On ionization energy losses of relativistic particles created in matter

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Abstract

The ionization energy loss of high-energy particles created in matter is considered. The analogy between interference phenomena in ionization energy losses after photoproduction in matter of ultrarelativistic electron–positron pairs and after decay of fast molecules in thin target is noted. It is demonstrated that after generation of ultrarelativistic electron in matter, for impact parameters making the main contribution to the ionization energy losses, the interference of the electrons own Coulomb field and the radiated electromagnetic wave is substantial. However, this interference phenomenon practically has no effect on the ionization energy losses of the particle.

1. Introduction

The ionization energy losses of fast charged particles located on small distances from each other, can substantially differ from the sum of ionization energy losses of particles separated far from each other. This fact is connected with interference of electromagnetic fields of particles on the distances making substantial contribution into ionization energy losses. Such a situation arises, for example, under high-energy electron–positron pair production in matter [1]. The characteristic angles between the components of the pair are rather small. Hence the transversal distance between particles of the pair will be small for a long time in comparison with the maximum value of impact parameter $\rho_{max} = v/\omega_p$, where v is the velocity of the particle and ω_p is the plasma frequency, giving substantial contributions to the ionization energy loss of separated particles of the pair. In this case the electromagnetic fields of the electron and positron of the produced pair will reduce each other at transversal distances from the pair of the order of v/ω_p, and, consequently, ionization energy losses of such particles will be decreased in comparison to ionization energy losses of separated particles of the pair. For example, for the photon energy $\hbar\omega \sim 100$ GeV the characteristic angles between electron and positron of a pair are $\theta_\pm \sim 4mc^2/\hbar\omega \sim 2 \times 10^{-5}$ rad and the effect of the decrease of ionization energy losses will be manifest itself on longitudinal distances from the point of pair production corresponding to $l \sim \rho_{max}/\theta_\pm \sim 0.05$ cm. This effect had been observed in cosmic rays [1–3].

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The analogous effect takes place under Coulomb explosion of fast molecules in a thin layer of substance [4]. The analogy between these processes was noted in [5].

Under production of a high energy charged particle (electron) in matter its surrounding electromagnetic field can for a long time from the moment of production differ substantially from the normal field belonging to a particle moving rectilinearly and uniformly in the same direction all over the time [6,7]. The effect is due to interference of the Coulomb field of the electron and the field of the radiated wave. The radiation of ultrarelativistic electrons takes place mainly at small angles $\theta \sim mc^2/E$ to the direction of its velocity, where E is the energy of the electron. So the electron with its Coulomb field and the radiated electromagnetic wave will be located at a small distance to each other for a long time, and, consequently, there will be substantial interference between these fields. This effect is important for radiation on the distances $l_c \approx 2E(E - \hbar\omega)/m^2c^3\omega$ along the particle momentum, where $\hbar\omega$ is the photon energy. The length l_c is called the coherence length [7,8]. Such interference effects manifest themselves in many processes connected with the radiation of relativistic electrons in matter, such as the process of coherent Bremsstrahlung in oriented crystals and Landau–Pomeranchuk effect of Bremsstrahlung suppression of ultrahigh energy electrons in amorphous medium (see, for instance, [7]). Hence the question about the possibility of manifestation of such effects in the process of ionization energy losses of high-energy electrons created in matter arises naturally. This paper is devoted to consideration of this problem. We use the simplest approach based on the classical electrodynamics.

We would like to note that the considered problem has some parallels to the gradual outset of the Fermi density effect behind the entrance surface of a solid target by the relativistic charged particles (see [9,10]). It should also be noted that for an electron at high energy, ionization plays only a minor role in the energy loss process, which is dominated by Bremsstrahlung. But, the ionization energy losses are important for formation of particle tracks in matter.

2. Electromagnetic field of high energy charged particle produced in matter

Let us consider some peculiarities of evolution of the field of high energy charged particle after its production in matter. Initially, we shall neglect dielectric properties of the medium.

Let the charged particle be created instantaneously at the time momentum $t = 0$ with the finite velocity $\boldsymbol{v}$ close to the velocity of light c. Such situation arises, for example, after scattering of an ultrarelativistic electron on an atom of a medium, or in the process of high-energy pair production in the case of large separation angles of the components of the pair. The potentials of the particle's field under $t > 0$ are determined by the equations

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\varphi(\boldsymbol{r}, t) = -4\pi e\delta(\boldsymbol{r} - \boldsymbol{v}t)\Theta(t), \tag{1}$$

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\boldsymbol{A}(\boldsymbol{r}, t) = -4\pi e\boldsymbol{v}\delta(\boldsymbol{r} - \boldsymbol{v}t)\Theta(t), \tag{2}$$

where $\Theta(t)$ is the Heaviside step function. (We use later the system of units in which the speed of light c is taken as 1.) The solutions of these equations can be written for retarded potentials in the form of Fourier integrals (see [7])

$$\varphi(\boldsymbol{r}, t) = \frac{e}{2\pi^2} Re \int \frac{d^3k}{k(k - \boldsymbol{k}\boldsymbol{v})} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\boldsymbol{r}} \times \left(\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\boldsymbol{v}t} - \mathrm{e}^{-\mathrm{i}kt}\right)\Theta(t), \tag{3}$$

$$\boldsymbol{A}(\boldsymbol{r}, t) = \frac{e}{2\pi^2} \boldsymbol{v} Re \int \frac{d^3k}{k(k - \boldsymbol{k}\boldsymbol{v})} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\boldsymbol{r}} \times \left(\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\boldsymbol{v}t} - \mathrm{e}^{-\mathrm{i}kt}\right)\Theta(t). \tag{4}$$

After integrating we obtain

$$\varphi(\boldsymbol{r}, t) = \Theta(t - r)\varphi_0(\boldsymbol{r}, t), \tag{5}$$

$$\boldsymbol{A}(\boldsymbol{r}, t) = \Theta(t - r)\boldsymbol{A}_0(\boldsymbol{r}, t), \tag{6}$$

where φ_0 and $\boldsymbol{A}_0$ determine the common Coulomb field of a charged particle moving with the velocity $\boldsymbol{v}$,

$$\varphi_0(\boldsymbol{r},t) = \frac{e}{\left[(z-vt)^2 + \boldsymbol{\rho}^2\gamma^{-2}\right]^{1/2}}, \tag{7}$$

$$\boldsymbol{A}_0(\boldsymbol{r},t) = \frac{e\boldsymbol{v}}{\left[(z-vt)^2 + \boldsymbol{\rho}^2\gamma^{-2}\right]^{1/2}}. \tag{8}$$

Here $\gamma = (1-\boldsymbol{v}^2)^{-1/2}$ is the Lorentz-factor of the particle, z-axis is parallel to $\boldsymbol{v}$, and $\boldsymbol{\rho}$ is the radius vector in the plane orthogonal to $\boldsymbol{v}$. Equipotential surfaces of the potential (5) for relativistic electron are shown in Fig. 1.

The first terms in (3) and (4) describe the potentials of the own Coulomb field of a particle moving with velocity $\boldsymbol{v}$. The second terms describe for $r \to \infty$ the field of the radiated wave [7]. In fact, the Fourier components of the vector potential

$$\boldsymbol{A}_\omega = \int \mathrm{d}t\ \boldsymbol{A}(\boldsymbol{r},t)\mathrm{e}^{\mathrm{i}\omega t}$$

at large distances from a charge ($r \gg \boldsymbol{v}t$) have, according to retarded potential (4), the form [7]

$$\boldsymbol{A}_\omega = \frac{e\boldsymbol{v}}{r}\mathrm{e}^{\mathrm{i}\omega r}\int_0^\infty \mathrm{d}\tau\ \mathrm{e}^{\mathrm{i}\omega(1-\boldsymbol{n}\boldsymbol{v})\tau},$$

where $\boldsymbol{n} = \boldsymbol{r}/r$ is the unit vector in the direction $\boldsymbol{r}$. Such an asymptotic value of $\boldsymbol{A}_\omega$ is connected with the second term in (4).

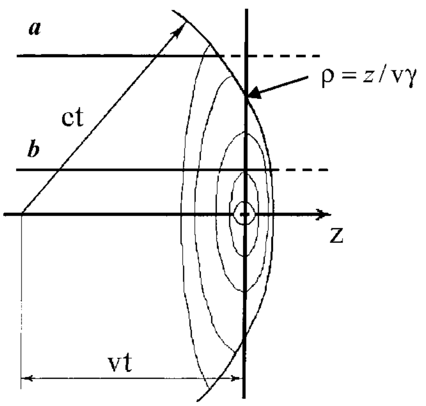

Fig. 1. Equipotential surfaces of the field surrounding the electron after its production in matter.

The main contribution to the integral over $\boldsymbol{k}$ for every term in (3) and (4) is made by that values of $\boldsymbol{k}$, whose directions are close to $\boldsymbol{v}$, namely, when characteristic angles θ between $\boldsymbol{k}$ and $\boldsymbol{v}$ are of the order of $\theta \sim \gamma^{-1}$. For such values of θ according to (3) and (4) during the time interval from $t = 0$ to $t < (k - \boldsymbol{k}\boldsymbol{v})^{-1} \sim 2\gamma^2/k$ the corresponding Fourier components of the field surrounding the particle are suppressed compared to the case $t > 2\gamma^2/k$. This means that during the time interval $\Delta t \sim 2\gamma^2/k$ the particle is in the 'half-bare' state without its normal Coulomb field [6,7]. Taking into account that the basic contribution to ionization energy losses of a particle is made for values $k > \omega_\mathrm{p}/v$, one should expect that during the time interval $\Delta t \sim 2\gamma^2/k$ the ionization energy losses will be weakened. For the electron with the energy $E \sim 100$ GeV we get $v\Delta t \sim 10^2$ cm.

3. Ionization energy losses of high energy charged particles produced in matter

Consider now the ionization energy losses of high energy electrons created in matter. At first let us use the Bohr method of computation of ionization energy losses of fast particle in matter (see, for example, [11]). According to this method, the energy transmitted by the moving particle to one electron of the medium is determined by the relation

$$\Delta E = \frac{(\Delta \boldsymbol{p})^2}{2m}, \tag{9}$$

where m is the electron mass and $\Delta \boldsymbol{p}$ is the momentum transmitted to the electron

$$\Delta \boldsymbol{p} = -e\int_{-\infty}^{\infty} \mathrm{d}t\ \nabla\varphi(\boldsymbol{r},t). \tag{10}$$

The energy loss per unit time is determined by

$$T = 2\pi v n \int_{\rho_{\min}}^{\rho_{\max}} \rho\ \mathrm{d}\rho\ \Delta E(\rho), \tag{11}$$

where n is the density of electrons in the medium and ρ is the impact parameter (distance between the particle's trajectory and the electron of the medium). The values $\rho_{\min}$ and $\rho_{\max}$ are the minimal and the maximal values of the impact parameters giving substantial contributions to the ionization energy loss.

For the particle moving uniformly and rectilinearly all time the potential $\varphi(\boldsymbol{r}, t)$ in (10) is the potential of the particle's own field (7). In this case

$$|\Delta \boldsymbol{p}| = \frac{2e^2}{v\rho} \tag{12}$$

and the expression for ionization energy losses takes the form

$$T = T_0 = \frac{4\pi n e^2}{mv} \ln \frac{\rho_{\max}}{\rho_{\min}}. \tag{13}$$

The value $\rho_{\min}$ is determined by quantum consideration and consists of order of $\hbar/p$, where p is the momentum of the projectile particle. The value $\rho_{\max}$ for high-energy electrons is of the order of $\rho_{max} \approx v/\omega_{\mathrm{p}}$. In the case of absence of the Fermi density effect $\rho_{\max} \sim v\gamma/\bar{w}$, where $\bar{w}$ is the characteristic average frequency of the electron motion in the atom [12].

After creation of fast particle in matter the electromagnetic field surrounding it is determined by relations (5) and (6). Substituting (5) into (10) we obtain

$$\Delta \boldsymbol{p}_\perp = \Delta \boldsymbol{p}_\perp^{(1)} + \Delta \boldsymbol{p}_\perp^{(2)}, \tag{14}$$

where

$$\Delta \boldsymbol{p}_\perp^{(1)} = -e \int_{-\infty}^{\infty} \mathrm{d}t\ \Theta(t - r) \frac{\partial}{\partial \boldsymbol{\rho}} \varphi_0(\boldsymbol{r}, t), \tag{15}$$

$$\Delta \boldsymbol{p}_\perp^{(2)} = -e \int_{-\infty}^{\infty} \mathrm{d}t\ \varphi_0(\boldsymbol{r}, t) \frac{\partial}{\partial \boldsymbol{\rho}} \Theta(t - r). \tag{16}$$

The first term in (14) corresponds to the momentum transmitted to the electron of the medium by the truncated field of the particle. The second term is the momentum transmitted to the electron by the arising electromagnetic wave.

Substituting (7) into (15) and (16), it is easy to see that on distances $z \gg \boldsymbol{\rho}$ from the origin of the particle

$$\Delta \boldsymbol{p}_\perp^{(1)} = \frac{\boldsymbol{\rho}}{\rho} \frac{e^2}{v\rho} \left[1 + \frac{1 - (\boldsymbol{\rho}\gamma/z)^2}{1 + (\boldsymbol{\rho}\gamma/z)^2} \right] \times (1 + \mathrm{O}(\boldsymbol{\rho}^2/z^2)), \tag{17}$$

$$\Delta \boldsymbol{p}_\perp^{(2)} = \frac{\boldsymbol{\rho}}{\rho} \frac{e^2}{v\rho} \frac{2(\boldsymbol{\rho}\gamma/z)^2}{1 + (\boldsymbol{\rho}\gamma/z)^2} (1 + \mathrm{O}(\boldsymbol{\rho}^2/z^2)). \tag{18}$$

Summing these relations we obtain that in this case the momentum transfer to the electron of the medium stays the same as in the case of a particle moving uniformly in the forward direction all the time

$$|\Delta \boldsymbol{p}_\perp| = \left| \Delta \boldsymbol{p}_\perp^{(1)} + \Delta \boldsymbol{p}_\perp^{(2)} \right| = \frac{2e^2}{v\rho} (1 + \mathrm{O}(\boldsymbol{\rho}^2/z^2)). \tag{19}$$

We would like to note that the total transverse momentum $\Delta \boldsymbol{p}_\perp$ is determined by the term (17) for small values of ρ ($\rho \to 0$). It is easy to see that the delta-burst is delivering essentially all the momentum transferred to target electrons positioned such that $\rho \ll z \ll \gamma\rho$.

Namely, we have for this case

$$\Delta p \approx \left| \Delta p_\perp^{(2)} \right|.$$

The reason for that is illustrated by Fig. 1, where two lines are presented for electrons of the medium with $\rho > z/\gamma$ (a) and $\rho < z/\gamma$ (b) along which the integration in Eq. (15) is performed.

The longitudinal momentum transfer on distances $z \gg \rho$ is equal to

$$\left| \Delta \boldsymbol{p}_\| \right| = \frac{e^2}{vz} (1 + \mathrm{O}(\boldsymbol{\rho}^2/z^2)) \ll |\Delta \boldsymbol{p}_\perp|.$$

Therefore, the ionization energy losses of the particle arising in matter at distances $z \gg \rho_{\max}$ from the point of its production are the same as in the case of particle moving uniformly and rectilinearly all the time. In other words, the correction to the conventional expression $T = T_0$ for energy losses of the particle in matter connected with prolonged existence of the electron in a half-bare state is small. The fast increase of ionization energy losses compared with the time of restoring of the common Coulomb field of the electron is explained by the fact that the δ-burst of the field strength at $r = t$ compensates the decrease of the ionization energy losses connected with the absence of the field at distances $r > t$.

4. Accounting of dielectric properties of a medium

Let us consider now the influence of the dielectric properties of the medium on the effects under consideration. Eqs. (1) and (2) in this case take the form [12]

$$\hat{\varepsilon}\left(\nabla^2 - \hat{\varepsilon}\frac{\partial^2}{\partial t^2}\right)\varphi(\boldsymbol{r}, t) = -4\pi e\delta(\boldsymbol{r} - \boldsymbol{v}t)\Theta(t), \quad (20)$$

$$\left(\nabla^2 - \hat{\varepsilon}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{A}(\boldsymbol{r}, t) = -4\pi e\boldsymbol{v}\delta(\boldsymbol{r} - \boldsymbol{v}t)\Theta(t). \quad (21)$$

In the range of high frequencies the dielectric function of a medium is determined by the relation

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{(\omega + i0)^2}. \quad (22)$$

The solutions of Eqs. (20) and (21) in this case have the form

$$\varphi(\boldsymbol{r}, t) = \frac{e}{2\pi^2}\mathrm{Re}\int\frac{d^3k}{k^2} \mathrm{e}^{i\boldsymbol{k}\boldsymbol{r}}\left\{-\frac{\omega_p \mathrm{e}^{-i\boldsymbol{k}\boldsymbol{v}t}}{\omega_p - \boldsymbol{k}\boldsymbol{v}} + \frac{\omega_p \mathrm{e}^{-i\omega_p t}}{\omega_p - \boldsymbol{k}\boldsymbol{v}} + \frac{\sqrt{k^2+\omega_p^2}}{\sqrt{k^2+\omega_p^2} - \boldsymbol{k}\boldsymbol{v}}\mathrm{e}^{-i\boldsymbol{k}\boldsymbol{v}t} - \frac{\sqrt{k^2+\omega_p^2}}{\sqrt{k^2+\omega_p^2} - \boldsymbol{k}\boldsymbol{v}}\mathrm{e}^{-i\sqrt{k^2+\omega_p^2}t}\right\}\Theta(t), \quad (23)$$

$$\boldsymbol{A}(\boldsymbol{r}, t) = \frac{e}{2\pi^2}v\,\mathrm{Re}\int d^3k\,\mathrm{e}^{i\boldsymbol{k}\boldsymbol{r}} \times\left\{\frac{\mathrm{e}^{-i\boldsymbol{k}\boldsymbol{v}t}}{\sqrt{k^2+\omega_p^2}\left(\sqrt{k^2+\omega_p^2} - \boldsymbol{k}\boldsymbol{v}\right)} - \frac{\mathrm{e}^{-i\sqrt{k^2+\omega_p^2}t}}{\sqrt{k^2+\omega_p^2}\left(\sqrt{k^2+\omega_p^2} - \boldsymbol{k}\boldsymbol{v}\right)}\right\}\Theta(t). \quad (24)$$

It is easy to see that (23) and (24) will be transformed into (3) and (4) for $\omega_p \to 0$. Calculating the ionization energy losses by the method described in [12], we obtain

$$T = T_0 + \frac{e^2\omega_p}{v}\frac{\sin(\omega_p t)}{t} - \frac{e^2\omega_p^2}{v}\left[\mathrm{ci}(\omega_p t)\sin^2(\omega_p t) - \mathrm{si}(\omega_p t)\cos^2(\omega_p t)\right], \quad (25)$$

where

$$\mathrm{ci}(x) = -\int_x^\infty \frac{\cos t}{t}\,\mathrm{d}t = C + \ln x + \int_0^x \frac{\cos t - 1}{t}\,\mathrm{d}t$$

$$\mathrm{si}(x) = -\int_x^\infty \frac{\sin t}{t}\,\mathrm{d}t = -\frac{\pi}{2} + \int_0^x \frac{\sin t}{t}\,\mathrm{d}t,$$

$C = 0.577$ is the Euler's constant.

The value $X = (v/\omega_p^2 e^2)(T - T_0)$ expressing the difference of the ionization energy losses of the half-bare electron from the normal energy losses, according to expression (25), is plotted in Fig. 2 as a function of $\omega_p t$.

Note for estimation that for electrons with an energy $E \sim 1$ GeV the value $(v/(\omega_p^2 e^2))T_0) \sim 17$, so the difference of the value of T calculated by formula (25) from T_0 on the distances of some ω_p^{-1}

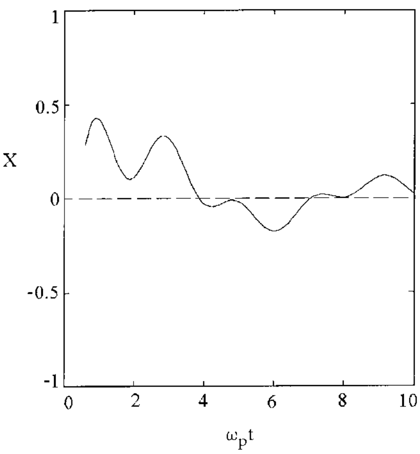

Fig. 2. The value X calculated according to Eq. (25) (solid line) as a function of $\omega_p t$.

(that is some ρ_{max} for high energy particles) consists not more than 2%.

Thus we see that the possibility of the prolonged existence of the particles without their regular Coulomb field practically has no effect on ionization energy losses by the particle in matter. Note, however, that this effect of the half-bare electron essentially manifests itself in the process of radiation by relativistic particles in matter [13].

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