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Fuzzy formalization of the multi-stage processes temporal description

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Abstract. The problem of constructing the periodization of socio-economic and organizational-technological processes, which makes it possible to divide them into stages for a more detailed study, is considered. The methodological basis of the study is the conceptual apparatus of L. Zadeh's theory of the linguistic variable in combination with expert assessment technologies. According to the specifics of the description of non-stationary processes, a linguistic variable has been developed, the values of which (temporal terms) correspond to the stages of the process. A new type of temporal terms that are semantically described by bitrapezoidal membership functions is proposed, and an expert procedure for constructing these terms is indicated. A system of algebraic operations on temporal terms is proposed and justified (by statements on properties), which allows formulating syntactic and semantic rules for the transformation of terms. The proposed formal linguistic description of the stages allows the analysis of socio-economic and organizational-technological processes using the procedure of fuzzy inference based on a fuzzy production rule about the subject area, which can help increase the scientific validity of management decisions.

1. Introduction

One of the general methodological approaches in the analysis of complex non-stationary processes of various nature is to divide the time interval of the review (process lifetime) into sub-intervals (stages, periods), each of which is determined by its own set of indicators, its limitations and its own evaluation criteria [1]. At the same time, an adequate description of the process requires using different mathematical (simulation) models, that is significantly more complicated when taking into account the interaction of several processes. This circumstance makes the problem of periodization very actual. Character for considered time (temporal) subinterval conditions of processes' flowing in various moments can be differently inherent to it, application of the formal apparatus of fuzzy logic [3] to the temporal description of processes.

The purpose of this work is to develop an algebraic formalization of the fuzzy linguistic temporal description of multi-stage non-stationary processes' stages.

2. Materials and methods

The methodological basis of the research is the conceptual apparatus of the L. Zadeh's linguistic variable theory [4] in combination with the methodology of expert evaluation [5,6]. When describing the system of operations on terms of the proposed linguistic variable constructed in work, the apparatus of the theory of context-free grammars are used [7].

The formal descriptions of multi-stage non-stationary processes developed in this paper represent the development of tools for decision support in conditions of uncertainty (for example, [8,9]) when planning (managing) processes.



3. Fuzzy linguistic description of the stage

The lifetime of the process (as an assembly of stages and relations between them) can be formalized by a linguistic variable

$$\langle Stages, T, TT_{Base} G, M \rangle,$$

where *Stages* is the name of the linguistic variable «stages of life»; *T* is the time continuum; *TT_{Base}* is the base vector of temporal terms; *G* is a syntactic rule that allows generating names of temporal terms from the names of *TT_{Base}* elements; *M* is a semantic rule that establishes a correspondence between temporal terms and fuzzy subsets of *T*.

The semantics of the temporal term *tt*, which corresponds to the stage of the process, is defined by a fuzzy interval having a bitrapezoidal membership function (Figure 1). In comparison with terms whose semantics is determined by the usual trapezoidal membership function the proposed type of semantics of temporal terms allows describing the stages of the process more accurately because of taking into account transient processes between the stages.

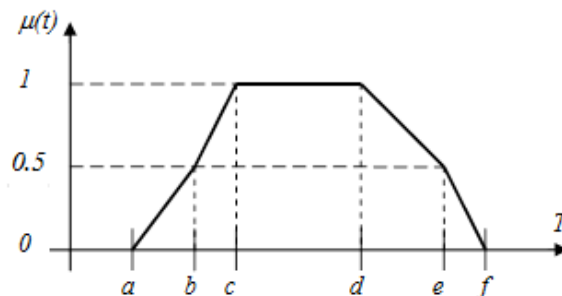


Figure1. Bitrapezoidal membership function of the temporal term *tt*.

Formally, the bitrapezoidal membership function is a 6-parameter piecewise linear function $\mu_{a,b,c,d,e,f} : T \rightarrow [0, 1]$, where *T* - the time continuum; the parameters ($a < b < c < d < e < f$) are defined by the following formula:

$$\mu_{a,b,c,d,e,f}(t) = \begin{cases} 0, & \text{if } t < a \\ \frac{0.5}{b-a} \cdot t + \frac{-0.5 \cdot a}{b-a}, & \text{if } a \leq t < b \\ \frac{0.5}{c-b} \cdot t + \frac{0.5 \cdot c - b}{c-b}, & \text{if } b \leq t < c \\ 1, & \text{if } c \leq t < d \\ \frac{-0.5}{e-d} \cdot t + \frac{e - 0.5 \cdot d}{e-d}, & \text{if } d \leq t < e \\ \frac{-0.5}{f-e} \cdot t + \frac{0.5 \cdot f}{f-e}, & \text{if } e \leq t < f \\ 0, & \text{if } f \leq t \end{cases}$$

It is important to note that it is the natural interpretation for parameters $a, b, c, d, e < f \in T$ and a survey of experts can obtain them:

- parameters a, f correspond to moments of time – the boundaries of the temporal region, which in no way can be attributed to the considered stage of the process (the segment $[a, f]$ is the carrier of a fuzzy interval);
- parameters c, d correspond to moments of time – the boundaries of the temporal region, which can be fully attributed to the consideration stage of the process (the segment $[c, d]$ is the core of the fuzzy interval);
- parameters b and e correspond to moments of time – the boundary between the consideration and neighbouring (previous and subsequent) stages (transition points of the membership function).

At the same time, the issues of organizing team examinations and analysis to assess the stability of results to possible changes in expert judgments (for example, [10]) require additional research.

For further formalization, it is convenient to use the characteristics of the duration of the temporal term tt (process stage):

- optimistic duration: $\delta_{opt} = d - c$;
- standard duration: $\delta_{norm} = e - b$;
- pessimistic duration: $\delta_{pes} = f - a$.

The durations of transient processes correspond to the lengths of segments $[a,b]$, $[b,c]$, $[c,d]$ and $[d,e]$, which we denote as $\delta_{e_pre}, \delta_{i_pre}, \delta_{i_post}, \delta_{e_post}$. These values are called the durations of the external prestage, internal prestage, internal poststage, and external poststage, respectively. Figure 2 illustrates the introduced concepts.

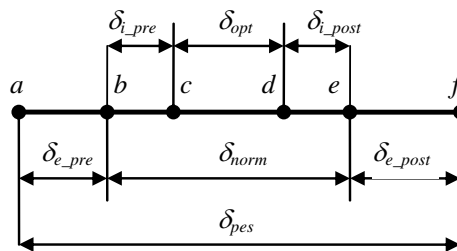


Figure 2. Interrelationship of durations and parameters of the bitrapezoidal function belonging to the temporal term tt

Thus, the membership function for the temporal term tt_N can be redefined in terms of durations as a 6-component vector

$$\mu_N = \langle \tau^{(N)}, \delta_{e_pre}^{(N)}, \delta_{i_pre}^{(N)}, \delta_{norm}^{(N)}, \delta_{i_post}^{(N)}, \delta_{e_post}^{(N)} \rangle$$

Here N is the name of the temporal term, τ is the initial moment of time that coincides with the parameter a .

The base vector of temporal terms is a finite linearly ordered set in which the initial stage, some intermediate stage, and the final stage. The membership functions of these temporal terms must meet the requirements clearly shown in Figure 3.

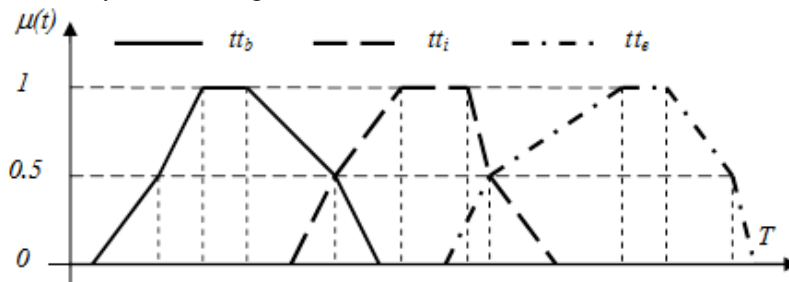


Figure 3. Membership functions of elements of temporal terms base vector

4. Description of operations on temporal terms

The syntactic rule G and semantic rule M , which allow switching from the temporal terms base vector TT_{BASE} to the set of all terms TT , built based on the entered operations on temporal terms $O1 \rightarrow O10$.

Introduced operations have the common syntax:

1. The alphabet (set of terminal meta symbols) includes the following subsets:
 - names of elements of the temporal terms base vector;
 - the set of rational numbers greater than 0 and written in the 10th number system;
 - set of integer numbers greater than 0 and written in the 10th number system;
 - $\{*, e_pre^*, i_pre^*, e_post^*, i_post^*, >>, +, /, :, .1, .2, (,)\}$ – set of special symbols.
2. The grammar of the syntactic rule, presented in the form of Backus-Naur [7]:

$$prop := n : m.1 | n : m.2, \text{ где } n, m \in \mathbb{Z}_+;$$

$$term := N, \text{ где } N \in N_{Base};$$

$$\begin{aligned} \text{term} := & (\text{term}) * \alpha \mid (\text{term})e_pre * \alpha \mid (\text{term})i_pre * \alpha \mid (\text{term})e_post * \alpha \mid (\text{term})i_post * \alpha \mid \\ & (\text{term}) \gg \alpha \mid (\text{term} + \text{term}) \mid (\text{term}) / \text{prop} \end{aligned}$$

where $\alpha \in R_+$.

The introduced operations are semantically defined through setting / changing the values of the temporal term bitrapezoidal membership function parameters:

O1. Basic temporal term initialization operation:

$$\mu_N = \langle \tau^{(N)}, \delta_{e_pre}^{(N)}, \delta_{i_pre}^{(N)}, \delta_{norm}^{(N)}, \delta_{i_post}^{(N)}, \delta_{e_post}^{(N)} \rangle,$$

where the values $\tau^{(N)}$, $\delta_{e_pre}^{(N)}$, $\delta_{i_pre}^{(N)}$, $\delta_{norm}^{(N)}$, $\delta_{i_post}^{(N)}$, $\delta_{e_post}^{(N)}$ for the basic temporal term and the name N are set in dialogue with the expert.

O2. Time shift operation on the temporal term:

$$\mu_{(N) \gg \alpha} = \langle \tau^{(N)} + \alpha, \delta_{e_pre}^{(N)}, \delta_{i_pre}^{(N)}, \delta_{norm}^{(N)}, \delta_{i_post}^{(N)}, \delta_{e_post}^{(N)} \rangle.$$

O3. Multiplication operation (increase/reduction of the regulatory duration) on the temporal term:

$$\mu_{(N) * \alpha} = \langle \tau^{(N)}, \delta_{e_pre}^{(N)}, \delta_{i_pre}^{(N)}, \delta_{norm}^{(N)} \cdot \alpha, \delta_{i_post}^{(N)}, \delta_{e_post}^{(N)} \rangle,$$

$$\text{when } \alpha \geq \frac{\delta_{i_pre}^{(N)} + \delta_{i_post}^{(N)}}{\delta_{norm}^{(N)}}.$$

O4. The operation of changing the duration of the external pre-stage of the temporal term:

$$\mu_{(N)e_pre * \alpha} = \langle \tau^{(N)} + \delta_{e_pre}^{(N)} \cdot (1 - \alpha), \delta_{e_pre}^{(N)} \cdot \alpha, \delta_{i_pre}^{(N)}, \delta_{norm}^{(N)}, \delta_{i_post}^{(N)}, \delta_{e_post}^{(N)} \rangle.$$

O5. The operation for changing the duration of the internal pre-stage of the temporal term:

$$\mu_{(N)i_pre * \alpha} = \langle \tau^{(N)}, \delta_{e_pre}^{(N)}, \delta_{i_pre}^{(N)} \cdot \alpha, \delta_{norm}^{(N)}, \delta_{i_post}^{(N)}, \delta_{e_post}^{(N)} \rangle,$$

$$\text{when } \alpha \leq \frac{\delta_{norm}^{(N)} - \delta_{i_post}^{(N)}}{\delta_{i_pre}^{(N)}}.$$

O6. The operation for changing the duration of the internal post-stage of the temporal term:

$$\mu_{(N)i_post * \alpha} = \langle \tau^{(N)}, \delta_{e_pre}^{(N)}, \delta_{i_pre}^{(N)}, \delta_{norm}^{(N)}, \delta_{i_post}^{(N)} \cdot \alpha, \delta_{e_post}^{(N)} \rangle,$$

$$\text{when } \alpha \leq \frac{\delta_{norm}^{(N)} - \delta_{i_pre}^{(N)}}{\delta_{i_post}^{(N)}}.$$

O7. The operation for changing the duration of the external post-stage of the temporal term:

$$\mu_{(N)e_post * \alpha} = \langle \tau^{(N)}, \delta_{e_pre}^{(N)}, \delta_{i_pre}^{(N)}, \delta_{norm}^{(N)}, \delta_{i_post}^{(N)}, \delta_{e_post}^{(N)} \cdot \alpha \rangle.$$

O8. The operation of emphasizing the initial sub-stage of the temporal term in the proportion $n:m$:

$$\mu_{(N)/n:m.1} = \langle \tau^{(N)}, \delta_{e_pre}^{(N)}, \delta_{i_pre}^{(N)}, \delta_{norm}^{(N)} \cdot \frac{n}{n+m}, \delta_{i_post}^{(N)}, \delta_{e_post}^{(N)} \rangle,$$

$$\text{when } \frac{n}{n+m} \geq \frac{\delta_{i_pre}^{(N)} + \delta_{i_post}^{(N)}}{\delta_{norm}^{(N)}}.$$

O9. The operation of emphasizing the final sub-stage of the temporal term in the proportion $n:m$:

$$\mu_{(N)/n:m.2} = \langle \tau^{(N)} + \frac{\delta_{norm}^{(N)} \cdot n}{n+m}, \delta_{e_pre}^{(N)}, \delta_{i_pre}^{(N)}, \delta_{norm}^{(N)} \cdot \frac{m}{n+m}, \delta_{i_post}^{(N)}, \delta_{e_post}^{(N)} \rangle,$$

$$\text{when } \frac{m}{n+m} \geq \frac{\delta_{i_pre}^{(N)} + \delta_{i_post}^{(N)}}{\delta_{norm}^{(N)}}.$$

O10. The operations of temporal terms' join:

$$\mu_{(N1+N2)} = \langle \tau^{(N)}, \delta_{e_pre}^{(N1)}, \delta_{i_pre}^{(N1)}, \delta_{norm}^{(N1)} + \delta_{norm}^{(N2)}, \delta_{i_post}^{(N2)}, \delta_{e_post}^{(N2)} \rangle$$

$$\text{when } \tau^{(N1)} + \delta_{e_pre}^{(N1)} + \delta_{norm}^{(N1)} = \tau^{(N2)} + \delta_{e_pre}^{(N2)}.$$

The following statements are valid for the introduced operations:

Statement 1. Any temporal term $tt \in TT$ can be generated using unary operations $O2 \div O7$ from an arbitrary base temporal term $tt^* \in TT_{base}$ initiated by operation $O1$. The exclusion of any operation from the $O2 \div O7$ list result in the loss of this property.

Statement 2. The order of performing unary operations $O2 \div O7$ does not matter.

Statement 3. Applying the operation of join on temporal terms $O10$ to terms obtained by applying operations $O8$ (selection of the initial sub-stage) and $O9$ (selection of the final sub-stage) to the same term $tt \in TT$ with the same parameters of operations result in the same term.

The obvious statements given in this paper without proof can serve as a justification for the type of introduced operations.

5. Conclusion

The proposed formal linguistic description of the stages allows analyzing socio-economic and organizational-technological processes using the procedure of fuzzy logical inference based on a fuzzy production model of knowledge about the subject area, which can contribute to improving the scientific validity of management decisions. In particular, one of the promising applications of the developed tools is to solve problems of optimal synthesis of organizational and technological dynamic discrete systems [11].

Acknowledgments

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