Spectral–Angular Distributions of Radiation from Relativistic Electrons in a Thin Layer of Matter

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Abstract—Expressions for the spectral–angular density of bremsstrahlung from a relativistic electron in a thin layer of matter are obtained. The effect that the multiple scattering of electrons by medium atoms exerts on the spectral–angular features of radiation in a thin amorphous target is studied. It is shown that, if the root-mean-square angle of multiple scattering is much larger than the characteristic angle of relativistic-electron radiation, there occurs the bremsstrahlung-suppression effect, which is similar to the Landau–Pomeranchuk–Migdal effect.

1. The multiple scattering of high-energy electrons in a medium can have a considerable effect on bremsstrahlung. The case where the radiation coherence length is much greater than the target thickness is of particular interest. In [1–3], it was shown that the bremsstrahlung-suppression effect, which is similar to the Landau–Pomeranchuk–Migdal effect, can arise in this case. Experimentally, this effect was studied at the SLAC accelerator [4, 5], the spectral properties of bremsstrahlung in the low-frequency region being explored there.

Here, we consider the spectral–angular properties of bremsstrahlung from high-energy electrons in a thin layer of matter. Particular attention is given to the conditions under which the bremsstrahlung-suppression effect is more pronounced in the angular than in the spectral distribution. It is shown that, if the root-mean-square angle of multiple scattering is much larger than the characteristic angle of relativistic-electron radiation, $\theta \sim \gamma^{-1}$ (θ is the angle between the wave-propagation and the electron-velocity vectors, and γ is the electron Lorentz factor), the angular distribution of electron radiation within the region around the angle $\theta \sim \gamma^{-1}$ with respect to the projectile-beam direction is virtually independent of the target thickness, with its maximum being at an angle of $\theta_m \approx \gamma^{-1}$.

2. Within classical electrodynamics (see [6, 7]), the spectral–angular density of radiation from an electron moving along the trajectory $\mathbf{r}(t)$ is given by

$$\frac{d^2E}{d\omega do} = \frac{e^2}{4\pi^2}[\mathbf{k} \times \mathbf{I}]^2, \qquad (1)$$

where

$$\mathbf{I} = \int\limits_{-\infty}^{\infty} \dot{\mathbf{r}}(t) e^{i(\omega t - \mathbf{k}\cdot\mathbf{r}(t))} dt, \qquad (2)$$

and $\mathbf{k}$ and ω are, respectively, the wave vector and the frequency of the emitted wave (we use the system of units where the speed of light in a vacuum is taken to be unity).

In a thin layer of matter, characteristic values of the relativistic-electron-scattering angle ϑ_e are much smaller than unity. If, concurrently, the radiation coherence length is much greater than the target thickness T,

$$l_c \approx \frac{2\gamma^2}{\omega}\frac{1}{1+\gamma^2\theta^2+\gamma^2\vartheta_e^2} \gg T, \qquad (3)$$

then the quantity $\mathbf{I}$ can be represented in the form (see [7])

$$\mathbf{I} \approx \frac{i}{\omega}\left(\frac{\mathbf{v}'}{1-\mathbf{n}\cdot\mathbf{v}'} - \frac{\mathbf{v}}{1-\mathbf{n}\cdot\mathbf{v}}\right), \qquad (4)$$

where $\mathbf{v}$ and $\mathbf{v}'$ are the electron velocities before and after scattering, respectively, and $\mathbf{n} = \mathbf{k}/\omega$. In this case, the spectral–angular density of radiation from an electron depends only on the particle-scattering angle in matter. Substituting (4) into (1), we find that, at small values of the scattering and radiation angles,

$$\frac{d^2E}{d\omega do} = \frac{e^2\gamma^2}{\pi^2}\left\{\frac{1+\alpha^2+\alpha^2\beta^2+2\alpha\beta\cos\varphi}{1+\alpha^2+\beta^2-2\alpha\beta\cos\varphi}\right. \qquad (5)$$

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$$\times \frac{1}{(1+\alpha^2)^2} - \frac{1}{(1+\alpha^2+\beta^2-2\alpha\beta\cos\varphi)^2}\Bigg\},$$

where $\alpha = \gamma\theta$, θ and φ being, respectively, the polar and the azimuthal angle of radiation, and $\beta = \gamma\vartheta_e$, ϑ_e being the electron-scattering angle. The angles θ and ϑ_e are reckoned from the direction of the initial electron velocity $\mathbf{v}$, while φ is the angle between the vectors $\mathbf{k}_\perp$ and $\mathbf{v}'_\perp$ in the plane orthogonal to $\mathbf{v}$.

Expression (5) must be averaged over the angles of particle scattering within the target. Given the angular distribution of scattered particles, $f(\vartheta_e)$, we can determine the averaged spectral–angular density of radiation according to the formula

$$\left\langle \frac{d^2E}{d\omega do} \right\rangle = \int d^2\vartheta_e f(\vartheta_e) \frac{d^2E}{d\omega do}, \qquad (6)$$

which is valid for any type of target, provided that its thickness is much smaller than the bremsstrahlung coherence length. The specific features of the scatterer would affect only the form of the distribution function $f(\vartheta_e)$.

For an amorphous target, the scattering-angle distribution of particles is described by the Bethe–Molière function (see [8, 9]),

$$f_{\mathrm{BM}}(\vartheta_e) = \frac{1}{2\pi}\int\limits_0^\infty \eta d\eta J_0(\eta\vartheta_e) \qquad (7)$$

$$\times \exp\left\{-nT\int \chi d\chi \sigma(\chi)[1-J_0(\eta\chi)]\right\},$$

where n is the medium-atom density and $\sigma(\chi)$ is the differential cross section for electron scattering by a single medium atom at a small angle χ.

The distribution function (7) is independent of φ; therefore, integration with respect to φ in expression (6) can be performed in a general form. The result is

$$\left\langle \frac{d^2E}{d\omega do} \right\rangle = \int\limits_0^\infty \vartheta_e d\vartheta_e f_{\mathrm{BM}}(\vartheta_e)\Phi(\theta,\vartheta_e), \qquad (8)$$

$$\Phi(\theta,\vartheta_e) = \frac{e^2\gamma^2}{\pi^2}\left\{\frac{2+\beta^2}{(1+\alpha^2)q^{1/2}} \right. \qquad (9)$$

$$\left. - \frac{1+\alpha^2+\beta^2}{q^{3/2}} - \frac{1}{(1+\beta^2)^2}\right\},$$

where $q = (1+\alpha^2+\beta^2)^2 - 4\alpha^2\beta^2$.

For the case of a screened atomic potential, Bethe derived a somewhat simplified expression for the distribution function (see [9]); that is,

$$f_{\mathrm{B}}(\vartheta_e) = \sum_{n=0}^{\infty} f_n(\vartheta_e)\frac{1}{B^n}, \qquad (10)$$

$$f_n(\vartheta_e) = \frac{1}{2\overline{\vartheta^2}}\int\limits_0^\infty udu J_0\left(u\frac{\vartheta_e}{\chi_c B^{1/2}}\right) \qquad (11)$$

$$\times e^{-u^2/4}\frac{1}{n!}\left(\frac{u^2}{4B}\ln\frac{u^2}{4}\right)^n,$$

where $\overline{\vartheta^2} = \chi_c^2 B$ is the mean-square value of the multiple-scattering angle, $\chi_c^2 = 4\pi nTZ^2e^4/(pv)^2$, and B is obtained from the relation

$$B - \ln B = \ln\frac{\chi_c^2}{\chi_1^2} + 1 - 2C. \qquad (12)$$

Here, $Z|e|$ is the medium-atom charge; p is particle momentum; $\chi_1 = 1/pR$, R being the radius of atomic-potential screening; and $C = 0.577...$ is the Euler constant.

Expression (8), involving the distribution function (10), describes the spectral–angular distribution of bremsstrahlung from relativistic electrons in a thin amorphous-medium layer.

3. We will now focus on some special features of the spectral–angular distributions of radiation from a relativistic electron in a thin amorphous-medium layer. First, we consider the angular distribution of radiation from an electron in the $(\mathbf{v},\mathbf{v}')$ plane. Rewriting (5) in terms of the Cartesian coordinates (x, y, z), with the z axis being directed along $\mathbf{v}$ and the y axis being orthogonal to the $(\mathbf{v},\mathbf{v}')$ plane, we find that the distribution of radiation in the $(\mathbf{v},\mathbf{v}')$ plane is

$$\frac{d^2E}{d\omega do} = \frac{e^2\gamma^2}{\pi^2}\left[\frac{\alpha_x}{1+\alpha_x^2} - \frac{\alpha_x-\beta}{1+(\alpha_x-\beta)^2}\right]^2, \qquad (13)$$

where $\alpha_x = \alpha\cos\varphi$.

Let us examine the asymptotic behavior of this expression at small and large scattering angles.

For small scattering angle ($\beta \ll 1$), we have

$$\frac{d^2E}{d\omega do} = \frac{e^2\gamma^2}{\pi^2}\beta^2\frac{(1-\alpha_x^2)^2}{(1+\alpha_x^2)^4}. \qquad (14)$$

This expression shows that, for $\beta \ll 1$, the angular distribution of radiation peaks at $\alpha_x = 0$ and that the spectral–angular density of radiation from an electron vanishes at $\alpha_x = \pm 1$. In the case being considered, the bulk of the radiation spectral density is concentrated within the angular region $\theta_x \le \gamma^{-1}$, where $\theta_x = \theta\cos\varphi$.

For large values of the scattering angle ($\beta \gg 1$), the angular distribution (13) of radiation has maxima at $\alpha_x \approx 1$ and $\alpha_x \approx \beta - 1$ and vanishes at $\alpha_x \approx -1/\beta$ and $\alpha_x \approx \beta + 1/\beta$. Expression (13) also shows that the angular density of radiation decreases fast in the angular intervals corresponding to the regions $\alpha_x \le -1$ and $\alpha_x \ge \beta + 1$. In the region $1 \le \alpha_x \le \beta$, the

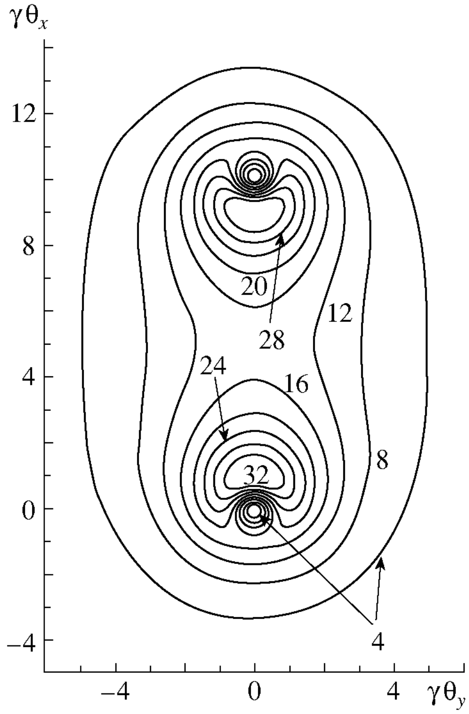

Fig. 1. Angular distribution of radiation from an electron in the xy plane, which is orthogonal to the direction of initial electron velocity at $\beta = 10$. The figures on the curves correspond to the values of the angular-distribution density (5) in units of $10^{-2}e^2\gamma^2/\pi^2$.

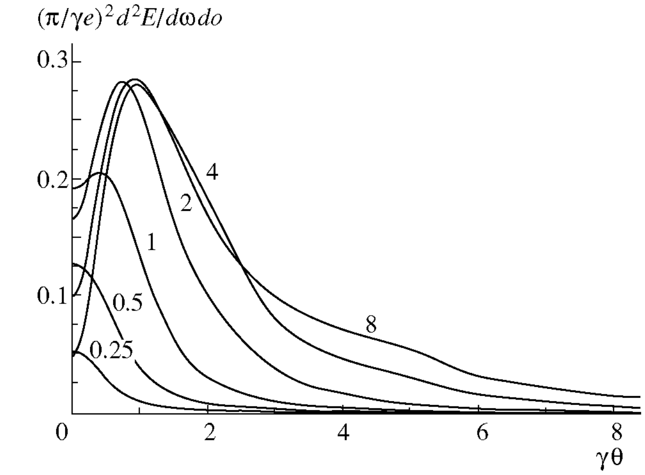

Fig. 2. Spectral–angular density of electron radiation as a function of the polar angle θ with respect to the direction of initial electron velocity. Figures on the curves correspond to the values of the parameter $\sqrt{\overline{\beta^2}}$.

angular density of radiation takes commensurate values over a rather broad interval of scattering angles. By way of example, we indicate that, for $\beta = 10$, the angular density of radiation has a minimum at $\alpha_x = \beta/2$, where the radiation intensity is about half its value at the maxima. This means that, at $\beta \gg 1$, the bulk of the spectral density of radiation from an electron is concentrated within the angular range $0 \leqslant \vartheta_x \leqslant \vartheta_e$.

For $\beta \gg 1$, it follows from (5) that, in directions close to that of the initial particle-motion velocity $\mathbf{v}$, the angular density of radiation can be represented as

$$\frac{d^2E}{d\omega do} \approx \frac{e^2\gamma^2}{\pi^2}\frac{\alpha^2}{(1+\alpha^2)^2} \quad (\alpha \ll \beta). \qquad (15)$$

In this case, the angular density of radiation is independent of the scattering angle.

These results are illustrated in Fig. 1, which displays, for $\beta = 10$, isolines of the angular density of radiation from an electron in the xy plane.

We now consider the multiple-scattering effect on the angular distribution of bremsstrahlung. For small scattering-angle values such that the condition $\overline{\beta^2} = \gamma^2\overline{\vartheta_e^2} \ll 1$ is satisfied, the function $\Phi(\theta, \vartheta_e)$ in (8) can be expanded in β. In the first order of this expansion, the spectral–angular density of radiation is given by the expression

$$\left\langle \frac{d^2E}{d\omega do} \right\rangle = \frac{2e^2\gamma^2}{\pi^2}\overline{\beta^2}\frac{1+\alpha^4}{(1+\alpha^2)^4}, \qquad (16)$$

which coincides with the corresponding result for the spectral–angular density of radiation in Bethe–Heitler theory, according to which the radiation intensity grows linearly with increasing target thickness {see, for example, expression (5.9) in [10]}.

The inequality $\overline{\beta^2} \ll 1$ is a condition under which the dipole approximation is valid in describing radiation from a particle in a medium [7]. As the target thickness is increased, the condition $\overline{\beta^2} \ll 1$ is violated (since $\overline{\beta^2} \sim T$); therefore, effects associated with a nondipole character of radiation must be taken into account. For arbitrary values of $\overline{\beta^2}$, averaging in (8) can be performed only on the basis of numerical methods.

The spectral–angular density of radiation as a function of its polar angle θ is given in Fig. 2 for various values of the parameter $\sqrt{\overline{\beta^2}}$. The displayed curves show that, for $\overline{\beta^2} < 1$, the angular distribution of radiation has a maximum in the direction of the initial particle velocity and decreases fast with increasing θ [see Eq. (16)]. At $\overline{\beta^2} \sim 1$, the maximum in the angular distribution of radiation is shifted into the region of angles around $\theta \sim \gamma^{-1}$. Concurrently, the growth of radiation intensity with increasing target thickness becomes slower than a linear one, which is typical of Bethe–Heitler theory. For $\overline{\beta^2} > 1$, the maximum in the angular distribution of radiation occurs in the angular region around $\theta \sim \gamma^{-1}$, with the radiation intensity at this maximum being virtually independent of $\overline{\beta^2}$ (that is, of the target thickness). However, the angular distribution broadens with increasing target thickness. For $\overline{\beta^2} > 1$, the radiation intensity in the angular region $\theta \ll \gamma^{-1}$ decreases fast with increasing $\overline{\beta^2}$. For $\overline{\beta^2} \gg 1$, the distribution of radiation in the angular region $\theta \ll \sqrt{\overline{\beta^2}}$ is given by (15). Within this region of radiation angles, the angular distribution is of a universal form; that is, it is independent of the form of scattering-angle distribution of particles.

Thus, we see that, for $\overline{\beta^2} \gg 1$, bremsstrahlung is suppressed in the angular region $\theta \ll \sqrt{\overline{\vartheta_e^2}}$ in relation to the corresponding result in Bethe–Heitler theory. This bremsstrahlung-suppression effect is similar to the effect of suppression of the spectral density of bremsstrahlung in a thin layer of matter (see [2, 3]). For the spectral density of radiation as a function of the target thickness, however, there occurs a transition from a linear to a logarithmic target-thickness dependence of the radiation intensity, while, for the spectral–angular distribution of radiation, a linear growth of the radiation intensity with the target thickness at small $\overline{\beta^2}$ gives way to a constant value at large $\overline{\beta^2}$. This means that the bremsstrahlung-suppression effect is more pronounced in the spectral–angular distribution of radiation than in its spectral density.

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