Dynamic Chaos Phenomenon at Axial Channeling and Over-Barrier Motion of Relativistic Ions in Crystals

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Abstract

Characteristics of relativistic ion motion through a crystal near one of the crystal axes are discussed. The main features of such motion in a crystal are compared for protons and for highly charged ions. The possibility of bending of relativistic ion beams due to their multiple scattering on atomic strings in a bent crystal is discussed.

1. Introduction

Since a crystal has a periodic structure, it seems that any motion of particles in it should be regular and quasi-periodic. In reality along with the regular motion an irregular chaotic motion of a particle may be realized in a crystal. This fact was mentioned in Refs. [1,2]. In these papers the motion of high energy positively and negatively charged particles (protons, electrons etc) moving near one of the crystal axes was investigated.

In the present paper we study some characteristics of relativistic proton and ion motion in a crystal near one of its axes. The main attention is paid to the comparison of proton and highly charged ion motion through a crystal. We show that the mechanisms of motion for protons and highly charged ions through a bent crystal may be essentially different.

2. The motion of relativistic ions and protons in the periodical continuous field of crystal atomic strings

The motion of fast charged particles in a crystal at small angle ψ with respect to one of the crystal axes (z-axis) is mainly determined by the continuous potential of crystal atomic strings oriented parallel to the z-axis. In the field of this potential the particle momentum component, which is parallel to the z-axis, is conserved. In this case the motion in a plane orthogonal to the z-axis is defined by a two-dimensional Newton equation, in which the particle energy plays the role of the mass [1–3]:

$$\ddot{\boldsymbol{\rho}} = -\frac{1}{\varepsilon}\frac{\partial}{\partial \boldsymbol{\rho}} U(\boldsymbol{\rho}) \qquad (1)$$

Here $\boldsymbol{\rho} = (x,y)$ is the particle coordinates in the plane orthogonal to the z-axis. We use the system of units in which the velocity of light is unity.

The potential energy $U(\boldsymbol{\rho})$ is the sum of potential energies of particle interactions with separate atomic strings. The axes of these strings are arranged periodically in the (x,y)-plane.

Figure 1 shows the equipotential surfaces of the continuous potential $U(x,y)$ for uranium ions U^{+92} moving in a silicon crystal near the $\langle 100\rangle$ crystallographic axis. The function $U(x,y)$ for positively charged particles has maxima $U_{\max}$ at points corresponding to the positions of atomic strings in the orthogonal plane and shallow potential wells in the regions between neighbor strings. The function $U(x,y)$ in the vicinity of every atomic string has cylindrical symmetry. Figure 1 also shows typical particle trajectories of hyper-channeling (a) and over-barrier (b) motion of positively charged particles in the field $U(x,y)$. Let us consider some details of the ions motion in the continuous potential of crystal atomic strings.

The ion motion in such a field may be both finite and infinite in the plane orthogonal to the z-axis. The motion is finite if the energy of transverse motion $\varepsilon_\perp$ is less than the value of the potential energy in the saddle point U_H. The value $\varepsilon_\perp$ is the first integral of equation (1) and is defined by the equation $\varepsilon_\perp = \varepsilon\dot{\boldsymbol{\rho}}^2/2 + U(\boldsymbol{\rho})$. The depth of the potential wells for ions is Z_i times deeper than for protons, where $Z_i|e|$ is the charge of an ion. That is why the fraction of ions involved in the axial hyper-channeling mode is considerably larger than that for protons. For example, for beams with energy $E = 450$ GeV and divergence $\Delta\theta \sim 2\cdot 10^{-5}$ rad the finite motion fraction of ions in the crystal field (Fig. 1) is about 25%, whereas for protons this value is less than 1%.

For $\varepsilon_\perp > U_H$ the ion motion in the field $U(x,y)$ will be over the barrier in the plane orthogonal to the atomic strings. The critical angle of axial channeling $\psi_c = \sqrt{4ZZ_i e^2/\varepsilon d}$ is an

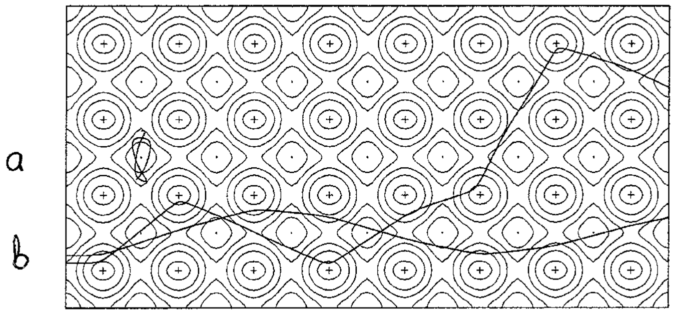

Fig. 1. Equipotential surfaces of the continuous potential energy of the interaction of uranium ions U^{+92} with a silicon crystal for particles moving at small angle to the $\langle 100\rangle$ axis and typical trajectories of the axial hyper-channeling (a) and over-barrier (b) particles in the field $U(x,y)$. Dots correspond to $U(x,y) = 0$, the equipotential surfaces correspond to values $U(x,y) = 100$, 250, 500, 1000 and 3000 eV. Crosses indicate the axes of atomic strings (at room temperature $U(x,y) = 8482$ eV at these points).

important parameter that determines the particle motion in a crystal near the crystal axis, where $Z|e|$ is the atomic charge of a crystal atom and d is the distance between atoms along the z-axis. If $\psi \leq \psi_c$ particles cannot move close to the axes of atomic strings. Therefore, the multiple scattering on the thermal oscillations of the lattice atoms is suppressed in this case in comparison with multiple scattering in amorphous matter, and the energy $\varepsilon_\perp$ varies for $\psi \leq \psi_c$ only due to weak multiple scattering at the electron subsystem of the lattice. Since the value of the critical axial channeling angle ψ_c is proportional to $\sqrt{Z_i}$, this effect takes place for ions in a much wider range of the angle ψ, than for protons.

The multiple scattering on the thermal oscillations of the lattice atoms becomes essential at $\varepsilon_\perp \geq U_{\max}$, i.e. when a particle can approach closely to the axes of atomic strings. The average value of the multiple scattering angle for ions in this case is Z_i times greater, than for protons.

Therefore, the motion of ions in a crystal is mainly determined by the continuous potential of the crystal atomic strings in a wide range of ψ values.

he problem of the particle motion in the periodical field U $.y)$ is a typical problem in the theory of nonlinear systems, where the dynamic chaos phenomenon may develop. In other words, in such a field along with the regular particle motion an essentially irregular chaotic motion of particles relative to the crystal atomic strings is possible. It is important to note that the dynamic chaos phenomenon is possible both for hyper-channeling and over-barrier particles. We can consider the collisions of particles with different crystal atomic strings as random if the dynamic chaos phenomenon exists for over-barrier motion. Consequently we come to the problem of multiple scattering of particles by crystal atomic strings. The simulation of a positively charged particle motion in a crystal shows that such regime takes place for a considerable fraction of particles in a beam up to the value $\psi \sim 2\psi_c$. For ions this scattering mechanism exists in a wider range of angles ψ, than for protons. The mean square value of the angle of multiple scattering of ions by the crystal atomic strings may exceed the corresponding value for amorphous matter by $\eta \sim a/4\psi_c d$, where a is the screened radius of the atom [2]. For protons this factor goes up to $\eta \sim 100$, for uranium ions it is about $\eta \sim 10$.

3. Motion of relativistic ions and protons through bent crystal

The process of multiple scattering of fast charged particles in a periodic field of bent crystal strings was considered in [4,5]. There was shown that beam deflection is possible for this case as well as for the case of a beam passing closely to the bent crystal plane [6]. The bending mechanisms for these two cases are principally different. Namely, plane deflection is possible only for plane channeling particles whereas axial deflection is possible both for channeling and over-barrier particles.

The over-barrier particles beam deflection was shown to be possible under condition [7]

$$\alpha = \frac{L}{R\psi_c}\frac{l_\perp}{R\psi_c} < 1 \qquad (2)$$

where L is the crystal length, R is the crystal bending curvature radius and $l_\perp$ is the length of beam transverse momentum

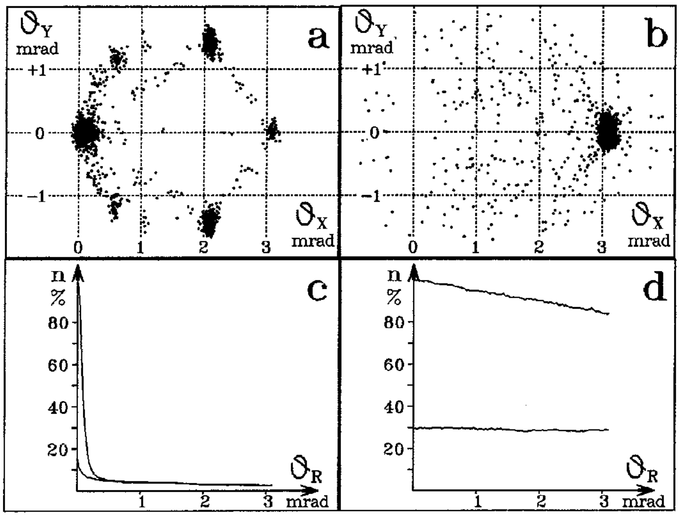

Fig. 2. Passage of the proton (a,c) and U^{+92}-ion (b,d) beams with energy $E = 450$ GeV through a bent silicon crystal with length $L = 3.1$cm and curvature radius R=10 m near the ⟨110⟩ axis. (a,b) – angular distribution of particles after beam passage through the crystal; (c,d) – fractions of all deflected particles which are in the limits of the critical axial channeling angle around the current direction of the crystal axis (upper curves) and hyper-channeling (down curves) particles as functions of passage length ($\vartheta_R = L/R$); the initial beam center coordinates are $(\vartheta_X, \vartheta_Y) = (0.,0.)$, the beam divergence is $3{\cdot}10^{-6}$ rad; the coordinates of the axis direction at the outcome of the crystal are $(\vartheta_X, \vartheta_Y) = (3.1,0.0)$ mrad.

equalization due to multiple scattering. For $\psi \leq \psi_c$ one may estimate $l_\perp \sim (\psi_c nda)^{-1}$, where n is the atomic density [3]. Due to relations $\alpha \sim \psi_c^{-3}$ and $\psi_c \sim \sqrt{Z_i}$ the motion of protons and highly charged ions through the bent crystal seems to be different too. For instance, for U^{+92} beam with energy E = 450 GeV which passes near the ⟨110⟩ axis through a Si crystal with the length L = 3.1 cm and curvature radius R = 10 m the value of the parameter α is small ($\alpha_U = 0.1$), while for protons for the same case it is large ($\alpha_P = 83$). Fig. 2 presents the simulation results for this case. The simulation takes into account real geometry of atomic strings in crystal, hyper-channeling and over-barrier motion as well as noncoherent scattering of particles caused by thermal oscillations of lattice atoms.

The presented results show that the angular distribution of protons (Fig. 2a) and ions (Fig. 2b) passed through the bent crystal are essentially different. The final direction of the bent crystal axis in this case is $\vartheta_R = 3.1$ mrad. For uranium ions the main fraction of the beam ions deflects to this angle. The particles follow the bent axis direction in this case. Deflection takes place both for hyper-channeling and over-barrier particles. It is important that the over-barrier fraction of the deflected beam is very large (Fig. 2d). For protons, in the presented case, the deflection is realized only for hyper-channeling particles (Fig. 2c), which fraction is smaller than for ions due to the smaller potential well depth (the initial beam divergence is $\psi \sim 3 \cdot 10^{-6}$rad). Symmetry maxima in the proton angular distribution are caused by particles, which were trapped by plane channels.

Note that the presented simulation result for protons (Fig. 2a) is in a good agreement with CERN experimental data [8]. The large value of the parameter $\alpha_P = 83$ (for the conditions of this experiment) violates the inequality (2). Thus, the experiment [8] could not detect the over-barrier deflection. On the other hand, the same conditions for uranium ions seems to be suitable for detecting over-barrier deflection, because the value of the parameter α satisfies the condition (2) in this case ($\alpha_U = 0.1$).

Therefore, one may conclude the following: The dynamical chaos effect at multiple scattering of charged particles on atomic strings leads to the doughnut scattering effect and then to the over-barrier deflection in a bent crystal. A high value of ion charge essentially simplifies the fulfillment of the over-barrier bending condition that makes possible the deflection of highly charged ion beams by a bent crystal.

The presented results show that the effect of beam axial deflection can be investigated at the GSI ion accelerator.

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References

1. Bolotin, Yu. L., Gonchar, V. Yu., Truten', V. I. and Shul'ga, N. F., Phys. Lett. A **123**, 357 (1987).
2. Akhiezer, A. I., Truten', V. I. and Shul'ga, N. F., Phys. Rep. **203**, 289 (1991).
3. Lindhard, J., Dansk. Vid. Selsk. Math. Phys. Medd. **34** (1965).
4. Greenenko, A. A. and Shul'ga, N.F., J. Exp. Theor. Phys Lett. **54**, 524 (1991).
5. Akhiezer, A. I., Shul'ga, N. F., Truten', V. I., Greenenko, A. A. and Syshchenko, V. V., Phys. Uspekhi **38**, 1119 (1995).
6. Tsyganov, E. N., Fermilab TM-682, TM-684 (Batavia, 1976).
7. Shul'ga, N. F. and Greenenko, A. A., Phys. Lett. B **353**, 373 (1995).
8. Baurichter, A. *et al.*, Nucl. Instr. Meth. B **115**, 172 (1996).