A Film Model of Sound Propagation in Gas–Liquid Foams: 2. The Sound Absorption

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Abstract—The concepts of the sound energy dissipation in gas–liquid foams are described within the framework of the film model of sound propagation. The high absorption of sound in foams is explained by hydrodynamic losses in foam films. The experimental dependence of the absorption coefficient on the foam expansion is explained. The calculated expansion corresponding to the absorption maximum is close to the experimental value.

In the first part of this work [1], we proposed a new (film) mechanism of sound propagation in gas–liquid foams. This model qualitatively explains the principal experimental facts concerning the sound velocity in foams. The second most important characteristic of this process is the attenuation of sound. In this respect, a foam differs fundamentally from its constituting components: the gas and the liquid. Measurements show [2, 3] that the absorption of sound in foams is 7–10 orders of magnitude (!) greater than its absorption in water or in the air. An approximate list of the possible mechanisms of the sound energy dissipation in gas–liquid media is given in [4]; it includes (i) heat exchange between the liquid and the gas in bubbles due to the phase difference between the sound pressures in these media; (ii) the scattering of sound at the surfaces of gas bubbles; (iii) viscous friction in the liquid during volume oscillations of the gas bubbles; and (iv) molecular energy exchange, where the kinetic energy of molecules is transformed into the potential energy (vibrational or dissociation energy) or used for a structural rearrangement of the medium. It was also noted that the attenuation of the sound signal at low expansions depends on the gas content in the foam (its expansion) and on the viscosity of the liquid component.

As was established in [5], the foam films thicken when a foam is exposed to sound. This filling of the foam films with a liquid proceeded at a constant expansion of the foam and was manifested both by the scattering of a laser beam on the foam and visually, by interference effects. That is, the action of sound on the foam causes a redistribution of the liquid in the foam, namely, a transport of the liquid from nodes and channels into films. This process, referred to as acoustic pumping, was attributed to the surface flows in films because of the Marangoni effect.

In [6], we proposed our approximation of the experimental results of [2, 5] on the basis of a film model of sound propagation in the gas–liquid foams. This work provides a development of these concepts and, to some degree, their mathematical substantiation.

A dependence of the absorption coefficient β on the foam expansion in the $K = 80$–1700 range was obtained in [2]. The $\beta(K)$ curve monotonically decreases with expansion. A monotonic decrease of the β value at $K \geq 300$ was also obtained by the authors of [7]. Since absorption in the pure liquid ($K = 1$) is very low, one should expect a maximum on this curve at expansions $K < 80$. Indeed, we revealed this maximum [3] near $K \approx 60$ in experiments with foams that had varying structural parameters. We also noticed a distinct correlation between the sound absorption in a foam and the thickness of interbubble films: the absorption maximum was observed at a certain (critical) film thickness.

According to the film model [1], the sound energy in medium- and high-expansion foams is transferred from one film to another via gas bubbles. In this process, the foam films vibrate with the frequency of the sound wave. Liquid films have a negligible shear elasticity. So what is the reason for the high absorption of the sound energy in foams? In our opinion, the high absorption is due to hydrodynamic losses during the motions of the liquid in the films. Let us try to substantiate this statement on the basis of the film mechanism of sound propagation in foams.

Figure 1a schematically represents a plane-parallel foam film of diameter d and thickness δ. Under the influence of a harmonically changing pressure $p = p_m \cos \omega t$, the film vibrates with the velocity $v = v_m \sin \omega t$ and amplitude $A = v_m/\omega = p_m/(\omega \rho c)$, where $v_m = p_m/\rho c$, ρ is the density of the gaseous phase, ω is the circular frequency of the sound vibrations, and c is the sound velocity in the gas. For example, under the experimental conditions of [5], the amplitude of the film vibrations was 50–100 μm, that is, one or two orders of magnitude greater than the film thicknesses (1–6 μm, see [1]).

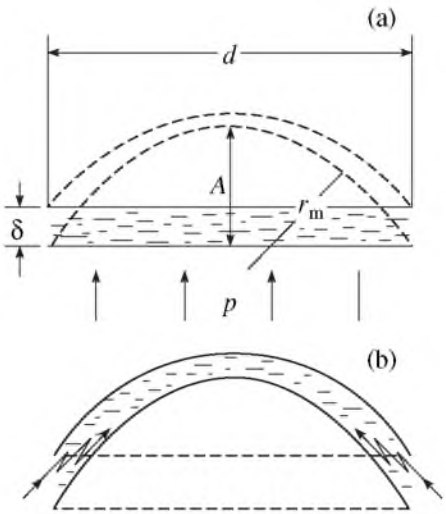

Fig. 1. On the calculation of sound losses in a foam: (a) film deformation because of sound pressure and (b) liquid motion in films.

As the film is deformed during the vibrations, its area increases, but the volume remains virtually constant because of the liquid inertia. As a result, the thickness of the film decreases in its middle, and the elasticity of the surface layers (which is due to the Marangoni effect) creates a certain excess rarefaction in the film in comparison with that in adjacent channels. This rarefaction drives the liquid from the periphery to the film. According to the data of [5], the thickness of the films significantly increases after several seconds of their exposure to sound. During this time, the films perform many thousands of oscillations.

The maximum rarefaction p_{m1} emerging in the film increases with increasing amplitude of its vibrations, which, in turn, is proportional to the sound pressure p_m. In the first approximation, this relationship may be presented in the form

$$p_{m1} = bp_m, \tag{1}$$

where b = const is an empirical coefficient whose value is determined from an experiment.

Thus, as sound propagates in the foam, an additional rarefaction $p_1 = p_{m1}\cos\omega_1 t$ emerges in the films. It may be regarded as a source of a harmonic pressure perturbation in the film-confined liquid with a doubled frequency ($\omega_1 = 2\omega$). Driven by this perturbation, the liquid makes reciprocating movements as schematically shown in Fig. 1b, creating a pulsing liquid flux from the periphery to the central part of the film. In our opinion, this is the essence of the acoustic pumping mechanism in foam films. Hydraulic losses during these movements are just the reason for the high absorption of the sound energy in films.

Let us estimate these losses using a simplified model that represents a foam as consisting of cubic gas bubbles (Fig. 2). A plane sound wave with the initial energy

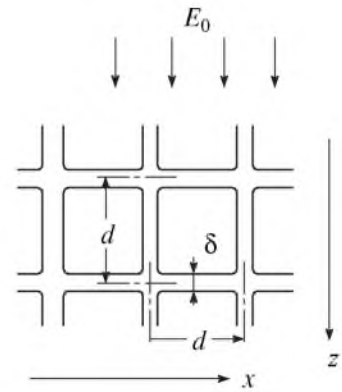

Fig. 2. The model of a cubic foam.

E_0 moves along the z axis against the resistance of a "stack" of liquid films having a linear size d and thickness δ each. The distance between the films is also equal to d.

As the sound propagates in the foam (in the direction of the z axis), the pressure amplitude and the energy of the sound wave exponentially decrease:

$$p_m(z) = p_0 e^{-\beta z}, \quad E(z) = E_0 e^{-2\beta z}, \tag{2}$$

where β is the sound absorption coefficient, which may be represented as the sum

$$\beta = \beta_g + \beta_l + \beta_f. \tag{3}$$

Here, β_g and β_l are the sound absorption coefficients in the gas bubbles and in the film liquid, respectively; they are negligible compared to the absorption coefficient β_f, which is due to hydraulic losses in the films. Therefore, hereafter we consider $\beta \approx \beta_f$.

The wave energy may be expressed via the energy density u in the sound wave:

$$E = uV = ucSt, \tag{4}$$

where $V = cSt$ is a certain cylindrical volume with the cross-sectional area S, which the wave passes at the velocity c during the time t. Using Eqs. (4) and (2), let us express the absorption coefficient via the volume energy density:

$$\beta = -\frac{1}{2uc}\frac{du}{dt}. \tag{5}$$

The energy density in a sound wave is determined (for example, see [8]) by the relationship

$$u = p_m^2/p_a, \tag{6}$$

where p_a is the atmospheric pressure.

A solution to the problem of a liquid motion between two parallel planes is known (e.g., see [9]).

The frictional force acting on a unit surface area during such a motion is equal to

$$f = -\frac{\delta}{2}\frac{dp_1}{dx}, \tag{7}$$

and the absolute value of the velocity averaged over the cross section is

$$v = -\frac{\delta^2}{12\eta}\frac{dp_1}{dx} = \frac{v_{m1}}{\sqrt{2}}, \tag{8}$$

where v_{m1} is the amplitude velocity value and η is the dynamic viscosity of the liquid.

Since the excess rarefaction created in the film assumes the highest value p_{m1} at its center but is nullified at its periphery, let us assume that the average pressure gradient along the film is equal to $dp_1/dx \approx -p_{m1}/(d/2)$ in the first approximation. Then,

$$v = \frac{\delta^2 p_{m1}}{6\eta d}$$

or, considering Eq. (1),

$$v = \frac{\delta^2 b p_m}{6\eta d}, \tag{9}$$

that is, the average velocity in such a flow increases in proportion with δ^2. On the other hand, the amplitude of the liquid velocity v_{m1} cannot exceed the velocity of the sound motion of particles in an infinite liquid—the acoustic velocity

$$v_{ac} = \frac{p_{m1}}{\rho_1 c_1}, \tag{10}$$

where ρ_1 is the liquid density and c_1 is the sound velocity in the liquid. Comparing Eqs. (8), (9), and (10), one can estimate the smallest film thickness δ_m up from which the average velocity of the liquid flow no longer depends on the thickness and becomes equal to the acoustic velocity

$$\delta_m = \sqrt{\frac{6\eta d}{\sqrt{2}\rho_1 c_1}} \approx 2\sqrt{\frac{\eta d}{\rho_1 c_1}}. \tag{11}$$

For aqueous foams with the bubble dimensions d = 0.1–2.0 mm, Eq. (11) yields $\delta_m \approx$ 0.5–2.3 μm. In our experiments [3], the critical film thickness was $\delta_m \approx$ 0.77 μm (for comparison, the thickness of equilibrium free films was 0.02–0.5 μm [10]).

Thus, foams may be said to form three categories with respect to sound absorption. (I) Spherical and low-expansion foams, for which $\delta \gg \delta_m$. As was noted in the first part of [1], the sound velocity in such foams is satisfactorily described by the relationships of the homogeneous model. The sound absorption in them is also mainly due to energy dissipation at the molecular level during the oscillations of the gas bubble volumes. These losses (let us call them acoustic) are relatively low. (II) Medium-expansion foams with the film thicknesses $\delta > \delta_m$ form the acousto-hydrodynamic category. In this intermediate region between low-expansion and "dry" foams, the films are already distinct enough but still have a significant thickness. (III) High-expansion ("dry") foams with the film thickness $\delta \ll \delta_m$. The average velocity of the liquid flow in such films is determined by Eq. (9). The sound energy losses in such foams will be referred to as hydrodynamic.

Acoustic losses in liquids with bubbles (gas–liquid emulsions) are well studied (e.g., see [11]). Sound absorption in spherical and low-expansion foams has a molecular nature and does not qualitatively differ from the sound absorption in gas–liquid emulsions. The processes of interest for us are the sound absorption in the (III) hydrodynamic and (II) acousto-hydrodynamic regions.

Excluding the dp_1/dx derivative from Eqs. (7) and (8), we find $f = 6\eta v/\delta$. Multiplying this value by the film surface area $2d^2$, we obtain an expression for the hydraulic frictional force within one cell: $F = 12\eta d^2 v/\delta$. Since F and v change according to the harmonic law, the average work of the frictional force per unit time (the loss power) equals $W = Fv/2 = 6\eta d^2 v^2/\delta$. Dividing this expression by the volume d^3 of a single cell, we find the rate of the sound energy dissipation per unit volume of the foam:

$$\frac{W}{d^3} = \frac{6\eta v^2}{d\delta} = -\frac{du}{dt}. \tag{12}$$

Substituting Eq. (12) into Eq. (5) and considering Eq. (6), we obtain

$$\beta = \frac{3p_a\eta v^2}{p_m^2 c d\delta}. \tag{13}$$

For high-expansion foams (in the hydrodynamic region), the average velocity of the liquid flow is determined by formula (9). Substituting Eq. (9) into Eq. (13), we obtain

$$\beta = \frac{p_a b^2 \delta^3}{12 c\eta d^3}. \tag{14}$$

Let us use the relationship proposed in [1]:

$$K = \frac{ad}{\delta}, \tag{15}$$

where a is a dimensionless empirical coefficient.[1] Substituting this expression into Eq. (14), we obtain

$$\beta = \frac{a^3 b^2 p_a}{12 c\eta K^3} \sim \frac{1}{K^3}. \tag{16}$$

[1] Estimation according to the results of [2] yields $a = 0.23 \pm 0.02$ (see [1]). In our experiments [3], this parameter was equal to 0.22.

This is just the dependence representing the right (descending) branch of the $\beta(K)$ curve at expansions $K > 80$, obtained in [2].

In the acousto-hydrodynamic region, the average velocity of the liquid flow is equal to $v \approx v_{ac}$ and determined by dependence (10). Substituting Eq. (10) into Eq. (13) and considering Eq. (1), we obtain

$$\beta = \frac{3b^2\eta p_a}{cd\delta\rho_1^2 c_1^2}$$

and, with allowance for Eq. (15),

$$\beta = \frac{3b^2\eta p_a K}{acd^2\rho_1^2 c_1^2} \sim K. \quad (17)$$

That is, starting from gas–liquid emulsions, the sound absorption increases with expansion. Dependence (17) represents the left (ascending) branch of the $\beta(K)$ curve obtained in our work [3]. At the boundary between regions II and III, this dependence has a maximum corresponding to the critical film thickness (11). The expansion value at this extremum can be estimated by equating dependences (16) and (17) and obtaining the relationship

$$K_m \approx a\sqrt{\frac{\rho_1 c_1 d}{6\eta}}. \quad (18)$$

Under the conditions of our experiment [3], we have $K_m \approx 55$, a value close enough to the experimental value $K_m \approx 60$.

Thus, the proposed film model makes it possible to qualitatively explain the main specific features of sound propagation in the gas–liquid foams. It provides an expression for the sound velocity in a foam that is consistent with the experiment, explains the absence of a dispersion at low sound frequencies and the presence of a maximum at the graphical dependence of the attenuation factor on the foam expansion, gives a correct value for the position of this maximum, explains the phenomenon of acoustic pumping in a foam, etc.

The model also enables us to make some predictions that can be experimentally verified. For example, the authors of [5] determined the sound velocity in a foam by the propagation of the first sound pulse. However, acoustic pumping leads to a thickening of the films after several seconds of exposure to sound. According to [1], the sound velocity must accordingly decrease. It is not difficult to verify this conclusion in an experiment, and its confirmation would be a serious argument in favor of the proposed model (unfortunately, the experimental setup used for these measurements does not exist any longer).

Some of the above conclusions do not fully agree with experimental facts. For example, the authors of [2] established that the sound absorption in the $K = 80$–1700 range only slightly depends on the expansion (as $K^{-0.5}$), whereas Eq. (16) gives a stronger dependence: $\beta \sim K^{-3}$. This fact may presumably be explained by two reasons: (i) the β_f coefficient in the high-expansion region becomes so small with increasing expansion that the contributions of the β_g and β_l coefficients to sound absorption become significant [see Eq. (3)], and these two coefficients are virtually independent of the expansion; (ii) during acoustic pumping, the liquid flows into the films but the expansion remains constant. The thickened films correspond to lower equilibrium expansions. With allowance for this fact, the $\beta(K)$ dependence would decrease more steeply.

Presently it is difficult to provide more precise (quantitative) estimates, because the geometry of real foams (in particular, the shapes and sizes of foam films) is complicated and insufficiently studied. Nevertheless, the film model may serve as the basis for further theoretical and experimental studies of the propagation of weak (sound) perturbations in the gas–liquid foams.

Note that foams are presumably the only substances with a powerful hydrodynamic mechanism of sound energy dissipation. This fact makes gas–liquid foams a unique and promising object for their practical use as media with controllable (regulated) acoustic properties.

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