EFFECT OF MUTUAL ORIENTATION OF THE LATTICE OF A SINGLE-CRYSTAL RADIATOR AND ITS OUTER SURFACE ON X-RAY TRANSITION RADIATION CHARACTERISTICS

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X-ray transition radiation (TR) of relativistic electrons crossing a single crystal target in the Bragg scattering geometry is considered within the framework of a dynamic diffraction theory. Analytical expressions are obtained for the TR spectral-angular distribution taking into account crystal surface orientation with respect to atomic planes. It is shown that at a fixed angle of electron incidence onto a system of atomic planes of the crystal the TR spectral-angular characteristics in the region of Bragg frequencies strongly depend on the orientation of the entrance surface of the target.

INTRODUCTION

When a fast charged particle crosses the interface between two different media, transition radiation (TR) is generated [1]. In the case where a charged particle crosses a crystal plate, there is interference of the waves emitted on its entrance and exit surfaces. It should be noted that TR waves emitted on the entrance surface of the target might be scattered by the system of atomic planes of the crystal, resulting in a change in the conditions of the above-mentioned interference [2]. The latter work discussed a crystal whose atomic planes are parallel to its surface.

The purpose of this work is to study the effect of orientation of the entrance surface of a single-crystal target with respect to the atomic crystal planes on the spectral-angular density of TR.

The work performs a detailed theoretical analysis of the TR waves in the Bragg scattering geometry. Using a dynamic diffraction theory [3], spectral-angular distributions of TR photons are obtained. It is shown that for a predetermined angle of electron incidence onto the system of atomic planes of the crystal in the Bragg geometry the interference of waves from the entrance and exit surfaces would strongly depend on the orientation of the outer surface, which is demonstrated in a sharp change of TR spectral-angular properties in the region of the Bragg scattering frequency. This implies that by cutting the single-crystal target in a variety of ways we could vary TR spectral-angular properties. This effect, which is similar to a well-known effect for free x-ray waves in a crystal [3], is accounted for by the variation in the frequency region of the complete reflection of waves generated on the entrance surface of the crystal by the system of its atomic planes. It is in this region of frequencies that the wave vectors assume complex values, which for the forward radiation corresponds to an exponential attenuation of wave intensity as they penetrate inside the crystal.

This work discusses the limiting cases of transformation of the formulas obtained into those reported in other works.

GENERAL EXPRESSIONS FOR SPECTRAL-ANGULAR DISTRIBUTION OF TRANSITION RADIATION IN A DYNAMIC APPROXIMATION

Let us consider radiation of a fast charged particle crossing a crystal of depth L at a constant velocity $\mathbf{V}$ (Fig. 1). In solving this problem, consider an equation for a Fourier image of the electromagnetic field

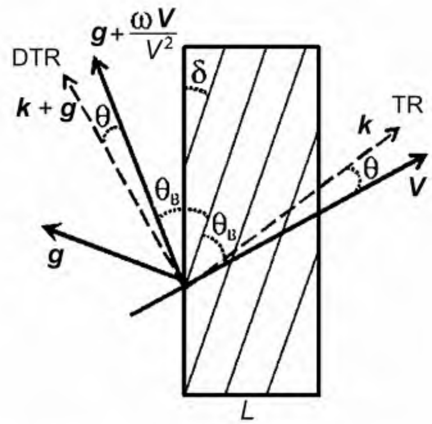

Fig. 1. Geometry of the radiation process.

$$\boldsymbol{E}(\boldsymbol{k},\omega)=\int dt d^3r \boldsymbol{E}(\boldsymbol{r},t)\exp\left(i\omega t-i\boldsymbol{k}\boldsymbol{r}\right). \tag{1}$$

Since the field of a relativistic particle could to a high accuracy be considered transverse, the incident $\boldsymbol{E}_0(\boldsymbol{k},\omega)$ and diffracted $\boldsymbol{E}_1(\boldsymbol{k},\omega)$ electromagnetic waves are determined by two amplitudes with different transverse polarization values

$$\begin{aligned}\boldsymbol{E}_0(k,\omega)&=E_0^{(1)}(\boldsymbol{k},\omega)\boldsymbol{e}_0^{(1)}+E_0^{(2)}(\boldsymbol{k},\omega)\boldsymbol{e}_0^{(2)},\\ \boldsymbol{E}_1(k,\omega)&=E_1^{(1)}(\boldsymbol{k},\omega)\boldsymbol{e}_1^{(1)}+E_1^{(2)}(\boldsymbol{k},\omega)\boldsymbol{e}_1^{(2)}.\end{aligned} \tag{2}$$

The unit polarization vectors $\boldsymbol{e}_0^{(1)},\boldsymbol{e}_0^{(2)},\boldsymbol{e}_1^{(1)}$ and $\boldsymbol{e}_1^{(2)}$ are selected in the following way. Vectors $\boldsymbol{e}_0^{(1)}$ and $\boldsymbol{e}_0^{(2)}$ are perpendicular to vector $\boldsymbol{k}$, while vectors $\boldsymbol{e}_1^{(1)}$ and $\boldsymbol{e}_1^{(2)}$ are parallel to vector $\boldsymbol{k}+\boldsymbol{g}$. Note that vectors $\boldsymbol{e}_0^{(2)}$ and $\boldsymbol{e}_1^{(2)}$ lie in the plane of vectors $\boldsymbol{k}$ and $\boldsymbol{k}+\boldsymbol{g}$ (π-polarization), while vectors $\boldsymbol{e}_0^{(1)}$ and $\boldsymbol{e}_1^{(1)}$ are perpendicular to this plane (σ–polarization), and $\boldsymbol{g}$ is the reciprocal lattice vector determining the system of reflecting atomic planes of the crystal. Using a two-wave approximation of the dynamic diffraction theory, let us write a well-known system of equations for a Fourier image of the electromagnetic field [4]

$$\begin{cases}(\omega^2(1+\chi_0)-k^2)E_0^{(s)}+\omega^2\chi_{-g}C^{(s)}E_1^{(s)}=8\pi^2 ie\omega\theta VP^{(s)}\delta(\omega-\boldsymbol{k}\boldsymbol{V}),\\ \omega^2\chi_g C^{(s)}E_0^{(s)}+(\omega^2(1+\chi_0)-(\boldsymbol{k}+\boldsymbol{g})^2)E_1^{(s)}=0,\end{cases} \tag{3}$$

where χ_g and χ_{-g} are the coefficients of Fourier expansion of dielectric susceptibility with respect to the reciprocal lattice vectors $\boldsymbol{g}$

$$\chi(\omega,\boldsymbol{r})=\sum_g\chi_g(\omega)e^{i\boldsymbol{g}\boldsymbol{r}}=\sum_g\left(\chi'_g(\omega)+i\chi''_g(\omega)\right)e^{i\boldsymbol{g}\boldsymbol{r}}. \tag{4}$$

The present work also addresses a crystal with the symmetry $(\chi_g=\chi_{-g})$. Here χ_g is determined by the expression

$$\chi_{\boldsymbol{g}} = \chi_0 \left(F(g)/Z \right) \left(S(\boldsymbol{g})/N_0 \right) \exp\left(-\frac{1}{2} g^2 u_\tau^2 \right), \tag{5}$$

where $\chi_0 = \chi_0' + i\chi_0''$ is the average dielectric susceptibility, $F(g)$ is the form factor of an atom containing Z electrons, $S(\boldsymbol{g})$ is the structural factor of a unit cell containing N_0 atoms, and u_τ is the r.m.s. amplitude of thermal vibrations of crystal atoms.

Quantities $C^{(s)}$ and $P^{(s)}$ have been determined in the system of equations (3) as follows:

$$C^{(s)} = \boldsymbol{e}_0^{(s)} \boldsymbol{e}_1^{(s)}, \quad C^{(1)} = 1, \quad C^{(2)} = \left| \cos 2\theta_{\mathrm{B}} \right|,$$

$$P^{(s)} = \boldsymbol{e}_0^{(s)} (\boldsymbol{\rho}/\rho), \quad P^{(1)} = \sin\varphi, \quad P^{(2)} = \cos\varphi. \tag{6}$$

Here $\boldsymbol{\rho} = \boldsymbol{k} - \omega \boldsymbol{V}/V^2$ is the component of the virtual photon pulse perpendicular to the particle velocity $\boldsymbol{V}$ ($\rho = \omega\theta/V$, where $\theta \ll 1$ is the angle between $\boldsymbol{k}$ and $\boldsymbol{V}$), θ_{B} is the angle between the electron velocity and the system of crystallographic planes (Bragg angle), φ is the azimuthal emission angle measured from the plane formed by vectors $\boldsymbol{V}$ and $\boldsymbol{g}$, and the value of the reciprocal lattice vector is determined by the expression $g = 2\omega_{\mathrm{B}} \sin\theta_{\mathrm{B}}/V$, where ω_{B} is the Bragg frequency. The system of equations (3) for $s = 1$ describes σ-polarized fields, and for $s = 2$ – π-polarized fields.

Solving the dispersion equation for x-ray waves in a crystal, which follows from the system of equations (3),

$$(\omega^2(1+\chi_0) - k^2)(\omega^2(1+\chi_0) - (\boldsymbol{k}+\boldsymbol{g})^2) - \omega^4 \chi_{-\boldsymbol{g}} \chi_{\boldsymbol{g}} C^{(s)2} = 0, \tag{7}$$

by conventional methods of the dynamic theory [3], we obtain an expression for wave vectors for the incident and diffracted waves

$$k^{(i,s)} = \omega\sqrt{1+\chi_0} + \frac{\omega\left|\chi_{\boldsymbol{g}}'\right| C^{(s)}}{2\varepsilon} \left(\xi^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2} \mp K^{(s)} \right), \tag{8a}$$

$$k_{\boldsymbol{g}}^{(i,s)} = \left|\boldsymbol{k}+\boldsymbol{g}\right| = \omega\sqrt{1+\chi_0} + \frac{\omega\left|\chi_{\boldsymbol{g}}'\right| C^{(s)}}{2} \left(\xi^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2} \pm K^{(s)} \right). \tag{8б}$$

Here we introduced the following notations:

$$K^{(s)} = \sqrt{\xi^{(s)2} - \varepsilon - \rho^{(s)} \left((1+\varepsilon)\xi^{(s)} + 2\frac{\chi_{\boldsymbol{g}}'' C^{(s)}}{\chi_0''} \frac{\left|\chi_{\boldsymbol{g}}'\right|}{\chi_{\boldsymbol{g}}'} \varepsilon \right) i - \rho^{(s)2} \left(\frac{(1+\varepsilon)^2}{4} - \frac{\chi_{\boldsymbol{g}}''^2 C^{(s)^2}}{\chi_0''^2} \varepsilon \right)},$$

$$\xi^{(s)} = \eta^{(s)} + \frac{\beta^{(s)}(1+\varepsilon)}{2}, \quad \rho^{(s)} = \frac{\chi_0''}{\left|\chi_{\boldsymbol{g}}'\right| C^{(s)}}, \quad \beta^{(s)} = \frac{1}{\left|\chi_{\boldsymbol{g}}'\right| C^{(s)}} \left(\theta^2 + \gamma^{-2} - \chi_0' \right),$$

$$\eta^{(s)} = \frac{(\boldsymbol{k}+\boldsymbol{g})^2 - k^2}{2\omega^2 \left|\chi_{\boldsymbol{g}}'\right| C^{(s)}} = \frac{2\sin^2\theta_{\mathrm{B}}}{V^2 \left|\chi_{\boldsymbol{g}}'\right| C^{(s)}} \left(\frac{\omega_{\mathrm{B}}(1+\theta\cos\varphi\cot\theta_{\mathrm{B}})}{\omega} - 1 \right), \tag{9}$$

where $\varepsilon = \sin(\theta_{\mathrm{B}} - \delta)/\sin(\theta_{\mathrm{B}} + \delta)$, with δ being the angle between the entrance surface of the target and the crystallographic plane.

In expressions (8), indices $i = 1, 2$ determine two branches of x-ray waves propagating in the crystal. From formulas (8) it is evident that in the case where the surface of the target and the reflecting crystallographic plane are parallel ($\varepsilon = 1$), the absolute values of wave vectors of the incident and diffracted waves coincide.

Since the inequality $2\sin^2\theta_B / V^2 \left|\chi'_g\right| C^{(s)} >> 1$ is satisfied in the region of x-ray frequencies, then $\eta^{(s)}(\omega)$ is a fast function of frequency ω, and it is, therefore, convenient for further analysis of the TR spectrum properties to consider $\eta^{(s)}(\omega)$ as a spectral variable characterizing frequency ω.

The solution to the first equation in the system (3) for the Coulomb field of a relativistic electron in vacuum ($\chi_{-g} = 0$) has the form

$$E_0^{(s)\text{vac}} = \frac{8\pi^2 ieV}{\omega} \frac{\theta P^{(s)}}{-\gamma^{-2} - \theta^2}, \tag{10a}$$

for the Coulomb field in the crystal

$$E_0^{(s)\text{cr}} = \frac{8\pi^2 ieV}{\omega} \frac{\theta P^{(s)}}{\chi_0 - \gamma^{-2} - \theta^2}. \tag{10b}$$

The radiation field formed on the entrance crystal surface has the form

$$E_0^{(s)\text{vac-cr}} = \frac{8\pi^2 ieV}{\omega} \theta P^{(s)} \left(\frac{1}{-\gamma^{-2} - \theta^2} - \frac{1}{\chi_0 - \gamma^{-2} - \theta^2} \right), \tag{10c}$$

and on the exit crystal surface it is

$$E_0^{(s)\text{cr-vac}} = \frac{8\pi^2 ieV}{\omega} \theta P^{(s)} \left(\frac{1}{\chi_0 - \gamma^{-2} - \theta^2} - \frac{1}{-\gamma^{-2} - \theta^2} \right). \tag{10d}$$

Then the forward electromagnetic field behind the crystal would be written as

$$E_{\text{Rad}}^{(s)\,TR} = E_0^{(s)\text{vac-cr}} U^{(s)} + E_0^{(s)\text{cr-vac}}$$

$$= \frac{8\pi^2 ieV}{\omega} \theta P^{(s)} \left(\frac{1}{\gamma^{-2} + \theta^2} - \frac{1}{\gamma^{-2} + \theta^2 - \chi_0} \right) \left(1 - U^{(s)}\right), \tag{11}$$

where the amplitude coefficient of propagation $U^{(s)}$ determines the amplitude of the TR field that passed through the crystal and appeared in the entrance surface of the crystal. Coefficient $U^{(s)}$ in the Bragg geometry considered here has the following form [3]:

$$U^{(s)} = \frac{-\sqrt{z^2 + q^{(s)}}}{(-z - \sqrt{z^2 + q^{(s)}}) e^{\frac{i\omega L}{2\cos(\psi_0)}\left(\chi'_0 - \theta^2 - \gamma^{-2} - z + \sqrt{z^2 + q^{(s)}}\right)} - (-z + \sqrt{z^2 + q^{(s)}}) e^{\frac{i\omega L}{2\cos(\psi_0)}\left(\chi'_0 - \theta^2 - \gamma^{-2} - z - \sqrt{z^2 + q^{(s)}}\right)}}, \tag{12}$$

with the following notations: $q^{(s)} = -C^{(s)2}\chi_g^2/\varepsilon$, $z = -\left(\alpha - (1+\varepsilon)\left(\chi_0 - \theta^2 - \gamma^{-2}\right)\right)/2\varepsilon$, and $\psi_0 = \pi/2 - (\theta_B + \delta)$ is the angle between the incident wave vector and the outer normal to the target of the surface. The amplitude coefficient (Eq.12) contains two branches of x-rays in the crystal.

The spectral-angular distribution of TR could be written as [4]

$$\frac{d^2W^{\mathrm{TR}}}{d\omega d\Omega} = \omega^2(2\pi)^{-6}\sum_{s=1}^{2}\left|E_{\mathrm{Rad}}^{(s)\,TR}\right|^2 . \tag{13}$$

Substituting (12) into (11), and then (11) into (13), we obtain an expression for the spectral-angular distribution of TR in the crystal

$$\omega\frac{d^2N^{\mathrm{TR}}}{d\omega d\Omega} = \frac{e^2}{\pi^2}\theta^2\sum_{s=1}^{2} P^{(s)2}\left(\frac{1}{\gamma^{-2}+\theta^2} - \frac{1}{\gamma^{-2}+\theta^2-\chi_0'}\right)^2$$

$$\times\left|1 + \frac{2K^{(s)}}{\left(\zeta^{(s)}-K^{(s)}\right)e^{\frac{i\omega L|\chi_g'|C^{(s)}}{2\cos(\psi_0)}\left(\frac{1}{\varepsilon}\left(\zeta^{(s)}+K^{(s)}\right)-\beta^{(s)}\right)} - \left(\zeta^{(s)}+K^{(s)}\right)e^{\frac{i\omega L|\chi_g'|C^{(s)}}{2\cos(\psi_0)}\left(\frac{1}{\varepsilon}\left(\zeta^{(s)}-K^{(s)}\right)-\beta^{(s)}\right)}}\right|^2 , \tag{14}$$

where $\zeta^{(s)} = \eta^{(s)} + \dfrac{\left(\beta^{(s)} - i\rho^{(s)}\right)(1+\varepsilon)}{2}$.

Expression (14) could be used for the investigation of manifestation of dynamic effects in transition radiation in a crystal of any thickness, taking into account absorption.

DEPENDENCE OF TR SPECTRAL-ANGULAR DISTRIBUTION ON TARGET ORIENTATION

Parameter $\varepsilon = \sin(\theta_{\mathrm{B}} - \delta)/\sin(\theta_{\mathrm{B}} + \delta)$ determines orientation of the entrance surface of the crystal target. The region of values $\varepsilon > 0$ corresponds to the Bragg orientation of the crystal. Note that the fraction of TR that is formed at the entrance surface, diffracting on the system of parallel crystallographic planes, leaves the crystal through the front boundary. For a fixed value of θ_{B}, parameter δ becomes negative as the angle of electron incidence $(\theta_{\mathrm{B}} + \delta)$ onto the target is decreased and begins to increase in its absolute value (limiting case $\delta \to -\theta_{\mathrm{B}}$), which results in an increase in ε. On the other hand, as the angle of incidence is increased, parameter ε is decreasing (limiting case $\delta \to \theta_{\mathrm{B}}$).

Let us consider the effect of crystal orientation on spectral-angular properties of TR in an asymptotic case of a thin crystal plate, where absorption could be neglected $(\chi_0'' = 0)$. Then expression (14) for the spectral-angular distribution of TR takes the form

$$\omega\frac{d^2N^{\mathrm{TR}}}{d\omega d\Omega} = \frac{e^2}{\pi^2}\theta^2\sum_{s=1}^{2} P^{(s)2}\left(\frac{1}{\gamma^{-2}+\theta^2} - \frac{1}{\gamma^{-2}+\theta^2-\chi_0'}\right)^2 T^{(s)},$$

where

$$T^{(s)}=\left[1+\frac{\xi^{(s)^2}-\varepsilon}{\xi^{(s)^2}-\varepsilon+\varepsilon\sin^2\left(b^{(s)}\frac{\sqrt{\xi^{(s)^2}-\varepsilon}}{\varepsilon}\right)}\left\langle 1-2\left(\cos\left(b^{(s)}\frac{\sqrt{\xi^{(s)^2}-\varepsilon}}{\varepsilon}\right)\cos\left(b^{(s)}\left(\frac{\xi^{(s)}}{\varepsilon}-\beta^{(s)}\right)\right)\right.\right.$$

$$\left.\left.+\frac{\xi^{(s)}}{\left(\xi^{(s)^2}-\varepsilon\right)^{1/2}}\sin\left(b^{(s)}\frac{\sqrt{\xi^{(s)^2}-\varepsilon}}{\varepsilon}\right)\sin\left(b^{(s)}\left(\frac{\xi^{(s)}}{\varepsilon}-\beta^{(s)}\right)\right)\right)\right\rangle\right]. \quad (15)$$

Here $\xi^{(s)}=\eta^{(s)}+\beta^{(s)}(1+\varepsilon)/2$ and $b^{(s)}=\omega L\left|\chi'_g\right|C^{(s)}/2\sin(\theta_{\rm B}+\delta)$ is the parameter that depends on crystal thickness and $T^{(s)}$ is the factor describing interference of radiation formed on both sides of the target. It is clear that formula (15) provides the major results of this work. It should be noted that in the case where the entrance surface is parallel to the system of crystallographic planes ($\delta=0$ or $\varepsilon=1$), expression (15) is converted into an expression for TR in a dynamic approximation obtained in Ref. [5].

Expression (15) is valid for all possible values of $\eta^{(s)}$ and is very different from the formula for TR from an amorphous target of the same thickness L. This difference is due to dynamic diffraction effects. It is substantial only in the vicinity of the Bragg frequency $\left|\eta^{(s)}+\beta^{(s)}(1+\varepsilon)/2\right|\le\varepsilon^{1/2}$. Outside this range, expression (15) assumes the form of a well-known expression for TR formed in an amorphous dielectric target [6]

$$\omega\frac{d^2N^{\rm TR}}{d\omega d\Omega}=2\frac{e^2}{\pi^2}\theta^2\sum_{s=1}^{2}P^{(s)2}\left(\frac{1}{\gamma^{-2}+\theta^2}-\frac{1}{\gamma^{-2}+\theta^2-\chi'_0}\right)^2$$
$$\times\left(1-\cos\left(\frac{\omega L}{2\sin(\theta_{\rm B}+\delta)}\left(\theta^2+\gamma^{-2}-\chi'_0\right)\right)\right). \quad (16)$$

Since we address here dynamic effects, we would be interested in the frequencies close to the Bragg frequency $\omega_{\rm B}$. From expression (16) it follows that the destructive interference of TR waves emitted from the entrance and exit surfaces of the crystal would totally suppress those far from the Bragg frequency, provided that [6]

$$\frac{\omega L}{2\sin(\theta_{\rm B}+\delta)}\left(\theta^2+\gamma^{-2}-\chi'_0\right)=2\pi n, \quad (17)$$

where n is a natural number.

Since expression (17) depends on the angle between the atomic planes and crystal surface δ, so variation in this angle for the condition of destructive interference maintained one should respectively vary either the angle of observation θ or the crystal thickness L.

Figure 2 shows the curves for the dependence of factor $T^{(s)}$ on spectral variable $\eta^{(s)}$, which were formed using formula (15) and describe a TR spectrum for certain polarization s, with the resonance condition (17) fulfilled for all of the curves. These curves had been constructed for different values of parameter ε at a fixed value of parameter $\beta^{(s)}$.

There is a frequency region of anomalous dispersion where TR waves emitted from the entrance surface are wholly reflected in the crystal from the atomic planes and do not propagate in the forward direction. The wave vector $k^{(i,s)}$ (see.

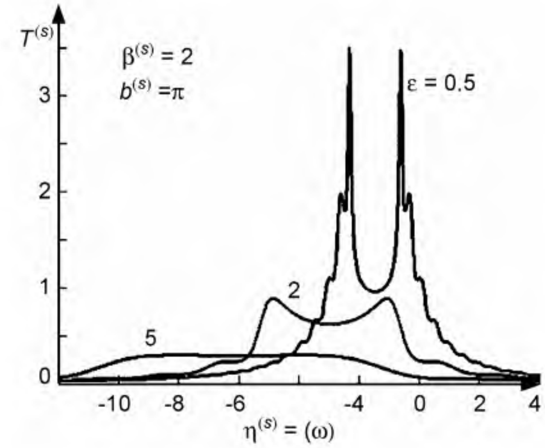

Fig. 2. Effect of crystal target orientation on TR spectrum at a fixed Bragg angle.

Eq. (8a)) in this region of TR from the target assumes complex values even in the absence of absorption $\left(\rho^{(s)} = 0\right)$. It is, therefore, evident that in this frequency range TR from the target would only consist of the waves emitted from the entrance surface. This frequency region is referred to as the region of complete reflection and is determined in accordance with an expression for $K^{(s)}$ from Eq. (9) as follows:

$$-\varepsilon^{1/2} - \beta^{(s)}(1+\varepsilon)/2 < \eta^{(s)} < \varepsilon^{1/2} - \beta^{(s)}(1+\varepsilon)/2 . \tag{18}$$

The width of this region is controlled by the value of $2\varepsilon^{1/2}$, and, which is worthy of mention, depends on the crystal orientation. It is evident from Fig. 2 that as parameter ε is increased, the spectral distribution of emission is shifted, the interference peaks tend to be lower, and for sufficiently large ε they practically diminish. As parameter ε is decreased, the region of complete reflection is decreased, resulting in the formation and growth of interference peaks and a decrease in the spectral width of TR.

SUMMARY

The work has presented a detailed theoretical analysis of TR waves in the Bragg scattering geometry. Using a dynamic diffraction theory, we have obtained expressions for the spectral-angular distribution of TR photon, which allow one to investigate the dependence of the spectral-angular distribution on the relative crystal lattice orientation and the entrance surface of a single crystal target. The investigations conducted have demonstrated that by varying the above-mentioned orientation one may noticeably change the spectral-angular properties of TR. This effect is due to a change in the region of anomalous dispersion (complete reflection). The results obtained could be applied to building of an x-ray source using transition radiation of a relativistic electron and to experimental investigations of other radiation mechanisms under the conditions where transition radiation is an interfering background.

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