X-ray bremsstrahlung by relativistic particles crossing a thin layer of a medium

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Abstract

Anomalous phenomena in the relativistic electron bremsstrahlung are investigated theoretically for the case when an emitting particle crosses a thin layer of amorphous medium. Peculiarities in the manifestation of Ter-Mikaelian effect as well as unexpected oscillations in the spectral–angular distribution of emitted photons are shown in this work.

Keywords: Bremsstrahlung; Transition radiation; Multiple scattering

1. Introduction

The fundamental aspects of the relativistic particle emission processes in condensed media have been under discussion for a long time, but the experimental verification of such theoretically predicted effects such as the Landau–Pomeranchuk effect [1] and Ter-Mikaelian effect [2] has been obtained within recent years only [3,4].

The results of experiment [4] performed for the case of a thick enough target are in good agreement with the theory [2] describing the bremsstrahlung of a particle moving in an unbounded medium. But the situation changes very essentially for the case of a relativistic particle bremsstrahlung in a thin layer of a medium. The anomalous manifestation of the Ter-Mikaelian effect has been shown in the experiment [5] devoted to 150 MeV electron bremsstrahlung in Al foil with a thickness of 30 μm. A complicated form of bremsstrahlung spectrum strongly different from that predicted by the theory of Ter-Mikaelian effect in an unbounded medium has been observed in this experiment. It is important to note that the result [5] is not explained by the traditional theory of relativistic particle bremsstrahlung in a target with a finite thickness [6]. Therefore the problem of adequate theoretical description of an emission by relativistic particles crossing a thin layer of a medium cannot be treated as solved.

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The last experimental research of relativistic electron bremsstrahlung [7] (particles with energies of 600–900 MeV and foils of different materials and thicknesses have been used in this experiment) confirmed the existence of peculiarities in the X-ray region of bremsstrahlung spectrum. The significant increase of the radiation intensity in the soft part of a spectrum has been observed for the case of a thin target with a thickness smaller than an emission formation length. In addition to this result, the oscillations of a spectral intensity have been observed in the wide spectral region. The authors of the mentioned work [7] assume the coherent bremsstrahlung of incident particles on macro non-uniformities of a target as the physical reason fof the observed phenomena.

The aim of this work consists in the detailed theoretical analysis of the emission process by relativistic particles crossing a thin layer of an amorphous medium. The strong difference between Ter-Mikaelian effect manifestation in the cases of thin and thick targets is shown within the framework of the developed theory as well as the essential influence of an interference between transition radiation and bremsstrahlung giving rise to the existence of coherent oscillations of a spectrum in the emitted photon energy range more wider than that for the ordinary transition radiation manifestation.

2. General expressions

Let us consider the structure of an electromagnetic field emitted by the flux of relativistic electrons crossing a thin layer of amorphous medium. The solution of Maxwell equations for the Fourier transform of an electrical field $\boldsymbol{E}_{k\omega} = 1/(2\pi)^4 \int \mathrm{d}^3 r\, \mathrm{d}t\, \boldsymbol{E}(\boldsymbol{r},t) \mathrm{e}^{\mathrm{i}\omega t - \mathrm{i}\boldsymbol{k}\boldsymbol{r}}$,

$$(k^2 - \omega^2 \varepsilon(\omega))\boldsymbol{E}_{k\omega} - \boldsymbol{k}(\boldsymbol{k}\boldsymbol{E}_{k\omega}) = 4\pi \mathrm{i}\omega \boldsymbol{j}_{k\omega}, \quad (1)$$

where $\varepsilon(\omega) = 1 - \omega_0^2/\omega^2$ is the dielectric permeability in the X-ray region, ω_0 the plasma frequency of a medium, $\boldsymbol{j}_{k\omega}$ the Fourier transform of an emitting particle current density $\boldsymbol{j}(\boldsymbol{r},t) = e\boldsymbol{V}_{\mathrm{e}}(t)\delta(\boldsymbol{r} - \boldsymbol{r}_{\mathrm{e}}(t))$, $\boldsymbol{r}_{\mathrm{e}}(t)$ the particle's trajectory, $\boldsymbol{V}_{\mathrm{e}}$ is its velocity is found in the form

$$E^{(1)}_{\lambda \boldsymbol{k}} = \frac{\mathrm{i}\omega e}{4\pi^3} \frac{1}{k^2 - \omega^2} \int \mathrm{d}t\, \boldsymbol{e}_{\lambda \boldsymbol{k}} \boldsymbol{V}_{\mathrm{e}}(t) \mathrm{e}^{\mathrm{i}\omega \mathrm{t} - \mathrm{i}\boldsymbol{k}\boldsymbol{r}_{\mathrm{e}}(\mathrm{t})}, \quad (2a)$$

$$E^{(2)}_{\lambda \boldsymbol{k}} = \frac{\mathrm{i}\omega e}{4\pi^3} \frac{1}{k^2 - \omega^2 \varepsilon(\omega)} \int \mathrm{d}t\, \boldsymbol{e}_{\lambda \boldsymbol{k}} \boldsymbol{V}_{\mathrm{e}}(\mathrm{t}) \mathrm{e}^{\mathrm{i}\omega \mathrm{t} - \mathrm{i}\boldsymbol{k}\boldsymbol{r}_{\mathrm{e}}(t)} + b_{\lambda \boldsymbol{k}_\parallel} \delta\left(k_\perp - \sqrt{\omega^2 \varepsilon(\omega) - k_\parallel^2}\right), \quad (2b)$$

$$E^{(3)}_{\lambda \boldsymbol{k}} = \frac{\mathrm{i}\omega e}{4\pi^3} \frac{1}{k^2 - \omega^2} \int \mathrm{d}t\, \boldsymbol{e}_{\lambda \boldsymbol{k}} \boldsymbol{V}_{\mathrm{e}}(t) \mathrm{e}^{\mathrm{i}\omega \mathrm{t} - \mathrm{i}\boldsymbol{k}\boldsymbol{r}_{\mathrm{e}}(t)} + a_{\lambda \boldsymbol{k}_\parallel} \delta\left(k_\perp - \sqrt{\omega^2 - k_\parallel^2}\right). \quad (2c)$$

Expressions (2a)–(2c) determine the transverse components of the electric field $\boldsymbol{E}^{\mathrm{tr}}_{k\omega} = \sum_{\lambda=1}^{2} \boldsymbol{e}_{\lambda \boldsymbol{k}} E_{\lambda \boldsymbol{k}}$ ($\boldsymbol{k}\boldsymbol{e}_{\lambda \boldsymbol{k}} = 0$, $\boldsymbol{e}_{\lambda \boldsymbol{k}}$ is the polarization vector, $\boldsymbol{k} = \boldsymbol{k}_\parallel + \boldsymbol{e}_\perp k_\perp, \boldsymbol{e}_\perp \boldsymbol{k}_\parallel = 0$, $\boldsymbol{e}_\perp$ is the normal to the target surface) in front of the target ($E^{(1)}_{\lambda \boldsymbol{k}}$), inside the target ($E^{(2)}_{\lambda \boldsymbol{k}}$) and behind the target ($E^{(3)}_{\lambda \boldsymbol{k}}$). Reflected waves are neglected in (2a)–(2c) in accordance with the general approach of X-ray transition radiation theory (see for example [6] and the literature presented there).

Determining unknown coefficients $b_{\lambda \boldsymbol{k}_\parallel}$ and $a_{\lambda \boldsymbol{k}_\parallel}$ by the use of ordinary boundary conditions, one should obtain the expression for an emission field by the calculation of Fourier integral $E^{\mathrm{Rad}}_{\lambda} = \int \mathrm{d}^3 k E^{(3)}_{\lambda \boldsymbol{k}} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\boldsymbol{r}}$. In order to find the field $E^{\mathrm{Rad}}_{\lambda}$ in the wave zone it is necessary to calculate this integral by the use of the stationary phase method. The result of the calculations has the form

$$E^{\mathrm{Rad}}_{\lambda \boldsymbol{k}} = A_\lambda \frac{\mathrm{e}^{\mathrm{i}\omega r}}{r}, \quad A_\lambda = \boldsymbol{e}_\lambda \boldsymbol{A}_{\boldsymbol{n}},$$

$$\boldsymbol{A}_{\boldsymbol{n}} = \frac{\mathrm{e}}{2\pi} \left[\frac{\boldsymbol{V}_{\mathrm{i}} - \boldsymbol{n}}{1 - \boldsymbol{n}\boldsymbol{V}_{\mathrm{i}}} - \frac{\boldsymbol{V}_{\mathrm{f}} - \boldsymbol{n}}{1 - \boldsymbol{n}\boldsymbol{V}_{\mathrm{f}}} \right.$$
$$\times \exp\left[\mathrm{i}\omega \left(L - \boldsymbol{n}_\parallel \boldsymbol{r}_\parallel(L) - \sqrt{n_\perp^2 + \frac{\omega_0^2}{\omega^2}} L \right) \right]$$
$$+ \mathrm{i}\omega \int_0^L \mathrm{d}t (\boldsymbol{V} - \boldsymbol{n}) \exp\left[\mathrm{i}\omega (t - \boldsymbol{n}_\parallel \boldsymbol{r}_\parallel(t) \right.$$

$$- \sqrt{n_\perp^2 + \frac{\omega_0^2}{\omega^2} r_\perp(t))} \Bigg] \Bigg]$$

$$\times \exp\left[\mathrm{i}\omega \left(\sqrt{n_\perp^2 + \omega_0^2/\omega^2} - n_\perp \right) L \right], \qquad (3)$$

where $\boldsymbol{n}$ is the unit vector along the direction of emitted photon propagation, $\boldsymbol{V}_\mathrm{i}$ the initial (before scattering in a target) particle's velocity, $\boldsymbol{V}_\mathrm{f}$ the final (after scattering) velocity, L is the target thickness (the difference between L/V_i and the time of a particle motion in a target is neglected because of $\boldsymbol{V}_\mathrm{i} = \boldsymbol{e}_\perp V \approx \boldsymbol{e}_\perp(1 - \frac{1}{2}\gamma^{-2}) \approx \boldsymbol{e}_\perp$ and a small enough value of a particle multiple scattering angle on the exit of a target).

Defining the angular variables Θ and Ψ_t by the formulae

$$\boldsymbol{n} = \boldsymbol{e}_\perp \left(1 - \frac{1}{2}\Theta^2 \right) + \Theta, \qquad \boldsymbol{e}_\perp \Theta = 0,$$
$$\boldsymbol{V}(t) = \boldsymbol{e}_\perp \left(1 - \frac{1}{2}\gamma^{-2} - \frac{1}{2}\Psi_t^2 \right) + \Psi_t, \ \boldsymbol{e}_\perp \Psi_t = 0, \qquad (4)$$

one can rewrite the expression for emission amplitude $A_{\boldsymbol{n}}$ from (3) in a more convenient form

$$A_{\boldsymbol{n}} = -\frac{e}{\pi} \left[\frac{\boldsymbol{u}_\mathrm{i}}{\gamma^{-2} + u_\mathrm{i}^2} - \frac{\boldsymbol{u}_\mathrm{f}}{\gamma^{-2} + u_\mathrm{f}^2} \right.$$
$$\times \exp\left(\frac{\mathrm{i}\omega}{2} \int_0^L \mathrm{d}t(\gamma_*^{-2} + u_t^2) \right) + \frac{\mathrm{i}\omega}{2} \int_0^L \mathrm{d}t \boldsymbol{u}_t$$
$$\left. \times \exp\left(\frac{\mathrm{i}\omega}{2} \int_0^t \mathrm{d}\tau(\gamma_*^{-2} + u_\tau^2) \right) \right] \exp\left(\frac{\mathrm{i}\omega_0^2}{2\omega} L \right), \qquad (5)$$

where $\boldsymbol{u}_t = \Theta - \Psi_t, \quad \gamma_* = \gamma/\sqrt{1 + \gamma^2\omega_0^2/\omega^2}$.

The last formula allows to obtain the following expression for the emission spectral–angular distribution:

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega \mathrm{d}^2\Theta}$$
$$= \frac{e^2}{\pi^2} \left\langle \frac{u_\mathrm{i}^2}{(\gamma^{-2} + u_\mathrm{i}^2)^2} + \frac{u_\mathrm{f}^2}{(\gamma^{-2} + u_\mathrm{f}^2)^2} \right.$$
$$- \frac{2\boldsymbol{u}_\mathrm{i}\boldsymbol{u}_\mathrm{f}}{(\gamma^{-2} + u_\mathrm{i}^2)(\gamma^{-2} + u_\mathrm{f}^2)} \cos\left(\frac{\omega}{2} \int_0^L \mathrm{d}t(\gamma_*^{-2} + u_t^2) \right)$$
$$- \omega \int_0^L \mathrm{d}t \frac{\boldsymbol{u}_\mathrm{i}\boldsymbol{u}_t}{\gamma^{-2} + u_\mathrm{i}^2} \sin\left(\frac{\omega}{2} \int_0^t \mathrm{d}\tau(\gamma_*^{-2} + u_\tau^2) \right)$$
$$- \omega \int_0^L \mathrm{d}t \frac{\boldsymbol{u}_\mathrm{f}\boldsymbol{u}_t}{\gamma^{-2} + u_\mathrm{f}^2} \sin\left(\frac{\omega}{2} \int_t^L \mathrm{d}\tau(\gamma_*^{-2} + u_\tau^2) \right)$$
$$+ \frac{\omega^2}{2} \int_0^L \mathrm{d}t \int_0^{L-t} \mathrm{d}\tau \boldsymbol{u}_t \boldsymbol{u}_{t+\tau}$$
$$\left. \times \cos\left(\frac{\omega}{2} \int_t^{t+\tau} \mathrm{d}\tau'(\gamma_*^{-2} + u_{\tau'}^2) \right) \right\rangle, \qquad (6)$$

where the brackets $\langle \ \rangle$ mean the averaging over particle's scattering angles.

Before beginning the analysis of the derived formula one should define the conceptions of thin and thick targets. In accordance with the general approach of relativistic particle emission theory a target is thin in case the total scattering angle of emitted particle Θ_sc (the angle of a particle multiple scattering on a target) is less than the characteristic emission angle of relativistic particles $\Theta_\mathrm{em} \approx \gamma^{-1}$. Since $\Theta_\mathrm{sc}^2 \approx (\varepsilon_k^2/m^2\gamma^2)(L/L_\mathrm{Rad})$ ($\varepsilon_k \approx 21$ MeV, L_Rad is the radiation length) a target may be assumed as thin under the condition

$$L \ll \frac{m^2}{\varepsilon_k^2} L_\mathrm{Rad} \equiv L_0, \qquad (7)$$

which does not depend on the particle energy.

In the case of a thick target $L \gg L_0$, the emission process has a non-dipole character caused by multiple scattering of an emitting particle and the Landau–Pomeranchuk effect takes place (suppression of an emission spectral intensity in the range of small emitted photon energies). Since the existing experiments are in agreement with the theory for the case of a thick target, and on the other hand, the essential discrepancies were observed in experiments devoted to the relativistic particle bremsstrahlung in thin target, condition (7) is assumed to be valid in the calculations of this work.

3. Anomalous Ter-Mikaelian effect

Let us consider the influence of the change of an emitted photon phase velocity due to polarization of a medium by an emitting particle electromagnetic field (Ter-Mikaelian effect described by the replacement $\gamma \to \gamma_*$ in (6)).

Formula (6) describes the total radiation of a fast particle; it includes both the bremsstrahlung and the X-ray transition radiation at the layer boundaries. In the general case, the interference between these two emission mechanisms may present essential difficulties for the experimental investigation of bremsstrahlung spectrum. In order to exclude the contribution of transition radiation the photon collimator with a small angular acceptance Θ_c ($\Theta_c^2 \ll \gamma^{-2}$) has been used in the experiment [5]. This method allows to use the characteristic deep gap in the transition radiation angular distribution [6] for the bremsstrahlung extraction (bremsstrahlung angular distribution has a maximum to the direction along an emitting particle velocity in contrast to transition radiation distribution).

The formula for the spectral distribution of the strongly collimated radiation following from (6) is given by

$$\omega\frac{dN}{d\omega} = \frac{e^2}{\pi}\Theta_c^2\left\langle \frac{\psi_L^2}{(\gamma^{-2}+\psi_L^2)^2} - \omega\int_0^L dt\,\frac{\psi_L\psi_t}{\gamma^{-2}+\Psi_L^2} \times \sin\left(\omega/2\int_t^L d\tau(\gamma_*^{-2}+\Psi_\tau^2)\right) + \frac{\omega^2}{2}\int_0^L dt\int_0^{L-t} d\tau\,\Psi_t\Psi_{t+\tau} \times \cos\left(\omega/2\int_t^{t+\tau} d\tau'(\gamma_*^{-2}+\Psi_{\tau'}^{2})\right)\right\rangle. \quad (8)$$

Taking into account condition (7) and assuming the phase deviation caused by the multiple scattering to be small enough,

$$\left\langle \frac{\omega}{2}\int_0^L dt\,\Psi_t^2 \right\rangle = \frac{1}{2}\frac{L}{L_0}\frac{L}{L_{coh}} \ll 1, \quad L_{coh} = \frac{2\gamma^2}{\omega}, \quad (9)$$

one can obtain from (8) the very simple expression

$$\omega\frac{dN}{d\omega} = \frac{e^2}{\pi}\gamma^2\Theta_c^2\frac{L}{L_0}\left[1 - 2\frac{\gamma_*^2}{\gamma^2}\left(1-\frac{\gamma_*^2}{\gamma^2}\right) \times \left(1 - \frac{2\gamma_*^2}{\omega L}\sin\frac{\omega L}{2\gamma_*^2}\right)\right]. \quad (10)$$

The obtained results valid in the region where conditions (7) and (9). It is easy to see that the target thickness L may be both less and more than the emission formation length L_{coh}.

The spectrum (10) is illustrated in Fig. 1 by curves 2 and 3 corresponding to the universal function $F(x,y)$ determined by the formula

$$\omega\frac{dN}{d\omega} = N_0F(x,y), \quad N_0 = \frac{e^2}{\pi}\gamma^2\Theta_c^2\frac{L}{L_0}, \quad F = 1 - \frac{2x^2}{(1+x^2)}\left(1 - \frac{\sin y(x+1/x)}{y(x+1/x)}\right), \quad (11)$$

where $x = \omega/\gamma\omega_0$, $y = \omega_0 L/2\gamma$.

The curve 1 describing the Ter-Mikaelian effect in an unbounded medium (this curve corresponds to the function $F_1 = x^4/(1+x^2)^2$, which follows from Ter-Mikaelian's theory [2] for the strongly collimated radiation) is presented in Fig. 1 as well as by the curve 4 corresponding to the absence of

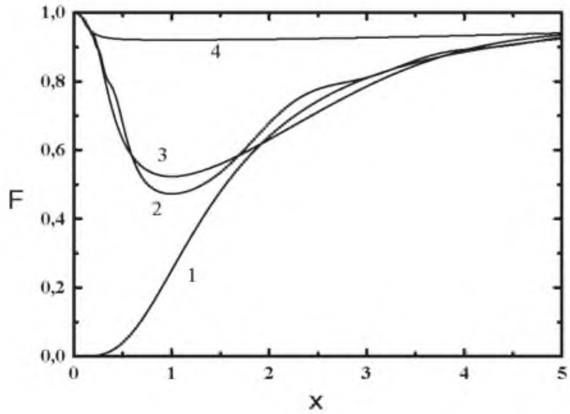

Fig. 1. >Spectrum of a collimated bremsstrahlung of a relativistic particle from a thin layer of a medium. $\omega dN/d\omega = N_0F(x,y), N_0=(e^2/\pi)\gamma^2\Theta_c^2L/L_0, x=\omega/\gamma\omega_0, y=\omega_0L/2\gamma$. Curve 1: $F_1=x^4/(1+x^2)^2$; curve 2: $y=1$; curve 3: $y=5$; curve 4: $y=0$.

Ter-Mikaelian effect in a thin target predicted by the traditional theory of a relativistic particle bremsstrahlung in a thin layer of a medium [6].

The obtained result consisting in a nonmonotonous shape of the spectral distribution (11) permits to explain the result of experimental researches of a relativistic particle bremsstrahlung in a thin layer of a medium [5]. The spectrum shape can be interpreted by the competition of two tendencies. On the one hand, the difference between the velocity of emitting particle $V \approx 1 - 1/2\gamma^{-2}$ and the phase velocity of emitted photon $V_{\rm ph} \approx 1 + \frac{1}{2}(\omega_0^2/\omega^2)$ increases in the frequency region $\omega < \gamma\omega_0$, with the decreasing of ω. This effect causes the bremsstrahlung suppression (Ter-Mikaelian effect). On the other hand, there is the growth of the emission formation length with the decreasing of ω. Therefore, in the $\omega \to 0$ case, for a rather thin layer the emission mechanism is effectively equivalent to that realizing for a particle moving in a vacuum along the trajectory of the angle form. As it is well known that (see for example [8]) the spectrum of this radiation does not depend on the frequency ω and is determined by the deviation angle of the particle velocity (according to formulae (8) and (10), this is the angle of multiple scattering $\Theta_{\rm sc} = \sqrt{\langle \Psi_L^2 \rangle} = (1/\gamma)\sqrt{L/L_0}$ for the case considered).

Thus the manifestation of the Ter-Mikaelian effect in bremsstrahlung for the real experiment can essentially differ from that predicted by the original theory [2]. At the same time the result [6] should be considered as asymptotic. As it follows from (11) the result [6] is correct, when $y \ll 1$.

4. The coherent oscillations in the emission spectrum

Let us return to the formula (6) and consider the emission properties in a more general case when the emission angle Θ is not much less than the characteristic emission angle of relativistic particles $\Theta_{\rm em} \approx \gamma^{-1}$. Assuming the conditions (7) and (9) to be valid one can obtain from (6) the following expression:

$$\omega \frac{dN}{d\omega\, d^2\Theta} = \frac{e^2}{\pi^2}\gamma^2 P(x,y,z), \tag{12a}$$

$$\begin{aligned} P = {} & 2z^2\left(\frac{1}{1+z^2} - \frac{1}{1+x^{-2}+z^2}\right)^2 (1-\cos\xi) \\ & + \frac{L}{L_0}\frac{1+z^4}{(1+z^2)^4} - 2\frac{L}{L_0}\frac{x^{-2}}{(1+z^2)(1+x^{-2}+z^2)^3} \\ & \times \left(\frac{1+x^{-2}}{1+z^2} + \frac{3z^4}{1+x^{-2}+z^2}\right)\left(1 - \frac{\sin\xi}{\xi}\right) \\ & - 2\frac{L}{L_0}\frac{z^2x^{-2}}{(1+z^2)(1+x^{-2}+z^2)} \\ & \times \left(\frac{2}{(1+z^2)^3} - \frac{1}{(1+x^{-2}+z^2)^2}\right)(1-\cos\xi) \\ & + 2\frac{L}{L_0}\frac{z^2x^{-4}(1+x^{-2}-z^4)}{(1+z^2)^3(1+x^{-2}+z^2)^4}\xi\sin\xi \\ & + \frac{2}{3}\frac{L}{L_0}\frac{z^4x^{-4}}{(1+z^2)^2(1+x^{-2}+z^2)^4}\xi^2\cos\xi, \end{aligned} \tag{12b}$$

where $z = \gamma\Theta$, $\xi = yx(1+x^{-2}+z^2)$.

The first term in (12) describes the spectral–angular distribution of transition radiation emitted by a particle crossing with a constant velocity a layer of a medium. The second one corresponds to the particle's bremsstrahlung without taking into account the influence of medium electrons' polarization. The last terms in (12) should be considered as the result of the interference between bremsstrahlung and transition radiation because they contain the coherent oscillations typical for the transition radiation in a layer of a medium and reduce to zero without taking into account the effect of medium polarization.

It is easy to see that the contribution of transition radiation to total emission yield is concentrated in the small frequency range $x \leqslant 1$ (this contribution is proportional to x^{-4} in the range $x > 1$). The most important property of the interference terms in (12) consists in the essential expansion of the frequency range, where the coherent oscillations in a spectrum take place. The simple formula

$$\begin{aligned} P \approx {} & \frac{L}{L_0}\frac{1}{(1+z^2)^2}\left\{1 + z^4 + \frac{2}{x^2}\left(\frac{1}{3}z^4y^2\cos\xi - \frac{1+3z^4}{1+z^2}\right.\right. \\ & \left.\left. \times \left(1 - \frac{\sin\xi}{\xi}\right) - z^2\frac{1-z^2}{1+z^2}(1-\cos\xi)\right)\right\}, \\ \xi = {} & xy(1+z^2), \end{aligned} \tag{13}$$

following from (12) in the frequency range $x^2 \gg L_0/L$, shows that the essential oscillations in the emission spectral–angular distribution may be observed at large angles ($z \approx 1$) in the case of $y^2 \gg 1$, when the transition radiation is formed because the thickness of a target exceeds the maximum value of the emission formation length in a medium.

The dependence $P(x)$ calculated by the general formula (12) for different values of the parameters y and z is presented in Figs. 2 and 3.

It is important to note that the emission spectrum has been measured in experiment [7] by the detector with an angular size Θ_c of the order of $\Theta_{em} \approx \gamma^{-1}$. Such a spectrum is described by the formula

$$\omega \frac{dN}{d\omega} = \frac{e^2}{\pi} Q(x, y, \gamma\Theta_c),$$
$$Q = \int_O^{\gamma^2\Theta_c^2} dz P(x, y, z), \tag{14}$$

where the function P is defined by (12).

The dependence $Q(x)$ presented in Fig. 4 gives the possibility to describe the oscillations in the emission spectrum measured in the experiment [7] without taking into account the coherent bremsstrahlung on the density fluctuations of target material as it was assumed in [7].

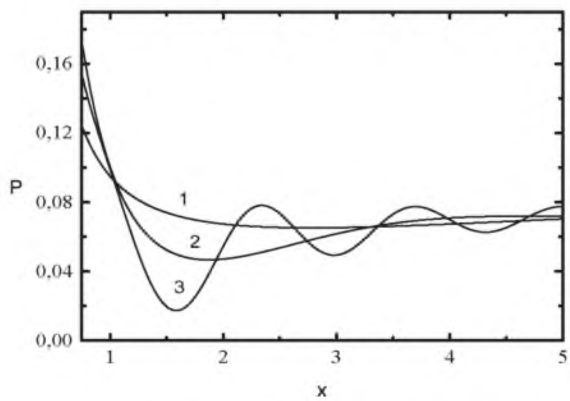

Fig. 2. Spectral–angular distribution of a relativistic particle X-ray emission from a thin layer of a medium. $\omega dN/(d\omega\, d^2\Theta) = (e^2/\pi^2)\gamma^2 P$, $x = \omega/\gamma\omega_0$, $y = \omega_0 L/2\gamma$, $z = \gamma\Theta = 0.8$. Curve 1: $y = 0,5$; curve 2: $y = 1$; curve 3: $y = 3$.

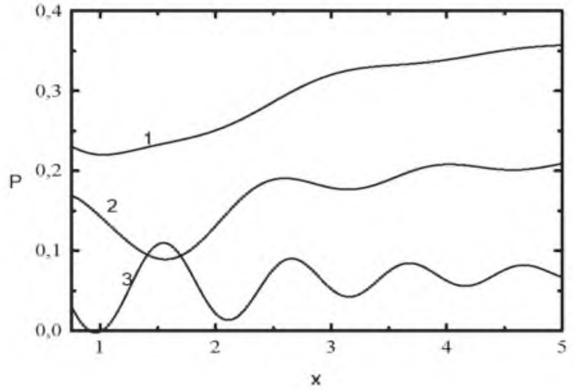

Fig. 3. The same as in Fig. 2 but for $y = 4$. Curve 1: $z = 0,1$; curve 2: $z = 0,4$; curve 3: $z = 0,8$.

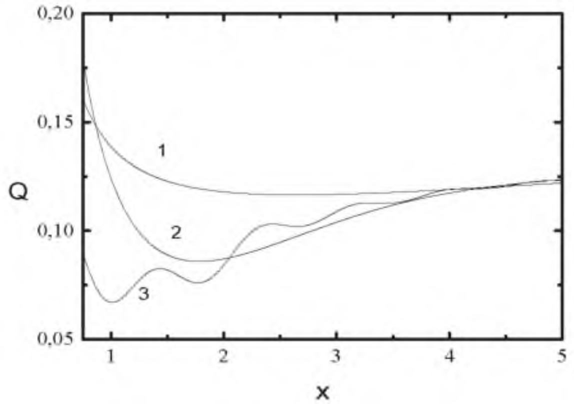

Fig. 4. The total spectrum of X-ray emission by relativistic particle from a thin layer of a medium. $\omega dN/d\omega = (e^2/\pi)Q$, $\gamma\Theta_c = 1$, $x = \omega/(\gamma\omega_0)$. Curve 1: $y = 0,5$; curve 2: $y = 1$; curve 3: $y = 4$.

5. On the spectrum of emitted photons with very small energies

Let us use the general result (12) for a discussion of the observed effect [7] consisting in the essential growth of emission spectral density in the small frequency range realized in the process of high energy particle emission in a very thin layer of a medium. The performed analysis shows that the main contribution to emission yield within the small frequency range $x < 1$ (more exactly $x^2 \ll 1$) is described by the first and second items in the general formula (12) corresponding to ordinary transition radiation and bremsstrahlung. In the condition under consideration the function Q has the simple form

$$Q \approx \gamma^4 \Theta_c^4 \left(1 - \cos\frac{\omega_0^2 L}{2\omega}\right) + \frac{L}{L_0}\gamma^2\Theta_c^2 \equiv Q^{TR} + Q^{BR}, \quad (15)$$

that is valid for collimated radiation ($\gamma^2\Theta_c^2 \ll 1$).

Two effects follow from this formula. The first of them is connected with the difference between angular distributions of transition radiation and bremsstrahlung (see (15)). Due to this difference, the relative contribution of the mentioned emission mechanisms depends strongly on the ratio between two parameters: $\gamma^2\Theta_c^2$ and L/L_0. It is easy to see that the transition radiation relative contribution located in the small frequency range increases when increasing of $\gamma\Theta_c$ or decreasing of L/L_0 is in agreement with the experimental results [7].

As it has been noted in [7], the spectrum shape was not changed with the decreasing of photon collimator angular size. This effect may be explained taking into account the initial angular divergence of an emitting electron beam. Such a divergence $\sqrt{\langle\Theta_0^2\rangle}$ causes the appearance of an additional item in the formula (15)

$$Q^{Ad} = \gamma^2\Theta_c^2 \frac{2\gamma^2\langle\Theta_0^2\rangle}{1+x^2}\left(1 - \cos\frac{\omega_0^2 L}{2\omega}\right) \quad (16)$$

as it has been shown in [5]. The formulae (15) and (16) show that the spectrum shape does not depend on the collimator acceptance Θ_c in the case $\Theta_c^0 < \langle\Theta_0^2\rangle$.

The second effect is connected with the influence of the density effect on the emission formation length (for collimated radiation) in a medium

$$l_{coh} \approx \frac{2\gamma^2}{\omega}\frac{1}{1+\gamma^2\omega_0^2/\omega^2} = \frac{2\gamma}{\omega_0}\frac{1}{x+x^{-1}}. \quad (17)$$

The formula (17) shows that the condition of transition radiation suppression due to the interference between waves emitted from *in* and *out* surfaces of a target $L/l_{coh} = y(x+x^{-1}) \ll 1$ is not valid in the total frequency range. In the case of

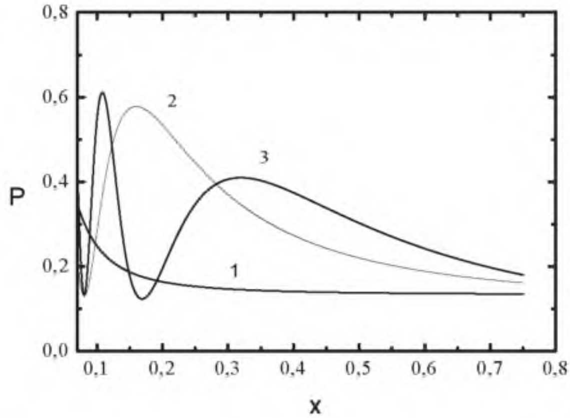

Fig. 5. Spectrum of a relativistic particle X-ray emission in the small frequency range. $\omega dN/d\omega = (e^2/\pi)Q$, $\gamma\Theta_c = 1$, $x = \omega/(\gamma\omega_0)$. Curve 1: $y = 0,1$; curve 2: $y = 0,5$; curve 3: $y = 1$.

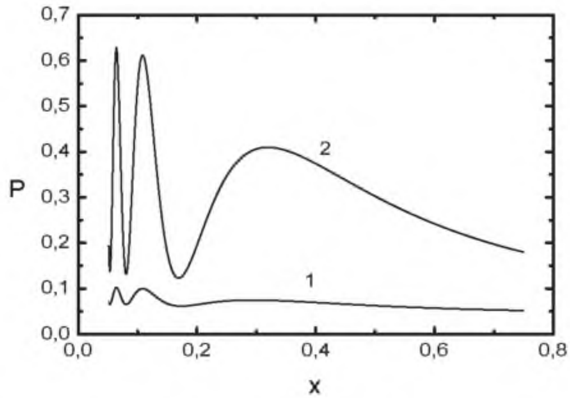

Fig. 6. The same as in Fig. 5, but for $y = 1$. Curve 1: $\gamma\Theta_c = 0,5$; curve 2: $\gamma\Theta_c = 1$.

very thin target $y \ll 1$, the transition radiation contribution is suppressed by the considered mechanism within the range $y < x < 1/y$ only.

The dependence $Q(x)$ calculated by the general formulae (12) and (14) is presented in Figs. 5 and 6 for the different values of the parameters $\gamma\Theta_c$ and y.

6. Conclusion

The study of this work gives the theoretical ground for an explanation of the peculiarities observed recently in the spectral–angular distribution of X-ray emission by relativistic electrons crossing a thin layer of a medium.

Suppression of the Ter-Mikaelian effect in a relativistic electron X-ray bremsstrahlung may be observed in the condition when a target thickness is smaller than an emission formation length.

Interference between bremsstrahlung and transition radiation of a relativistic particle crossing a thin layer of amorphous medium causes the manifestation of strong oscillations in an emission spectral–angular distribution.

Growth of an emission yield in a small frequency range observed in the case of a relativistic particle interaction with a very thin target may be explained by the contribution of transition radiation.

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