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Polarization and Diffraction Peculiarities of Optical Media Possessing Simultaneously the Properties of Right and Left Substances

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Abstract—With the simultaneous presence of positive and negative components both in the permittivity tensor ε_{ij} and in the tensor of magnetic permeability μ_{ij}, the propagation of an electromagnetic wave in optically homogeneous anisotropic media, and in periodically inhomogeneous media with a spiral structure is considered. It is shown that depending on polarization, such media exhibit the properties of both dextrorotatory and levorotatory substances. It is shown also that the medium with a spiral periodic structure may have the property of total diffraction reflection of the wave of any polarization. In the case of the presence of different signs of components of ε_{ij} and μ_{ij} in the spiral medium, it is revealed that two regions of diffraction reflection are formed, corresponding to the two waves with circular polarization with mutually opposite rotation of the polarization.

Keywords: spiral medium, magnetoactive medium, diffraction reflection

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1. INTRODUCTION

Until the second half of the last century optics studied media in which the components of the permittivity ε_{ij} and magnetic permeability μ_{ij} tensors are positive. The presence of a negative component in the mentioned tensors is perceived, at first glance, as a sign of the impossibility of wave propagation without strong attenuation, which, apparently, is caused by the fact that in the simplest case of isotropic nonmagnetic medium with the negative dielectric constant ($\mu_{ij} = \delta_{ij} > 0$, $\varepsilon_{ij} = \varepsilon\delta_{ij} < 0$), the wave propagation is impossible because the wave vector becomes imaginary.

For the first time an optically isotropic medium with $\varepsilon_{ij} = \varepsilon\delta_{ij}$, $\varepsilon < 0$ and $\mu_{ij} = \mu\delta_{ij}, \mu < 0$ was considered in [1], where it was also shown that the propagation of waves without attenuation is possible because the wave vector is real due to the reality of the refractive index $n = \pm\sqrt{\varepsilon\mu}$. $\mathbf{E}, \mathbf{H}, \mathbf{S}$ (where $\mathbf{S}$ is the Poynting vector) make up not the usual (right) triple of vectors, as in ordinary media, but the left. Currently, there is great interest in such media [2–5].

The possibility of wave propagation without attenuation in a nonmagnetic homogeneous anisotropic medium in which the tensor ε_{ij} has components of different signs is shown in [6], and a number of features of the optical properties of such media are established. As far as we know, the non-ordinary optical properties of such media were first paid attention in [7]. The results of systematic studies of media with different signs of the components are presented in [8, 9].

We also note the experimental work on determining the components with different signs of the tensor ε_{ij} [10] and the work [11] on the experimental confirmation of some conclusions of the theory developed in [8].

In [12–14], periodically inhomogeneous spiral structures were considered when all components ε_{ij} and μ_{ij} being negative, and a change in the sign of circular polarization of the wave undergoing diffraction reflection was revealed – when the sign of components ε_{ij} and μ_{ij} changes to the opposite. Note that the theory of propagation in spiral structures with positive values of material constants was developed in the second half of the last century [15, 16]. Amid the foregoing, the undoubted interest is the consideration of the optical properties of media in which both tensors ε_{ij} and μ_{ij} have both positive and negative components.

This work is devoted to the further study of the optical properties of such media.

Section 2 discusses the propagation of an electromagnetic wave in a homogeneous anisotropic medium with ε_{ij} and μ_{ij} tensors, which both have both positive and negative components. It is shown that in the case of wave propagation along one of the main directions of the commuting tensors ε_{ij} and μ_{ij}, such a medium is right for one of the two plane polarizations and the left one for the other plane polarization.

Section 3 shows that this property, a manifestation of the double right-handed rotation, is preserved in the case of magneto-optical activity, which results in the conversion of plane polarizations to the elliptic ones.

Section 4 considers the media with a spiral periodic structure. The most common representatives of such media are the cholesteric liquid crystals (CLC). One of the characteristic features of such media is the polarization-selective diffraction reflection of a wave incident on it with the right (left) circular polarization with the right (left) curling of the medium. As noted above, recent studies of the optical properties of magnetic media with a spiral structure have shown that when all tensor components ε_{ij} and μ_{ij} are negative, the sign of circular polarization at which the diffraction reflection takes place is changing [12, 13]. In the present paper, we consider the situation when both ε_{ij} and μ_{ij} tensors have both positive and negative components. It is shown that depending on the frequency of the incident wave in such a medium the diffraction reflection occurs for the incident wave of both right and left circular polarization. Thus, the spiral medium manifests itself as right-rotation at one frequency and left-rotation at another.

We will also demonstrate a situation of the absence of polarization selectivity in a medium with a spiral structure; in this case, there is a total reflection of the wave of any polarization.

2. ANISOTROPIC HOMOGENEOUS NON-GYROTROPIC MEDIA COMBINING THE PROPERTIES OF RIGHT AND LEFT SUBSTANCES

Let us consider the propagation of the plane monochromatic wave along the z axis

$$\mathbf{E}(z,t) = \mathbf{E}\exp i(k_z z - \omega t) \tag{1}$$

in a medium with the ε_{ij} and μ_{ij} tensors of the following form:

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}, \ \mu_{ij} = \begin{pmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{pmatrix}. \tag{2}$$

For the wave vectors, we obtain:

$$k_{1z}^2 = \frac{\omega^2}{c^2}\varepsilon_{xx}\mu_{yy}, \ k_{2z}^2 = \frac{\omega^2}{c^2}\varepsilon_{yy}\mu_{xx}. \tag{3}$$

In the wave with the index "1", the electric field has the x– component, the magnetic field has the y component, and in the wave with the index "2", the electric field has the y component, magnetic has the x component. The z components of Poynting vectors are equal to:

$$S_{1z} = \frac{c}{4\pi}E_{1x}H_{1y}, \quad S_{2z} = -\frac{c}{4\pi}E_{2y}H_{2x}. \tag{4}$$

Expressing H_{1y} through E_{1x}, and H_{2x} through E_{2y}, and using the equation

$$[\mathbf{kE}] = \frac{\omega}{c}\hat{\mu}\mathbf{H}, \tag{5}$$

we will obtain

$$k_{1z}E_{1x} = \frac{\omega}{c}\mu_{yy}\mathrm{H}_{1y}, \quad k_{2z}E_{2y} = \frac{\omega}{c}\mu_{xx}\mathrm{H}_{1x}. \tag{6}$$

With (4) and (6) we obtain:

$$S_{1z} = \frac{c}{4\pi} k_{1z} \frac{1}{\frac{\omega}{c}\mu_{yy}} E_{1x}^2, \quad S_{2z} = \frac{c}{4\pi} k_{2z} \frac{1}{\frac{\omega}{c}\mu_{xx}} E_{2y}^2. \tag{7}$$

Let

$$\varepsilon_{xx}, \mu_{yy} < 0, \quad \varepsilon_{yy}, \mu_{xx} > 0. \tag{8}$$

From (3), we have the following real expressions for waves propagating in the positive direction of the z axis for k_{1z} and k_{2z} equal to:

$$k_{1z} = \frac{\omega}{c}\sqrt{\varepsilon_{xx}\mu_{yy}}, \quad k_{2z} = \frac{\omega}{c}\sqrt{\varepsilon_{yy}\mu_{xx}}, \tag{9}$$

and for the z components of the Poynting vectors, we obtain:

$$S_{1z} = \frac{c}{4\pi}\frac{\sqrt{\varepsilon_{xx}\mu_{yy}}}{\mu_{yy}} E_{1x}^2, \quad S_{2z} = \frac{c}{4\pi}\frac{\sqrt{\varepsilon_{yy}\mu_{xx}}}{\mu_{xx}} E_{2y}^2. \tag{10}$$

According to (8), (9), and (10), we obtain

$$k_{1z} > 0, \quad S_{1z} < 0, \quad k_{2z} > 0, \quad S_{2z} > 0, \tag{11}$$

that is, in the wave with index 1 ($\varepsilon_{xx} < 0, \mu_{yy} < 0$) the vectors **k** and **S** are antiparallel, and in the wave with index 2 ($\varepsilon_{xx} > 0, \mu_{yy} > 0$) they are collinear. In other words, for a wave with one polarization (namely $\mathbf{E} = \mathbf{E}(E_x, 0, 0)$), the medium is left, and for another polarization ($\mathbf{E} = \mathbf{E}(0, E_y, 0)$), it is right:

$$\mathbf{S}_1 \uparrow\downarrow \mathbf{k}_1, \quad \mathbf{S}_2 \uparrow\uparrow \mathbf{k}_2. \tag{11a}$$

3. MAGNETOACTIVE MEDIA

Let us consider the manifestation of the optical properties of both right and left substances by an anisotropic homogeneous medium assuming the presence of an external constant uniform magnetic field directed along the z axis (one of the main directions of the commuting tensors ε_{ij} and μ_{ij}), which transforms plane polarizations in an anisotropic medium into the elliptic ones. We will proceed from the material equations

$$\mathbf{D} = \hat{\varepsilon}\mathbf{E} + i[\mathbf{g}\,\mathbf{E}], \quad \mathbf{g} = \mathbf{g}(0, 0, g_z), \quad \mathbf{B} = \hat{\mu}\mathbf{H}, \tag{12}$$

assuming magnetic permeability different from δ_{ij}. The components of the wave equation will have the form:

$$\left(\frac{\omega^2}{c^2}\varepsilon_{xx}\mu_{yy} - k_z^{\,2}\right)E_x - i\frac{\omega^2}{c^2}\mu_{yy}g_z E_y = 0, \quad i\frac{\omega^2}{c^2}\mu_{xx}g_z E_x + \left(\frac{\omega^2}{c^2}\varepsilon_{yy}\mu_{xx} - k_z^{\,2}\right)E_y = 0. \tag{13}$$

Limiting ourselves with the first-order terms $\frac{\mu_{xx}}{\varepsilon_{xx}}\frac{g_z^2}{\varepsilon_{xx}\mu_{yy} - \varepsilon_{yy}\mu_{xx}}$ and $\frac{\mu_{yy}}{\varepsilon_{yy}}\frac{g_z^2}{\varepsilon_{xx}\mu_{yy} - \varepsilon_{yy}\mu_{xx}}$,that is, assuming that the circular refraction induced by an external magnetic field is small as compared to the linear refraction, from (13) we obtain:

$$k_{1z} = \frac{\omega}{c}\sqrt{\varepsilon_{xx}\mu_{yy}}\left(1 + \frac{\mu_{xx}}{\varepsilon_{xx}}\frac{g_z^2}{\varepsilon_{xx}\mu_{yy} - \varepsilon_{yy}\mu_{xx}}\right), \quad k_{2z} = \frac{\omega}{c}\sqrt{\varepsilon_{yy}\mu_{xx}}\left(1 + \frac{\mu_{yy}}{\varepsilon_{yy}}\frac{g_z^2}{\varepsilon_{xx}\mu_{yy} - \varepsilon_{yy}\mu_{xx}}\right). \tag{14}$$

In (14) at $g_z = 0$, the coefficient k_{1z} coincides with the solution of the first, and k_{2z} of the second of equations (13). Substituting the expression k_{1z} in the first of equations (13), and k_{2z} in the second, we obtain the following relations determining the polarization of waves:

$$E_{1y} = i\frac{2\mu_{xx}g_z}{\varepsilon_{xx}\mu_{yy} - \varepsilon_{yy}\mu_{xx}}E_{1x}, \quad E_{2x} = i\frac{2\mu_{yy}g_z}{\varepsilon_{xx}\mu_{yy} - \varepsilon_{yy}\mu_{xx}}E_{2y}. \tag{15}$$

For z-components of the Poynting vectors, we obtain:

$$S_{1z} = \frac{c}{4\pi}\left(E_{1x}H_{1y} - E_{1y}H_{1x}\right), \quad S_{2z} = \frac{c}{4\pi}\left(E_{2x}H_{2y} - E_{2y}H_{2x}\right). \tag{16}$$

Using relations (15) and the equation $[\mathbf{k}\,\mathbf{E}] = \frac{\omega}{c}\hat{\mu}\mathbf{H}\pi$, let us express S_{1z} and S_{2z} through that component of the electric field in a given wave, which exists in the absence of magneto-optical activity (this is E_{1x} in the k_{1z} and E_{2y} in the wave with the z component of the wave vector k_{1z}). We obtain:

$$\begin{aligned} S_{1z} &= \frac{c}{4\pi\mu_{yy}}\left[1 - \frac{(\mu_{xx}g_z)^2}{(\varepsilon_{xx}\mu_{yy} - \varepsilon_{yy}\mu_{xx})^2}\frac{\mu_{yy}}{\mu_{xx}}\right]\sqrt{\varepsilon_{xx}\mu_{yy}}\left(1 + \frac{\mu_{xx}}{\varepsilon_{xx}}\frac{g_z^{\ 2}}{(\varepsilon_{xx}\mu_{yy} - \varepsilon_{yy}\mu_{xx})^2}\right)E_{1x}^2, \\ S_{2z} &= \frac{c}{4\pi\mu_{xx}}\left[1 - \frac{(\mu_{xx}g_z)^2}{(\varepsilon_{xx}\mu_{yy} - \varepsilon_{yy}\mu_{xx})^2}\frac{\mu_{xx}}{\mu_{yy}}\right]\sqrt{\varepsilon_{yy}\mu_{xx}}\left(1 - \frac{\mu_{yy}}{\varepsilon_{yy}}\frac{g_z^{\ 2}}{(\varepsilon_{xx}\mu_{yy} - \varepsilon_{yy}\mu_{xx})^2}\right)E_{1y}^2. \end{aligned} \tag{17}$$

Upon obtaining (17), it was believed that the expressions that stand for unity are small compared to unity. The second terms in square brackets in each of relations (17) are caused by the appearance of new field components, that is, the transformation of plane polarizations into the elliptic ones and the second terms in parentheses are because of the influence of magneto-optical activity on the refractive index. The basic cause of both effects is, surely, an external magnetic field.

According to (17), the medium property indicated in (11a) also remains when the polarization changes under the influence of a magnetic field, because, according to (14) and (17), we again have $\mathbf{S} \uparrow\uparrow \mathbf{k}$ for one of the waves and $\mathbf{S} \uparrow\downarrow \mathbf{k}$ for the other.

4. THE MEDIA WITH THE SPIRAL STRUCTURE

As indicated in the Introduction, in [12, 13] the diffraction reflection was considered in media with a supramolecular spiral structure (which are cholesteric liquid crystals), in which all the components of the tensors ε_{ij} and μ_{ij} in the plane perpendicular to the axis of swirling, are negative. Such media exhibit the optical properties of left-handed substances, which means the following: the diffraction reflection in a medium with swirling along the right (left) helix is experienced by a wave incident on the medium boundary and polarized along the left (right) circle. Below we consider media in which both the—ε_{ij} and μ_{ij} tensors have both positive and negative components. Recall that in well-known spiral media, the cholesteric liquid crystals, the right twisting of the circular polarization of the incident wave undergoing diffraction reflection coincides with the spiral along which the medium is twisted, that is, in a medium twisted along the right (left) spiral, the incident wave with right (left) circular polarization experiences the diffraction reflection.

Following the Oseen's approach [17] applied to nonmagnetic media, let us pass in the wave equation to the field components related to the x and y axes, halving the angles between the principal directions of the tensor ε_{ij} and the principal directions of the tensor μ_{ij} lying in planes perpendicular to the axis of helicity (z axis) for any z (Fig. 1).

For such a medium, the dispersion equation takes the form [8]:

$$K^4 - PK^2 - Q = 0, \tag{18}$$

where

$$\begin{aligned} Q &= -a^4 + \frac{\omega^2}{c^2}a^2\left[(\varepsilon_1\mu_1 + \varepsilon_2\mu_2)(\sin^4 W + \cos^4 W) + 2(\varepsilon_1\mu_2 + \varepsilon_2\mu_1)\sin^2 W\cos^2 W\right] - \frac{\omega^4}{c^4}\varepsilon_1\varepsilon_2\mu_1\mu_2, \\ P &= 2a^2 + \frac{\omega^2}{c^2}a^2\left[(\varepsilon_1\mu_1 + \varepsilon_2\mu_2)\sin^2 2W + (\varepsilon_1\mu_2 + \varepsilon_2\mu_1)\cos^2 2W\right]. \end{aligned} \tag{19}$$

Here ε_1 and ε_2 are the principal values of the permittivity tensor in planes perpendicular to the axis of twist of the medium, μ_1 and μ_2 are the same but for the magnetic tensor, $2W$ is the angle that the pair of principal directions of the tensor ε_{ij} lying in the plane $z = \text{const}$ is turned from the pair of principal directions of the tensor μ_{ij} lying in the same plane. With a change in z, both pairs rotate by the same angle $a\Delta z$ around the z axis (where $a = 2\pi/\sigma$, σ is the pitch of the spiral along which the medium is twisted).

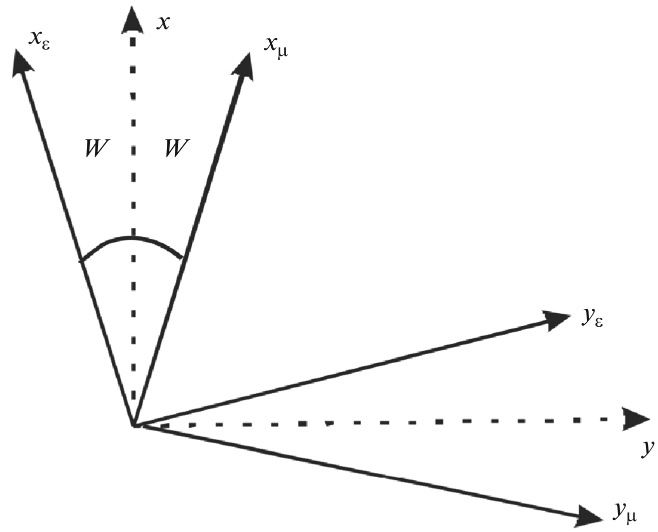

Fig. 1. To generalize the Oseen's approach to the transformation of the coordinate system: x_ε, y_ε are the main directions of the tensor ε_{ij}, and x_μ, y_μ are the main directions of the tensor μ_{ij}. All directions indicated in the figure are lying in a plane perpendicular to the direction around which the medium is twisted.

The diffraction reflection occurs at frequencies at which $K = 2\pi/\lambda' = 0$ [15], where λ' is the spatial period of the field in the x, y system, which rotates along with the principal directions of the tensors ε_{ij} and μ_{ij} in the plane perpendicular to the axis of the medium twisting: diffraction reflection occurs when the spatial period of the field in the laboratory system coincides with the period of the spiral structure. If these periods coincide, the period of the field in the system x, y that rotates with the structure will be infinite [8, 17]:

$$K = 0. \tag{20}$$

Substituting $K = 0$ in (18), we have $Q = 0$, and using the expression Q from (11), for the diffraction reflection frequencies we obtain:

$$\frac{\omega}{c}\sqrt{\varepsilon_{xx}\mu_{xx}} = a, \quad \frac{\omega}{c}\sqrt{\varepsilon_{yy}\mu_{yy}} = a. \tag{21}$$

When ε_{xx}, μ_{xx}, and ε_{yy}, are positive, then, as is known, the sign of the helicity of the twisting medium coincides with the sign of that circular polarization at which the diffraction reflection takes place [8]. If all ε_{ij} and μ_{xx} are negative, then the diffraction reflection takes place at the same frequencies but with the opposite sign of the circular polarization of the incident wave [12, 13]. Therefore, for $\varepsilon_{yy}, \mu_{xx} > 0$, and $\varepsilon_{xx}, \mu_{yy} < 0$, the diffraction reflection at the frequency ω_1 determined by relations (21) will take place with right (left) circular polarization in a right-swirling medium, and at the frequency ω_2 with reverse circular polarization.

Thus, a right-swirled (left-swirled) medium will reflect diffractively a wave with the right (left) circular polarization at one frequency $ac/\sqrt{\varepsilon_{xx}\mu_{xx}}$, and a wave with left (right) circular polarization at another frequency $ac/\sqrt{\varepsilon_{yy}\mu_{yy}}$, that is, this medium, depending on the frequency, can be considered both right and left.

Note that the roots (21) of equation (18) correspond to the case when $W = 0$. In the general case $W \neq 0$, these roots will depend on W, that is, the width and position of the diffraction reflection region will depend on W.

A situation of coincidence of the roots of equation (18) is possible – which means the coincidence of two frequencies, on one of which the wave with right circular polarization experiences diffraction reflection, on the other the left. Such a coincidence will mean that the medium reflects diffractively a wave with arbitrary polarization because a wave with arbitrary polarization can be decomposed into a superposition of waves with left and right circular polarizations.

5. CONCLUSIONS

One and the same medium, in which ε_{ij} and μ_{ij} tensors have both positive and negative components depending on the polarization and frequency, can manifest itself both the right substance and the left one. Such an extraordinary optical property is demonstrated by the example of anisotropic homogeneous media in the absence and presence of gyrotropy and periodically inhomogeneous media with a spiral structure. A similar medium with a spiral structure can reflect diffractively a wave with arbitrary polarization. The width of the region of diffraction reflection depends not only on ε_{ij} and μ_{ij}, and the pitch of the spiral, but also on the angle between the main directions of the tensor ε_{ij}, on the one hand, and the tensor μ_{ij}, on the other hand.

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CONFLICT OF INTEREST

The authors declare no conflict of interest.

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