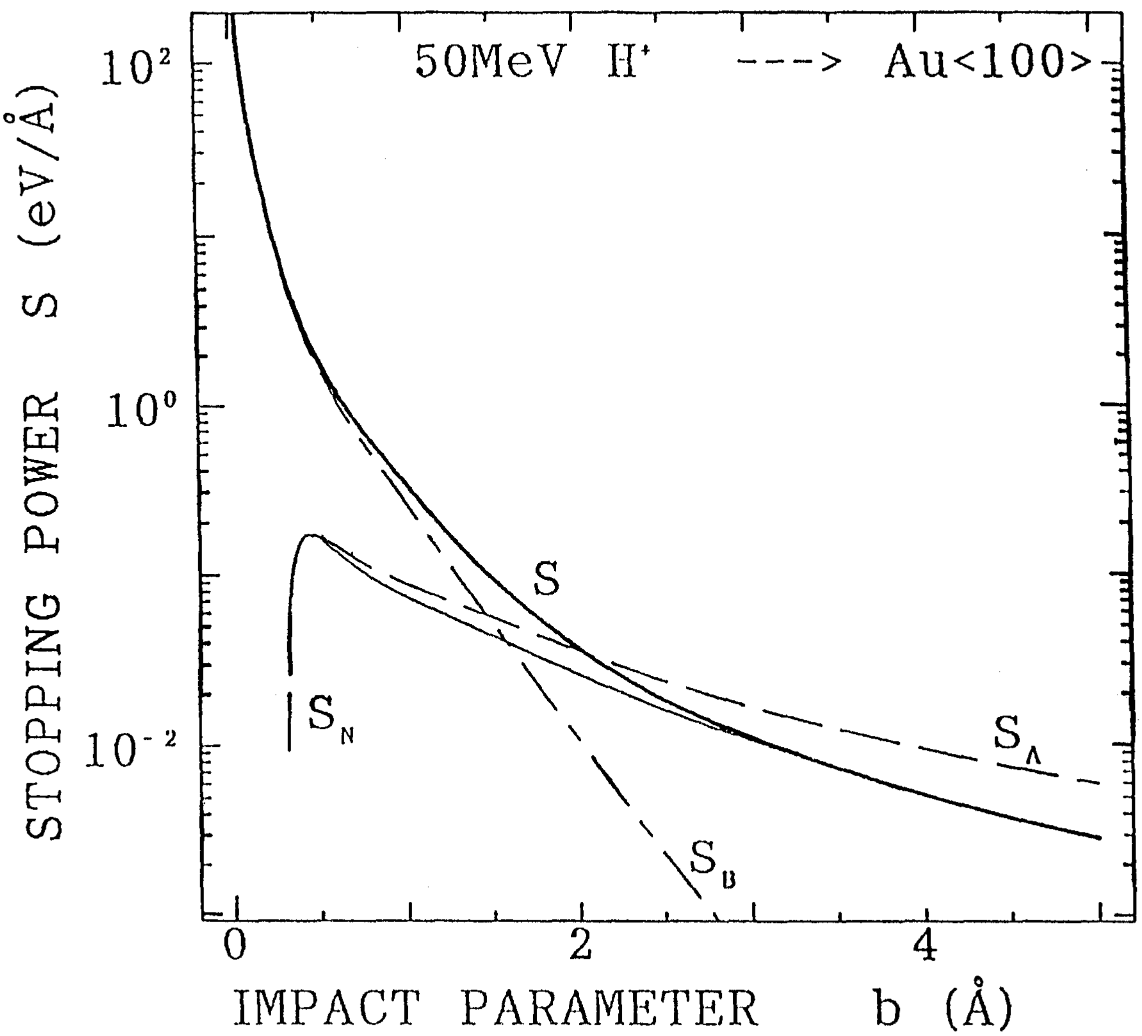
50MeV H+ ---> Au<100>
STOPPING POWER S (eV/Å)
IMPACT PARAMETER b (Å)
S
S_N
S_A
S_B
10²
10⁰
10⁻²
0
2
4

fig.3