Deformation of relativistic electron radiation spectra under conditions of multiple production of photons

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Abstract

Spectra of the relativistic electron radiation measured by detector of total energy absorption under conditions of multiple photon generation are considered. The calculations of spectra deformation under simultaneous registration of several photons by the single detector are carried out. The separation way of the contribution of double and triple photon coincidences is pointed up.

1. Introduction

Experimental research of radiation of the charged relativistic particles in the dense medium requires special approach because of the multiple photon production by the same particle in the target. A probability of simultaneous registration of two and more photons as one photon with a total energy by one detector takes place then.

For the first time the multiple photon production under the channeling was observed in the "Kristall" collaboration [1–3]. The well-known "Belkasem pik" in radiation of electrons of the 100 GeV energy [4] is the example of spectrum deformation caused by such an overlapping of events.

The effect of multiple photon production was observed also for electrons with the energy of about 1 GeV [5–7].

In the case of high incident beam flux density on a target, the effect of the overlapping of photon registration events can be observed also under absence of multiple productions of photons by a separate particle. One of the characteristic features of multiple photon production effect is impossibility to get rid of the overlapping by means of decreasing of flux intensity of particles emitting photons.

The probability of the production of several photons by one particle depends on target thickness. This effect can be insignificant in a thin target. However the investigations of gamma radiation production in optimal conditions must be carried out with the use of a target of about one radiation length thickness. In these conditions the measurement of spectral distribution of radiation is related either with necessity of the overlapping suppression or with its correct account.

The suppression of influence of the multiple photon production on the process of radiation spectral distribution measurement represents a task of the experiment. In particular, in [6] Compton scattering of photons on the target of a light material was used for this goal. Compton scattering allows decreasing the probability coincidence in the process of scattered photon registering at a small solid angle for a few orders. The unique dependence of the incident photon energy on energy of the photon Compton scattered at a given angle was allowed to recover the true spectrum of investigating primary radiation by measured spectrum of scattered photons.

Though, the influence suppression of the multiple photon production does not means absence of the overlapping in registration of radiation generated by different particles of the incident beam. These two cases should be considered independently.

In this work the results of investigation the task of recovering of radiation spectra deformed by the overlapping during the photon registration processes are presented.

2. Account of overlapping under registration of photons emitted in a target by different particles

Let $I(\omega)$ be the radiation spectrum generated by particles in the target and $I_{ms}(\omega, \tau)$ be result of its measurement in the overlapping conditions, τ is registration time. If it is considered that in the process of statistical data setting the one photon per unit time emitting probability is a constant value for any photon energy, then probability of n photons emitted in a time τ will obey a Poisson distribution: $P_n(\tau) = \frac{i}{n!} \cdot N(\tau)^n \cdot e^{-N(t)}$, where $N(\tau) = \lambda \cdot \tau$ is photons average, emitted in the time τ, a constant λ is equal to average number of photons emitted per unit time.

As a preliminary let us assume that effect of multiple photon production by one electron in the target is absent and take into account the overlapping of photons emitted by different incident particles only. Probabilities of these overlapping are defined by the value of the incident particle flux on the target and by the spatial registration angle.

The definition of the $N(\tau)$ value is a separate problem. Let us assume that this value is defined. With the use of the multiple overlapping probability distribution $P_n(\tau)$ the spectrum measured can be represented by the following expression:

$$I_{ms}(\omega, \tau) := P_1(\tau) \cdot I(\omega) + \sum_{n=2}^{\infty} P_n(\tau)$$
$$\cdot \int_0^{\omega} \int_0^{\omega - x_1} \cdots \int_0^{\omega - x_1 - \cdots x_{n-1}} I1(x_n) \, dx_n \cdots dx_2 \, dx_1,$$

(1)

where $I1(x_n) = I(x_1) \cdot I(\omega - x_1) \cdot (I(\omega - x_1 - x_2) \cdots I(\omega - x_1 - \cdots x_{n-1}))$.

As is seen from Eq. (2), the account of the overlapping of high multiplicity for the recovering spectrum $I(\omega)$ from the measured spectrum $I_{ms}(\omega, \tau)$ presents a difficult problem. However, the principal contribution into deformed spectrum in the sum (1) import the terms correspond to events with a multiplicity close to average number the photon emitted in the active time of the detector $N = N(\tau)$.

Let us consider the case of low radiation intensity when only the overlapping with multiplicity less than three will be essential. The expression (1) will thus be given by

$$I_{ms}(\omega, \tau) := P_1(\tau) \cdot I(\omega)$$
$$+ P_2(\tau) \cdot \int_0^{\omega} I(x) \cdot I(\omega - x) \, dx$$
$$+ P_3(\tau) \cdot \int_0^{\omega} I(x) \cdot \int_0^{\omega - x} I(y) \cdot I(\omega - x - y) \, dy \, dx.$$

(2)

The quantity $I(\omega)$ in (2) can be obtained by the numerical method. In such a manner the problem of recovering of the true radiation spectrum can be solved.

The measured spectrum obtained from Eq. (2) can be represented by the n combined equation for a series of discrete values of ω with a small remainder $\Delta\omega$. The system equations can be written in the form

$$I_{ms_i} := P(1,N) \cdot I_i + P(2,N) \cdot \Delta\omega \cdot \sum_{k=0}^{i-1} I_k \cdot I_{i-k}$$

$$+ P(3,N) \cdot \Delta\omega^2 \cdot \sum_{k=0}^{i-1} I_k \cdot \sum_{m=0}^{i-k-1} I_m \cdot I_{i-k-m},$$

$$(2a)$$

where $P(n,N)$ is the probability of emitting of n photons in the registration time (detector active time), N is average number of photons radiated in the same time, $\omega_i = i \cdot \Delta\omega$.

Righthand integrals of equation are presented as the sum appropriated to values of the photons energy ω, which define also the number of equations in the system.

The expression for the recovered spectrum obtained from (2a) have such a form:

$$I_i := a \cdot \left(I_{ms_i} - P(2,N) \cdot \Delta\omega \cdot \sum_{k=1}^{i-1} I_k \cdot I_{i-k} \right.$$

$$\left. - P(3,N) \cdot \Delta\omega^2 \cdot \sum_{k=0}^{i-1} I_k \cdot \sum_{m=1}^{i-k-1} I_m \cdot I_{i-k-m} \right),$$

$$(3)$$

where

$$a := \frac{1}{\left[P(1,N) + P(2,N) \cdot \Delta\omega \cdot I_0 + P(3,N) \cdot \Delta\omega^2 \cdot (I_0)^2 \right]}.$$

As an example of the account of deformation of the measuring spectrum by the coincidences caused by the intensity of radiating particles the calculation of the spectrum deformation by binary and triple overlapping of a step form spectrum (calculated by Eq. (2a)) and the recovered spectrum (calculated by Eq. (3)) is presented in Fig. 1(a). The same is presented in Fig. 1(b) for initial spectrum in form of normal distribution.

It is necessary to note that the overlapping caused by value of flux intensity of the emitting particle really can be taken into account with using a Poisson distribution and the Poisson distribution parameter $N(\tau)$ can be defined by the detector load ratio. It will be shown in the next section.

3. Account of multiplicity of photon radiation emitted by one projectile particle at a target under registration of photons by full absorption spectrometer

The overlapping related with multiple photon production can be calculated only in analyzing the specific process of one particle radiation on the target.

The probability distribution of multiple photon production may be found from experimental data by collating of this measurement results and measurement results of spectrum radiation, which scattered by the Compton effect at angle of significantly greater than the characteristic angle of the relativistic particle radiation $1/\gamma$. In this case the values of probability $P(n,N)$ of multiple production of photons of the multiplicity $n = 1$, $n = 2$ and $n = 3$ by the one projectile particle can be obtained as parameters, best appropriated to expression (2a) [6].

One can expect that this probability in amorphous solid will be met Poisson distribution too. This proposition holds because the probability of photon radiated by the particle is not change with the target depth. It means that the variation of the value of probabilities as independent parameters is not necessary i.e. it is enough to choose the parameter of Poisson distribution only.

At the case when charged particles product the photon emission in the oriented crystal one can expect an aberration in the overlapping probability distribution from Poisson law. It is related by the change of probability of photon emitting by the particle at the difference depths of the target defined by the changing of movement dynamics of particles in the crystal due to incoherent scattering. In this case it is necessary to vary all probability values of radiation multiplicity or find the appropriate form for the probability distribution in every concrete case.

In particular at the case of radiation of relativistic electron in the thin oriented crystal it is possible to hold that the probability of photon radiation along particle track change weakly. In this case the power-series expansion of the corresponding probability dependence by longitudinal coordinate of particle in the crystal, one can cast
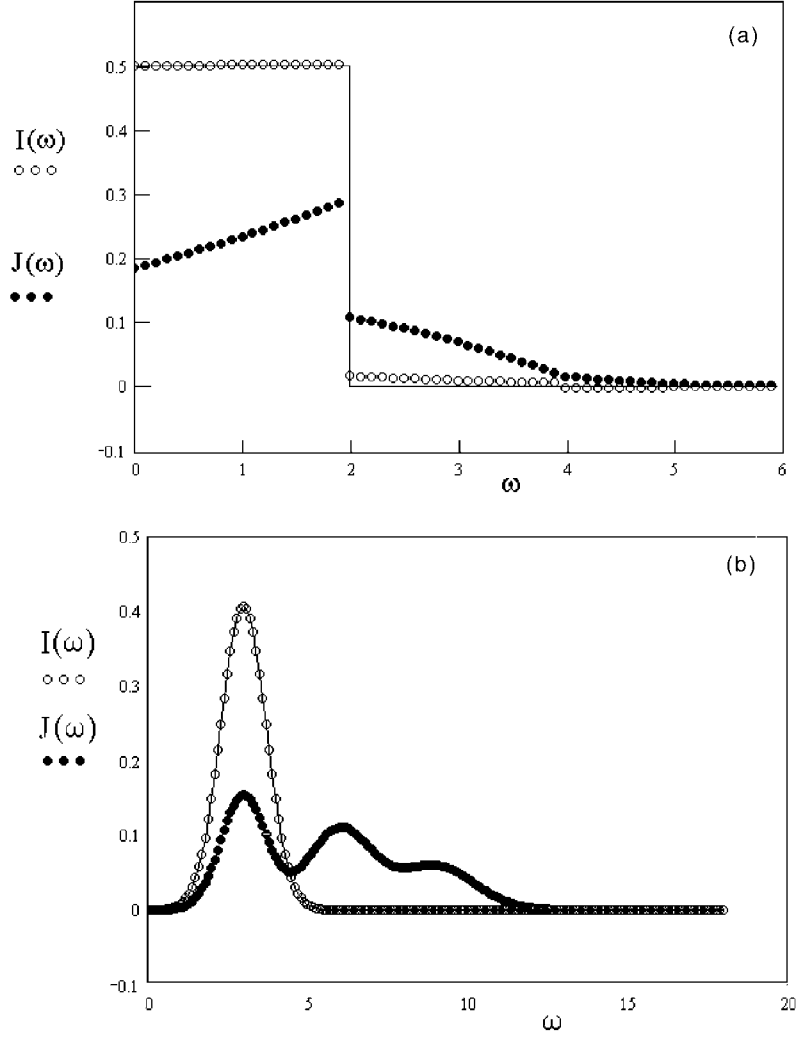
Fig. 1. Examples of recovering of radiation spectrum for initial spectrum of the step form (a) and initial spectrum of normal distribution form (b). Firm line is initial spectrum; ● ● ● – the spectrum deformed by the binary and triple overlapping $J(\omega)$ (calculated by Eq. (2a) under $N = 1$); ○○○ – recovered spectrum $I(\omega)$ (calculated by Eq. (3)).

out of series term, confining linear terms only. Probability of emitting of n photons by the one electron at this crystal can be represented by the following expression:

$$P(n, \lambda, t, \kappa)$$

$$:= \exp(-\lambda \cdot t) \cdot \frac{(\lambda \cdot t)^n}{n!} \cdot \exp\left(\frac{\kappa \cdot \lambda}{2} \cdot t^2\right)$$

$$\cdot \left[1 + \sum_{i=1}^{n} (-1)^i \cdot (\kappa \cdot t)^i \cdot \prod_{j=1}^{i} \frac{n-i+j}{2 \cdot j}\right], \quad (4)$$

where $\lambda \cdot (1 - \kappa \cdot t)$ is a probability of photon emitting on the unit thickness of the target, which differ from Poisson parameter on factor $(1 - \kappa \cdot t)$, κ – coefficient of proportionality, t – crystal thickness.

The coherent effects of relativistic charged particle radiation in the crystal usually are exponentially faded out from the particle trace due to multiple scattering of the particle. This situation can be written by exponential dependence of

probability of radiation over the unit length of the path $\lambda 1 = \lambda \cdot (1 + k \cdot e^{-\alpha \cdot t})$ i.e. $dP = \lambda \cdot (1 + k \cdot e^{-\alpha \cdot t}) \cdot dt$. In this case the following form can represent a probability of radiation of n photons on the crystal thickness t:

$$P(n, k, \alpha, \lambda, t) := \lambda^n \cdot \exp\left[-\lambda \cdot \left[t + \frac{k}{\alpha} \cdot (1 - e^{-\alpha \cdot t}) \right] \right]$$
$$\cdot \left[\left(\frac{t^n}{n!} \right) + S(n, k, \alpha, t) \right], \qquad (5)$$

where

$$S(n, k, \alpha, t) := \left[\sum_{i=0}^{n-1} \left[\left(\frac{k}{\alpha} \right)^{n-i} \cdot \frac{t^i}{i! \cdot (n-i)!} \right. \right.$$
$$+ (-1)^{i+1} \cdot \frac{e^{-\alpha \cdot t \cdot (i+1)}}{(i+1)!} \cdot \sum_{j=1}^{n-i} \left(\frac{k}{\alpha} \right)^{n-j+1}$$
$$\left. \left. \cdot \frac{t^{j-1}}{(n-i-j)! \cdot (j-1)!} \right] \right],$$

$$P(0, k, \alpha, \lambda, t) = \exp\left[-\lambda \cdot \left[t + \frac{k}{\alpha} \cdot (1 - e^{-\alpha \cdot t}) \right] \right].$$

This expression describes change of probability of relativistic particle radiation on crystal thickness (on unit length) from a maximum $\lambda \cdot k$ on the oriented crystal input to value that corresponds to the radiation in the disoriented crystal λ. The coefficient k is ratio of radiation intensities in thin oriented crystal and disoriented one, α is characteristic length of coherent effect attenuation in particle radiation.

4. Measurement of radiation spectra in experiments carried out on the impulse accelerator in conditions of event overlapping

Experimental research of radiation processes of relativistic charged particles on impulse accelerators is difficult because it is related with constraints in velocity of statistical data capture. These constraints are defined by low up-time ratio of accelerator work. Pulsing width of an accelerator practically consists 1–10 μs. Gamma-ray and X-ray detectors needs the registration time also about 10 μs for one event to get high resolution by energies of registered photons. Therefore during the one impulse of the accelerator a detector can register no more than one photon.

The frequency of pulse-repetition rate of accelerator does not exceed some hundred hertz and usually have the value about 50 Hz. To register event of one photon emitting with the high probability, the average frequency of events registered by detector must be less than frequency of accelerator impulse on more than an order. Naturally, in such experiments the capture of the set of statistical data will last many hours or days.

The problem of efficiency upgrading of impulse accelerator using in experiments on nuclear physics is so important that in order to solve the problem special large additional systems at the output of accelerator are built. These are storage rings, which transform the pulsed beam of accelerated particles in the quasi-continuous beam.

One of the ways of efficiency upgrading of impulse accelerator in the physical experiment is the carrying out of experiment under conditions of increased intensity of the flux of quantum registered with the subsequent correct recovering of spectra distorted by the overlapping.

In each concrete case it is necessary to determine correctly the distribution of probability of multiple overlapping of events of photon registration during one accelerator impulse to recover correctly the photon spectrum. Then further recovering of a spectrum will reduce to a simple calculating task.

The distribution of probability of the overlapping obeys Poisson distribution if the spectrum deformation because of the multiple photon production by one electron is absent in experiment. For this case the distribution parameter N (the time average number of quanta registered in one accelerator impulse) can be expressed by the quantity of the event registration frequency f (number of events registered by detector per a unit time).

Let us note that the registration frequency f is an easily measured quantity. On the other hand, the quantity f can be obtained out of probability to not radiate a photon in the one accelerator impulse:

$$f := F \cdot (1 - P(0, N)), \qquad (6)$$

where the F is the frequency of accelerator impulses. From the expression of the Poisson distribution at $n = 0$ one can write

$$N := \ln \left(\frac{F}{F - f} \right). \tag{7}$$

By inserting this quantity in the Poisson distribution we can write the expression of distribution probability of emitting n photons at the one event registration in the form of a function of the accelerator impulse frequency F and the frequency of events registered by detector f.

$$P(n, f, F) := \frac{1}{n!} \cdot \left(\frac{F - f}{F} \right) \cdot \left(\ln \left(\frac{F}{F - f} \right) \right)^{n}. \tag{8}$$

The obtained expression can be used for recovering of a spectrum deformed by the overlapping, as it was showed in the previous section.

If the measured spectrum contains deformations related to effect of multiple photon production, these distortions will stay also in a spectrum, recovered on an above technique. To obtain a true spectrum of photons, it is necessary in addition to apply procedure of transformation of a spectrum with use of probability distribution data of the multiple photon production. The data of distribution of probability can or be obtained in the special experiment similar mentioned above, or are calculated theoretically for a concrete process of interaction relativistic particle with a crystal target.

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