Effect of Anomalous Photoabsorption in Parametric X-Ray Radiation under Asymmetric Reflection Conditions

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Abstract—Parametric X-ray radiation (PXR) of a relativistic electron crossing a monocrystalline plate is considered in the Laue scattering geometry. Expressions describing the spectral–angular distributions of PXR and diffracted transient radiation (DTR), which are formed in atomic planes arranged at an arbitrary angle δ to the surface of a crystalline plane (asymmetric reflection), are derived in the two-wave approximation of the dynamic diffraction theory. The conditions under which the effect of anomalously weak photoabsorption (Borman effect) is manifested most clearly are determined. This effect can considerably enhance the intensity of the sources of tunable quasi-monochromatic X rays, which are based on the PXR mechanism.

INTRODUCTION

Anomalously weak photoabsorption is one of dynamic effects in scattering of free electrons in a crystal. This effect was experimentally detected for the first time by Borman [1] in experiments on scattering of free X rays in a crystal. The physical essence of the effect lies in the formation of an incident standing wave scattered by X-ray waves, whose antinodes lie in the middle of the space between adjacent atomic planes, where the electron density of the crystal (and, hence, photoabsorption) is minimal. In this case, two waves are formed in the crystal, one of which is absorbed anomalously strongly and the other is absorbed anomalously weakly. The linear absorption coefficient for both waves has the form [2]

$$\mu = \mu_0\left(1 \pm C^{(s)}\frac{\chi''_{\mathbf{g}}}{\chi''_0}\right),$$

where the plus and minus signs correspond to anomalously strong and weak absorption, respectively; $\chi''_{\mathbf{g}}$ and χ''_0 are imaginary parts of the coefficients in the expansion of permittivity into a Fourier series in the reciprocal lattice vectors,

$$\chi(\omega, \mathbf{r}) = \sum_{\mathbf{g}}(\chi'_{\mathbf{g}}(\omega) + i\chi''_{\mathbf{g}}(\omega))e^{i\mathbf{gr}};$$

and $C^{(s)}$ is the polarization factor. Factor $C^{(1)} = 1$ for the σ polarization and $C^{(2)} = \cos 2\theta_B$ for the π polarization, where θ_B is the Bragg angle. It follows from the above formula that the Borman effect is manifested more clearly for the σ polarization and the necessary condition for its emergence is the fulfillment of the relation $\chi''_{\mathbf{g}} C^{(s)}/\chi''_0 \approx 1$.

It would be interesting to find out whether an analogous effect exists in parametric X-ray radiation (PXR) emerging during Bragg's diffraction of pseudophotons of the Coulomb field of a fast particle moving in a crystal [3–5]. It was shown earlier in [6] that the additional contribution to the PRX from a thick absorbing crystal target on the side of transient radiation generated by a fast particle at the inlet surface of the crystal and diffracted from the same atomic planes, which are responsible for the emergence of PXR, can substantially increase due to the Borman effect observed for free electrons. In [7–9], the Borman effect was predicted precisely for scattering of the pseudophoton field of an emitting particle in the Bragg and Laue scattering geometries. However, PXR and diffracted transient radiation (DTR) were analyzed in the above-mentioned publications [6–9] for symmetric reflection and the Borman effect was considered in the limiting case of a semi-infinite crystal; in actual experiments, this may lead to suppression of this effect due to multiple scattering of emitting particles from the target. For symmetric reflection, the surface of the crystalline target is parallel ($\delta = 0$) to the system of diffracting atomic planes in the Bragg scattering geometry and perpendicular ($\delta = \pi/2$) to this system in the Laue geometry.

It was shown in [10, 11] that the spectral–angular distributions for PXR and DTR noticeably depend on angle δ. Manifestation of the Borman effect in PXR was considered in [12] for the case of a semi-infinite crystal in the Bragg scattering geometry under asymmetric reflection conditions.

Here, we consider coherent X rays emitted by a relativistic electron passing through a monocrystalline plate in the Laue scattering geometry and derive the expressions for the spectral–angular distribution of PXR and DTR, as well as the term describing the interference of these two emission mechanisms in the general case of asymmetric reflection (i.e., taking into account different orientations of atomic planes of the crystal relative to its surface (angle δ)). The conditions under which the effect of anomalously weak photoabsorption is manifested most clearly are determined. In particular, it is shown that when asymmetry becomes stronger in one direction (angle δ increases), the path length for a PXR photon can be increased to a value exceeding the photoabsorption length in the case when the electron path length in the plate is so short that multiple scattering can be ignored; this leads to a clear manifestation of the Borman effect. If, however, the asymmetry increases in the opposite direction (angle δ decreases), this effect is manifested less strongly; however, not only the amplitude increases substantially (which is obvious since absorption becomes weaker), but the spectral width also increases, which is unexpected.

This result can be used for developing sources of tunable quasi-monochromatic X rays based on the PXR mechanism.

SPECTRAL–ANGULAR DISTRIBUTION OF RADIATION

Let us consider radiation emitted by a fast charged particle passing through a monocrystalline plate at a constant velocity $\mathbf{V}$ (Fig. 1). We will consider the equations for the Fourier transform of an electromagnetic field,

$$\mathbf{E}(\mathbf{k}, \omega) = \int dt d^3\mathbf{r}\mathbf{E}(\mathbf{r}, t)\exp(i\omega t - i\mathbf{k}\mathbf{r}). \quad (1)$$

Since the field of the relativistic particle can be treated as transverse to a high degree of accuracy, the incident ($\mathbf{E}_0(\mathbf{k}, \omega)$) and diffracted ($\mathbf{E}_g(\mathbf{k}, \omega)$) electromagnetic waves are determined by two amplitudes with different values of transverse polarization,

$$\begin{aligned}\mathbf{E}_0(\mathbf{k}, \omega) &= E_0^{(1)}(\mathbf{k}, \omega)\mathbf{e}_0^{(1)} + E_0^{(2)}(\mathbf{k}, \omega)\mathbf{e}_0^{(2)},\\ \mathbf{E}_\mathbf{g}(\mathbf{k}, \omega) &= E_\mathbf{g}^{(1)}(\mathbf{k}, \omega)\mathbf{e}_1^{(1)} + E_\mathbf{g}^{(2)}(\mathbf{k}, \omega)\mathbf{e}_1^{(2)}.\end{aligned} \quad (2)$$

Unit vectors of polarization $\mathbf{e}_0^{(1)}$, $\mathbf{e}_0^{(2)}$, $\mathbf{e}_1^{(1)}$, and $\mathbf{e}_1^{(2)}$ are chosen as follows. Vectors $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_0^{(2)}$ are perpendicular to vector $\mathbf{k}$, while vectors $\mathbf{e}_1^{(1)}$ and $\mathbf{e}_1^{(2)}$ are perpendicular to vector $\mathbf{k}_\mathbf{g} = \mathbf{k} + \mathbf{g}$. Vectors $\mathbf{e}_0^{(2)}$ and $\mathbf{e}_1^{(2)}$ lie in the plane of vectors $\mathbf{k}$ and $\mathbf{k}_\mathbf{g}$ (π polarization), while vectors $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_1^{(1)}$ are perpendicular to it (σ polarization), and $\mathbf{g}$ is the reciprocal lattice vector determining the system of reflecting planes of the crystal. The system of equations for the Fourier transform of the electromagnetic field in the two-wave approximation of the dynamic theory of diffraction has the form [13]

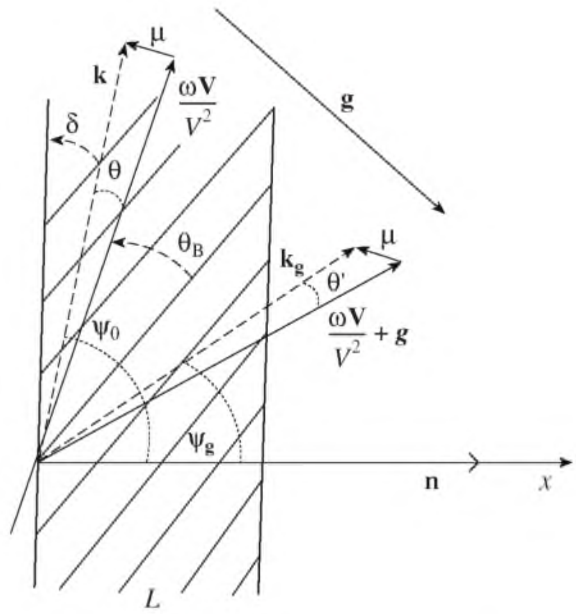

Fig. 1. Geometry of emission process: θ' is the emission angle, θ_B is the Bragg angle (angle between electron velocity $\mathbf{V}$ and atomic planes), δ is the angle between the surface and atomic planes of the crystal, and $\mathbf{k}$ and $\mathbf{k}_g$ are the wavevectors of incident and diffracted photon.

$$\begin{cases}(\omega^2(1+\chi_0) - k^2)E_0^{(s)} + \omega^2\chi_{-\mathbf{g}}C^{(s)}E_\mathbf{g}^{(s)}\\ = 8\pi^2 ie\omega\theta VP^{(s)}d(\omega - \mathbf{k}\mathbf{V}),\\ \omega^2\chi_\mathbf{g}C^{(s)}E_0^{(s)} + (\omega^2(1+\chi_0) - k_\mathbf{g}^2)E_\mathbf{g}^{(s)} = 0,\end{cases} \quad (3)$$

where $\chi_\mathbf{g}$ and $\chi_{-\mathbf{g}}$ are the coefficients of the Fourier expansion of the dynamic susceptibility of the crystal in reciprocal lattice vectors $\mathbf{g}$:

$$\chi(\omega, \mathbf{r}) = \sum_\mathbf{g}\chi_g(\omega)e^{i\mathbf{g}\mathbf{r}} = \sum_\mathbf{g}(\chi'_\mathbf{g}(\omega) + i\chi''_g(\omega))e^{i\mathbf{g}\mathbf{r}}. \quad (4)$$

We consider a symmetric crystal ($\chi_\mathbf{g} = \chi_{-\mathbf{g}}$); $\chi_\mathbf{g}$ is defined as

$$\chi_\mathbf{g} = \chi_0(F(g)/Z)(S(\mathbf{g})/N_0)\exp\left(-\frac{1}{2}g^2u_\tau^2\right), \quad (5)$$

where $\chi_0 = \chi'_0 + i\chi''_0$ is the mean dielectric susceptibility, $F(\mathbf{g})$ is the form factor of an atom containing Z electrons, $S(\mathbf{g})$ is the structural factor of a unit cell containing N_0 atoms, and u_τ is the root-mean-square amplitude of thermal oscillations of atoms in the crystal. We consider the X-ray frequency range ($\chi'_\mathbf{g} < 0$, $\chi'_0 < 0$).

The values of $C^{(s)}$ and $P^{(s)}$ in system (3) are given by

$$C^{(s)} = \mathbf{e}_0^{(s)}\mathbf{e}_1^{(s)}, \quad C^{(1)} = 1, \quad C^{(2)} = \cos 2\theta_B,$$
$$P^{(s)} = \mathbf{e}_0^{(s)}(\boldsymbol{\mu}/\mu), \quad P^{(1)} = \sin\varphi, \quad P^{(2)} = \cos\varphi, \tag{6}$$

where $\boldsymbol{\mu} = \mathbf{k} - \omega\mathbf{V}/V^2$ is the momentum component of a virtual photon, which is perpendicular to particle velocity $\mathbf{V}$ ($\mu = \omega\theta/V$, where $\theta \ll 1$ is the angle between vectors $\mathbf{k}$ and $\mathbf{V}$), θ_B is the angle between the electron velocity and the system of crystallographic planes (Bragg's angle), and φ is the azimuthal angle. Radiation is measured from the plane formed by vectors $\mathbf{V}$ and $\mathbf{g}$; the reciprocal lattice vector is defined as $g = 2\omega_B \sin\theta_B/V$, where ω_B is the Bragg frequency. The angle between vector $\frac{\omega\mathbf{V}}{V^2} + \mathbf{g}$ and wavevector $\mathbf{k}_\mathbf{g}$ of the diffracted wave is denoted by θ'. System of equations (3) describes σ-polarized fields for $s = 1$ and π-polarized fields for $s = 2$.

Let us solve the dispersion equation for X rays in a crystal,

$$(\omega^2(1+\chi_0) - k^2)(\omega^2(1+\chi_0) - k_\mathbf{g}^2) - \omega^4\chi_{-\mathbf{g}}\chi_\mathbf{g}C^{(s)^2} = 0, \tag{7}$$

which follows from system (3), using the standard methods of the dynamic theory [2].

We will seek the projections of wavevectors $\mathbf{k}$ and $\mathbf{k}_g$ onto the x axis coinciding with vector $\mathbf{n}$ (see Fig. 1) in the form

$$k_x = \omega\cos\psi_0 + \frac{\omega\chi_0}{2\cos\psi_0} + \frac{\lambda_0}{\cos\psi_0},$$
$$k_{\mathbf{g}x} = \omega\cos\psi_\mathbf{g} + \frac{\omega\chi_0}{2\cos\psi_\mathbf{g}} + \frac{\lambda_\mathbf{g}}{\cos\psi_\mathbf{g}}. \tag{8}$$

We will use the well-known relation connecting dynamic corrections λ_0 and $\lambda_\mathbf{g}$ for X-ray waves [2],

$$\lambda_\mathbf{g} = \frac{\omega\beta}{2} + \lambda_0\frac{\gamma_\mathbf{g}}{\gamma_0}, \tag{9}$$

where $\beta = \alpha - \chi_0\left(1 - \frac{\gamma_\mathbf{g}}{\gamma_0}\right)$, $\alpha = \frac{1}{\omega^2}(k_\mathbf{g}^2 - k^2)$, $\gamma_0 = \cos\psi_0$, $\gamma_\mathbf{g} = \cos\psi_\mathbf{g}$, ψ_0, is the angle between wavevector $\mathbf{k}$ of the incident wave and normal $\mathbf{n}$ to the plate surface, and $\psi_\mathbf{g}$ is the angle between wavevector $\mathbf{k}_\mathbf{g}$ and vector $\mathbf{n}$ (see Fig. 1). The magnitudes of vectors $\mathbf{k}$ and $\mathbf{k}_\mathbf{g}$ are given by

$$k = \omega\sqrt{1+\chi_0} + \lambda_0, \quad k_\mathbf{g} = \omega\sqrt{1+\chi_0} + \lambda_\mathbf{g}. \tag{10}$$

Substituting relations (8) onto (7) and taking into account expression (9) and estimates $k_\parallel \approx \omega\sin\psi_0$ and $k_{\mathbf{g}\parallel} \approx \omega\sin\psi_\mathbf{g}$, we obtain the following expressions for dynamic corrections:

$$\lambda_\mathbf{g}^{(1,2)} = \frac{\omega}{4}\left(\beta \pm \sqrt{\beta^2 + 4\chi_\mathbf{g}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_\mathbf{g}}{\gamma_0}}\right),$$
$$\lambda_0^{(1,2)} = \omega\frac{\gamma_0}{4\gamma_\mathbf{g}}\left(-\beta \pm \sqrt{\beta^2 + 4\chi_\mathbf{g}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_\mathbf{g}}{\gamma_0}}\right). \tag{11}$$

Since $|\lambda_0| \ll \omega$ and $|\lambda_\mathbf{g}| \ll \omega$, we can show that $\theta \approx \theta'$ (see Fig. 1; we will henceforth denote θ' by θ).

It is convenient to write the solution to system of equations (3) for the diffracted field in the crystal in the form

$$E_\mathbf{g}^{(s)\mathrm{cr}} = -\frac{8\pi^2 ieV\theta P^{(s)}}{\omega}$$
$$\times \frac{\omega^2\chi_\mathbf{g}C^{(s)}}{4\frac{\gamma_0^2}{\gamma_\mathbf{g}^2}(\lambda_\mathbf{g} - \lambda_\mathbf{g}^{(1)})(\lambda_\mathbf{g} - \lambda_\mathbf{g}^{(2)})}\delta\left(\frac{\omega\beta}{2} + \frac{\gamma_\mathbf{g}}{\gamma_0}\lambda_0^* - \lambda_\mathbf{g}\right) \tag{12}$$
$$+ E^{(s)^{(1)}}\delta(\lambda_\mathbf{g} - \lambda_\mathbf{g}^{(1)}) + E^{(s)^2}\delta(\lambda_\mathbf{g} - \lambda_\mathbf{g}^{(2)}),$$

where $\lambda_0^* = w\left(\frac{\gamma^{-2} + \theta^2 - \chi_0}{2}\right)$ and $\gamma = \sqrt{2 - V^2}$ are the Lorentz factors of the particle and $E^{(s)^{(1)}}$ and $E^{(s)^{(2)}}$ are the free fields corresponding to two solutions (11) of dispersion equation (7).

For the field in vacuum in front of the crystal, the solution to system (3) has the form

$$E_0^{(s)\mathrm{vac}} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{1}{-\chi_0 - \frac{2}{\omega}\lambda_0}\delta(\lambda_0^* - \lambda_0)$$
$$= \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{1}{\frac{\gamma_0}{\gamma_\mathbf{g}}\left(-\chi_0 - \frac{2\gamma_0}{\omega\gamma_\mathbf{g}}\lambda_\mathbf{g} + \beta\frac{\gamma_0}{\gamma_\mathbf{g}}\right)} \tag{13}$$
$$\times\delta\left(\frac{\omega\beta}{2} + \frac{\gamma_\mathbf{g}}{\gamma_0}\lambda_0^* - \lambda_\mathbf{g}\right);$$

here, we are using the relation following from Eq. (9),

$$\delta(\lambda_0^* - \lambda_0) = \frac{1}{\gamma_0/\gamma_\mathbf{g}}\delta\left(\frac{\omega\beta}{2} + \frac{\gamma_\mathbf{g}}{\gamma_0}\lambda_0^* - \lambda_\mathbf{g}\right).$$

For the field in vacuum behind the crystal, we have

$$E_\mathbf{g}^{(s)\mathrm{vac}} = E_\mathbf{g}^{(s)\mathrm{Rad}}\delta\left(\lambda_g + \frac{\omega\chi_0}{2}\right), \tag{14}$$

where $E^{(s)\mathrm{Rad}}$ is the radiation field.

The second equation from system (3) leads to the following expression connecting the diffracted and incident fields in the crystal:

$$E_0^{(s)\mathrm{cr}} = \frac{2\omega\lambda_\mathbf{g}}{\omega^2\chi_\mathbf{g}C^{(s)}}E_g^{(s)\mathrm{cr}}. \tag{15}$$

Using the conventional boundary conditions

$$\left.\begin{aligned}\int E_0^{(s)\mathrm{vac}} d\lambda_{\mathbf{g}} &= \int E_0^{(s)\mathrm{cr}} d\lambda_{\mathbf{g}},\\ \int E_{\mathbf{g}}^{(s)\mathrm{cr}} \exp\left(i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}L\right) d\lambda_{\mathbf{g}} &= \int E_{\mathbf{g}}^{(s)\mathrm{vac}} \exp\left(i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}L\right) d\lambda_{\mathbf{g}},\\ \int E_{\mathbf{g}}^{(s)\mathrm{cr}} d\lambda_{\mathbf{g}} &= 0,\end{aligned}\right\} \quad (16)$$

we derive the expression for the radiation field,

$$E_{\mathbf{g}}^{(s)\mathrm{Rad}} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \frac{\omega^2 \chi_{\mathbf{g}} C^{(s)} \exp\left(i\left(\frac{\omega\chi_0}{2} + \lambda_{\mathbf{g}}^*\right)\frac{L}{\gamma_{\mathbf{g}}}\right)}{2\omega(\lambda_{\mathbf{g}}^{(1)} - \lambda_{\mathbf{g}}^{(2)})}$$
$$\times\left[\left(\frac{\omega}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}(-\chi_0\omega - 2\lambda_0^*)} + \frac{\omega}{2\frac{\gamma_0^2}{\gamma_{\mathbf{g}}^2}(\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)})}\right)\right.$$
$$\times\left(1 - \exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)}}{\gamma_{\mathbf{g}}}L\right)\right) \quad (17)$$
$$-\left(\frac{\omega}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}(-\chi_0\omega - 2\lambda_0^*)} + \frac{\omega}{2\frac{\gamma_0^2}{\gamma_{\mathbf{g}}^2}(\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)})}\right)$$
$$\left.\times\left(1 - \exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)}}{\gamma_{\mathbf{g}}}L\right)\right)\right],$$

where $\lambda_{\mathbf{g}}^* = \frac{\omega\beta}{2} + \frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda_0^*$.

From expression (17) for the radiation field of a rectilinearly moving electron, we can separate two terms,

$$E_{\mathbf{g}}^{(s)\mathrm{Rad}} = E_{\mathbf{g}}^{(s)PXR} + E_{\mathbf{g}}^{(s)DTR}, \quad (18a)$$

$$E_{\mathbf{g}}^{(s)PXR} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \frac{\omega^2 x_{\mathbf{g}} C^{(s)}}{2\pi(\lambda_{\mathbf{g}}^{(1)} - \lambda_{\mathbf{g}}^{(2)})\frac{\gamma_0}{\gamma_{\mathbf{g}}}}$$
$$\times\left[\left(\frac{\omega}{2\frac{\gamma_0}{\gamma_{\mathbf{g}}}(\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)})} - \frac{\omega}{2\lambda_0^*}\right)\left(1 - \exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)}}{\gamma_{\mathbf{g}}}L\right)\right)\right. \quad (18b)$$
$$\left.-\left(\frac{\omega}{2\frac{\gamma_0}{\gamma_{\mathbf{g}}}(\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)})} - \frac{\omega}{2\lambda_0^*}\right)\left(1 - \exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)}}{\gamma_{\mathbf{g}}}L\right)\right)\right]$$
$$\times \exp\left(i\left(\frac{\omega\chi_0}{2} + \lambda_{\mathbf{g}}^*\right)\frac{L}{\gamma_{\mathbf{g}}}\right),$$

$$E_{\mathbf{g}}^{(s)DTR} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \frac{\omega^2 x_{\mathbf{g}} C^{(s)}}{2\omega(\lambda_{\mathbf{g}}^{(1)} - \lambda_{\mathbf{g}}^{(2)})\frac{\gamma_0}{\gamma_{\mathbf{g}}}}$$
$$\times\left(\frac{\omega}{-\omega\chi_0 - 2\lambda_0^*} + \frac{\omega}{2\lambda_0^*}\right)\left(\exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)}}{\gamma_{\mathbf{g}}}L\right)\right. \quad (18c)$$
$$\left.- \exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)}}{\gamma_{\mathbf{g}}}L\right)\right)\exp\left(i\left(\frac{\omega\chi_0}{2} + \lambda_{\mathbf{g}}^*\right)\frac{L}{\gamma_{\mathbf{g}}}\right).$$

Expression (18b) describes the field of PXR. In this case, the first branch of PXR is significant since the real part of the denominator corresponding to this branch can vanish ($\mathrm{Re}(\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)}) = 0$), while that corresponding to the second branch cannot vanish ($\mathrm{Re}(\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)}) \neq 0$). Expression (18c) describes the field of DTR emerging as a result of diffraction of transient radiation emerging at the inlet surface from the system of atomic planes of the crystal, which are responsible for PXR.

Substituting expressions (11) into relations (18b) and (18c) and retaining only the first branch of PXR, we an write these relations as

$$E_{\mathbf{g}}^{(s)PXR} = -\frac{4\pi^2 ieV}{\omega}\frac{\theta P^{(s)}}{\theta^2 + \gamma^{-2} - \chi_0}$$
$$\times \frac{\exp\left(i\left(\frac{\omega\chi_0}{2} + \lambda_{\mathbf{g}}^*\right)\frac{L}{\gamma_{\mathbf{g}}}\right)}{K^{(s)}} \frac{\xi^{(s)}(\omega) - \frac{i\rho^{(s)}(1-\varepsilon)}{2} - K^{(s)}}{\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega) - K^{(s)}}{\varepsilon}} \quad (19a)$$
$$\times\left(1 - \exp\left(-ib^{(s)}\left(\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega) - K^{(s)}}{\varepsilon}\right)\right)\right),$$

$$E_{\mathbf{g}}^{(s)DTR} = -\frac{4\pi^2 irV}{\omega}\theta P^{(s)}\left(\frac{1}{\theta^2 + \gamma^{-2}} - \frac{1}{\theta^2 + \gamma^{-2} - \chi_0}\right)$$
$$\times \frac{\exp\left(i\left(\frac{\omega\chi_0}{2} + \lambda_{\mathbf{g}}^*\right)\frac{L}{\gamma_{\mathbf{g}}}\right)}{\frac{K^{(s)}}{\varepsilon}} \quad (19b)$$
$$\times\left(\exp\left(-ib^{(s)}\left(\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega) - K^{(s)}}{\varepsilon}\right)\right)\right.$$
$$\left.- \exp\left(-ib^{(s)}\left(\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega) - K^{(s)}}{\varepsilon}\right)\right)\right),$$

where

$$K^{(s)} = \sqrt{\xi^{(s)}(\omega)^2 + \varepsilon - 2i\rho^{(s)}\left(\frac{(1-\varepsilon)}{2}\xi^{(s)}(\omega) + \kappa^{(s)}\varepsilon\right) - \rho^{(s)^2}\left(\frac{(1-\varepsilon)^2}{4} + \kappa^{(s)^2}\varepsilon\right)},$$

$$\xi^{(s)}(\omega) = \frac{\alpha}{2|\chi'_g|C^{(s)}} - \frac{\chi'_0(1-\varepsilon)}{2|\chi'_g|C^{(s)}} = \eta^{(s)}(\omega) + \frac{(1-\varepsilon)}{2\nu^{(s)}},$$

$$\nu^{(s)} = \frac{|\chi'_g|C^{(s)}}{|\chi'_0|}, \quad \rho^{(s)} = \frac{\chi''_0}{|\chi'_g|C^{(s)}},$$

$$\eta^{(s)}(\omega) = \frac{\alpha}{2|\chi'_g|C^{(s)}}$$

$$= \frac{2\sin^2\theta_B}{V^2|\chi'_g|C^{(s)}}\left(1 - \frac{\omega(1-\theta\cos\varphi\cot\theta_B)}{\omega_B}\right),$$

$$\varepsilon = \frac{\gamma_g}{\gamma_0}, \quad \kappa^{(s)} = \frac{\chi''_g C^{(s)}}{\chi''_0},$$

$$\sigma^{(s)} = \frac{1}{|\chi'_g|C^{(s)}}(\theta^2 + \gamma^{-2} - \chi'_0), \quad b^{(s)} = \frac{\omega|\chi'_g|C^{(s)}}{2}\frac{L}{\gamma_0}. \tag{20}$$

Parameter $b^{(s)}$ determines the role of electron path length L/γ_0 in the crystal during emission. Since the inequality $2\sin^2\theta_B/(V^2|\chi'_g|C^{(s)}) \gg 1$ holds on the X-ray frequency range, $\eta^{(s)}(\omega)$ is a rapid function of frequency ω; consequently, in analysis of the PXR and DTR spectra, it will be convenient to treat $\eta^{(s)}(\omega)$ as a spectral variable characterizing frequency ω. It should be noted that the formulas contain not only $\xi^{(s)}(\omega) = \eta^{(s)}(\omega) + \frac{(1-\varepsilon)}{2\nu^{(s)}}$, where the second term appears due to refraction in the case of asymmetric reflection. In the case of symmetric reflection ($\varepsilon = 1$), this term vanishes.

We can present parameter ε in the form $\varepsilon = \sin(\delta + \theta_B)/\sin(\delta - \theta_B)$, where δ is the angle between the inlet surface of the target and the crystallographic plane. Parameter ε increases with decreasing angle δ, and vice versa.

Substituting expressions (19a) and (19b) into the well-known [13] expression for the spectral–angular density of X rays,

$$\frac{d^2W}{d\omega d\Omega} = \omega^2(2\pi)^{-6}\sum_{s=1}^{2}\left|E_{\text{Rad}}^{(s)}\right|^2, \tag{21}$$

we obtain the formulas for the spectral–angular density of PXR, DTR, and the term which is the result of interference of these two emission mechanisms:

$$\omega\frac{d^2N_{PXR}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2}\frac{\theta^2 P^{(s)^2}}{(\theta^2+\gamma^{-2}-\chi'_0)^2}$$

$$\times\left|\frac{1}{K^{(s)}}\frac{\xi^{(s)}(\omega) - \frac{i\rho^{(s)}(1-\varepsilon)}{2} - K^{(s)}}{\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}}\right. \tag{22a}$$

$$\left.\times\left(1-\exp\left(-ib^{(s)}\left(\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}\right)\right)\right)\right|^2,$$

$$\omega\frac{d^2N_{DTR}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2}\theta^2 P^{(s)^2}$$

$$\times\left(\frac{1}{\theta^2+\gamma^{-2}} - \frac{1}{\theta^2+\gamma^{-2}-\chi'_0}\right)^2$$

$$\left|\times\frac{\exp\left(-ib^{(s)}\left(\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}\right)\right)}{K^{(s)}/\varepsilon}\right. \tag{22b}$$

$$\left.-\frac{\exp\left(-ib^{(s)}\left(\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}\right)\right)}{K^{(s)}/\varepsilon}\right|^2,$$

$$\omega\frac{d^2N_{INT}^{(s)}}{d\omega d\Omega} = \frac{e^2}{2\pi^2}\frac{\theta^2 P^{(s)^2}}{(\theta^2+\gamma^{-2}-\chi'_0)}$$

$$\times\left(\frac{1}{\theta^2+\gamma^{-2}} - \frac{1}{\theta^2+\gamma^{-2}-\chi'_0}\right)\frac{\varepsilon}{|K^{(s)}|^2}$$

$$\times\text{Re}\left(\frac{\xi^{(s)}(\omega) - \frac{i\rho^{(s)}(1-\varepsilon)}{2} - K^{(s)}}{\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}}\left(1-\exp\left(-ib^{(s)}\right.\right.\right. \tag{22c}$$

$$\left.\left.\left.\times\left(\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}\right)\right)\right)\right)$$

$$\times\left(\exp\left(ib^{(s)}\left(\sigma^{(s)}+\frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon}+\frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}\right)\right)\right.$$
$$\left.-\exp\left(ib\left(\sigma^{(s)}-\frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon}+\frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}\right)\right)\right),$$

where $K^{(s)}$ is the expression complex-conjugate to $K^{(s)}$.

The expressions for the spectral–angular density of PXR and DTR were derived in the two-wave approximation of the dynamic theory of diffraction taking into account absorption of radiation in the medium and the possibility of different orientations of diffracting atomic planes of the crystal relative to the surface of the crystalline plate.

In the particular case when the atomic planes of the crystal are perpendicular to the inlet surface ($\delta = \pi/2$, $\varepsilon = 1$), expressions (22a) and (22b) are transformed into the relations derived in [8].

EFFECT OF ANOMALOUS PHOTOABSORPTION FOR PXR

To analyze the Borman effect on the spectral–angular parameters of PXR in the general asymmetric case, we write expression (22a) in the following convenient form:

$$\omega\frac{d^2N_{PXR}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2}\frac{P^{(s)^2}}{|\chi_0'|}F_{PXR}^{(s)},$$

$$F_{PXR}^{(s)} = \frac{\dfrac{\theta^2}{|\chi_0'|}}{\left(\dfrac{\theta^2}{|\chi_0'|}+\dfrac{1}{\gamma^2|\chi_0'|}+1\right)}\left|\frac{1}{K^{(s)}}\right. \tag{23}$$

$$\times\frac{\xi^{(s)}(\omega)-\dfrac{i\rho^{(s)}(1-\varepsilon)}{2}-K^{(s)}}{\sigma^{(s)}-\dfrac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon}+\dfrac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}}\left(1-\exp\left(-ib^{(s)}\right.\right.$$

$$\left.\left.\left.\times\left(\sigma^{(s)}-\frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon}+\frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}\right)\right)\right)\right|^2.$$

Let u consider the possibility of manifestation of the Borman effect for various orientation of the plate surface relative to the system of parallel diffracting atomic planes (which is controlled by parameter ε). We assume that angle θ_B between the electron velocity and reflecting planes, as well as length L/γ_0 of the path traversed by an electron in the plate, are fixed.

Figure 2 shows three from the multitude of possible orientations of the crystal plate surface relative to the system of parallel diffracting atomic planes corresponding to a preset length of the rectilinear trajectory

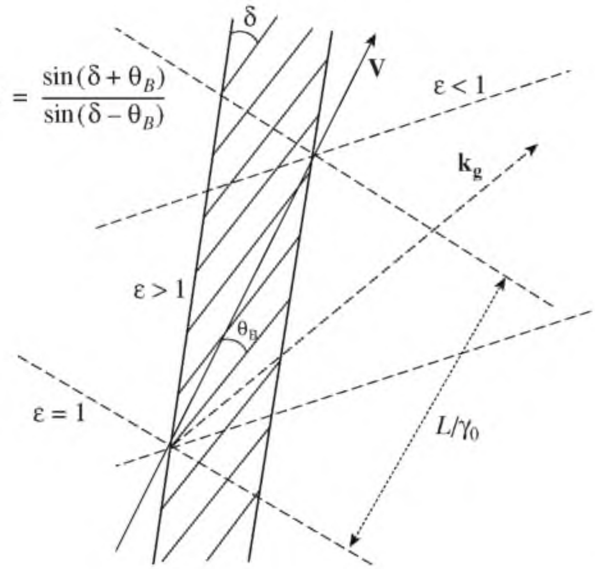

Fig. 2. Symmetric ($\varepsilon = 1$) and asymmetric ($\varepsilon > 1$ and $\varepsilon < 1$) reflection of the particle field.

of a relativistic electron. It is important to note that to have a fixed path length L/γ_0 traversed by the electron, it is necessary that the plate thickness vary upon an increase in parameter ε; in this case, the path length traversed by a free PXR photon in the plate also decrease.

Figure 3 shows the curves plotted using formula (23) and demonstrating the possibility of manifestation of the Borman effect in the case of a finite-thickness crystal with the above asymmetry coefficient. As parameter $\kappa^{(s)} = \dfrac{\chi_g''C^{(s)}}{\chi_0''}$ approaches unity, the manifestation of this effect for PXR (as well as for free X-ray waves) becomes significant (Borman effect is not observed for $\kappa^{(s)} = 0$ and is the strongest for $\kappa^{(s)} = 1$). It should be recalled that this parameter depends on the choice of the system of parallel diffracting atomic planes of the crystal, radiation frequency, and polarization of radiation. This effect is manifested more clearly for the σ polarization ($C^{(1)} = 1$) than for π polarization ($C^{(2)} = \cos 2\theta_B$). It should be noted that it is impossible in real experiment to select conditions under which $\kappa^{(s)}$ is equal to unity; the maximum possible value is $\kappa^{(s)} \approx 0.9$.

Figure 4 shows the curves analogous to those shown in Fig. 3 for a different asymmetry coefficient ε. It can be seen that the Borman effect in this case is manifested weakly since the PXR photon path length in the plane with such a coefficient becomes shorter and the amplitude of the spectrum increases. The spectral width increases thereby since the real part of the denominator of the fraction in expression (23a) varies less strongly for high values of ε,

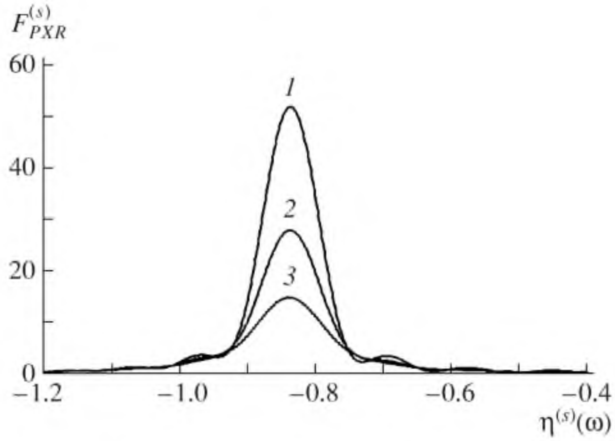

Fig. 3. Effect of asymmetry on the extent of manifestation of the Borman effect in the case of a finite-thickness crystal: $\kappa^{(s)} = 0.9$ (*1*); 0.5 (*2*); and 0 (3). Parameters of the system: $\varepsilon = 0.5$, $b^{(s)} = 20$, $\theta/|\chi_0'| = 1$, $\rho^{(s)} = 0.1$, $1/(\gamma\sqrt{x_0'}) = 0.5$, and $\nu^{(s)} = 0.8$.

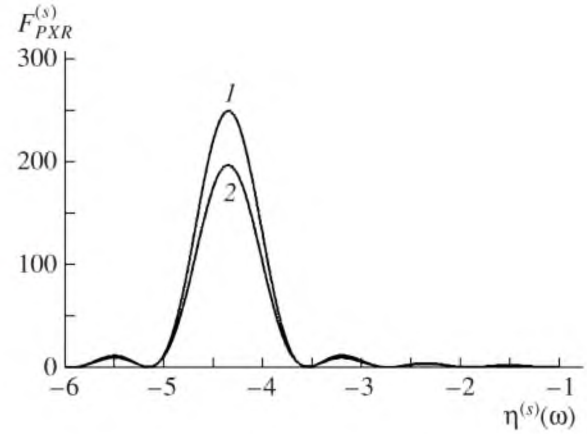

Fig. 4. The same as in Fig. 3 for $\kappa^{(s)} = 0.9$ (*1*) and 0 (*2*). Parameters of the system: $\varepsilon = 5$; the remaining parameters are the same as in Fig. 3.

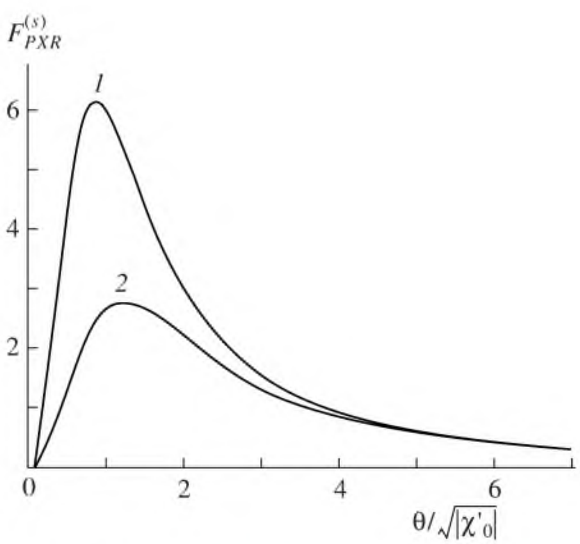

Fig. 5. Influence of the Borman effect on the angular density of PXR for asymmetry coefficient $\varepsilon = 0.5$; $\kappa^{(s)} = 0.9$ (*1*) and 0 (*2*).

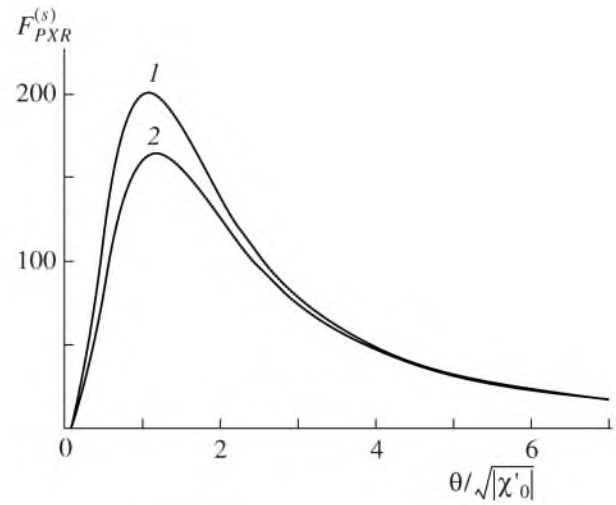

Fig. 6. The same as in Fig. 5 for $\varepsilon = 5$.

$$\mathrm{Re}\left[\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega) - K^{(s)}}{\varepsilon}\right] = 0. \quad (24)$$

The solution to this equation defines the frequency in the vicinity of which the spectrum of PXR photons emitted at a fixed angle of observation is concentrated.

Concluding the section, let us consider the influence of the Borman effect on the angular density of PXR. For this purpose, we integrate expression (23) with respect to frequency function $\eta^{(s)}(\omega)$:

$$\frac{dN_{PXR}^{(s)}}{d\Omega} = \frac{e^2\nu^{(s)}P^{(s)^2}}{8\pi^2\sin^2\theta_B}R_{PXR}^{(s)},$$
$$R_{PXR}^{(s)} = \int_{-\infty}^{+\infty} F_{PXR}^{(s)} d\eta^{(s)}(\omega). \quad (25)$$

The $R_{PXR}^{(s)}$ curves presented in Figs. 5 and 6 demonstrate the influence of the Borman effect on the angular density for two different asymmetry coefficients under the same conditions as in Figs. 3 and 4, respectively.

CONCLUSIONS

Analytic expressions for the spectral-angular distribution of PXR and DTR emitted by a relativistic electron passing through a crystalline plate of an arbitrary length are derived using the two-wave approximation in the dynamic theory of diffraction in the Laue scattering geometry for the general case of asymmetric reflection. These expressions contain angle δ between the crystalline plane surface and diffracting atomic planes of the crystal and make it possible to analyze the extent of the Borman dynamic effect in PXR as a function of this angle. It is shown that by varying the degree of asymmetry of reflection, it is possible to create the conditions in which the electron path length in the plate is short enough to disregard multiple scattering of the electron; in this case, the PXR photon path length becomes larger than the photoabsorption length, which will lead to a clearer manifestation of the Borman effect for PXR and will make it possible to detect this effect experimentally. This effect makes it possible to substantially increase the intensity of sources of tunable quasi-monochromatic X rays, which are based on the PXR mechanism.

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