Appropriateness of kinematical approach in description of parametric X-radiation of relativistic electron in a single crystal

Abstract. Contrastive analysis of the formulas for parametric x-ray radiation (PXR) in the dynamic and kinematic approximation is made in the present work. The criteria for the usage of the kinematic formulas are determined on the basis of the dynamic theory. Specifically it was shown that the dynamic effects of PXR must be taken into account even in case of a thin non-absorptive crystal. The published results of the experiment on relativistic electron PXR on Mainz microtron MAMI are discussed. The manifestation of the effect of abnormal photo absorption (the Borrmann effect) in PXR is demonstrated in the experiment.

1. Introduction

When a fast charged particle crosses a single crystal, its Coulomb field scatters on a system of parallel atomic planes of the crystal, generating parametric X-radiation (PXR) [1-3].

Nowadays there are two approaches of PXR description: kinematic [4,5] and dynamic [2,3,6]. It should be noted that kinematic approach takes into account the interaction of each atom only with a primary or refracted wave in the crystal. In contrast to the dynamic one this approach does not take into account multiwave scattering. A considerable progress has been made recently in the description of coherent radiation of relativistic electrons in crystals using dynamic approach [7-11]. The parametric radiation reflection in the direction of electron velocity [12], observed in the experiments and unpredicted by the kinematic theory, clearly proves the validity of the dynamic theory of the coherent x-radiation. A considerable influence of the asymmetry of the radiation field reflection relative to the surface of the crystal plate on the spectral-angular density of the radiation should be noted. According to the kinematic theory the asymmetry influences only the relation of the path of the charged particle and the radiated photon in the plate. The PXR dynamic theory predicts a considerable influence of the reflection asymmetry on the process of radiation, which leads to a change in PXR spectrum [10]. Thus, the problem of the limits of applicability of the kinematic theory of relativistic electron PXR the paper deals with is rather topical.

Previously an expression was derived in two-wave approximation describing PXR spectral-angular density for the case of asymmetric reflection on the crystal plate [10] and the peculiarities of the process of coherent radiation connected with it were revealed.

In the present work a contrastive analysis of the formulas for dynamic and kinematic approach is made both for a thin and thick crystal in order to determine the criteria limiting the applicability of the kinematic theory formulas.

2. PXR spectral – angular distribution

Let us consider the radiation of a fast charged particle crossing a single crystal plate with a constant velocity V (the radiation process geometry look in figure 1. in [13]).

Let us write the expression for PXR spectral-angular density based on the dynamic diffraction theory [10] in the following convenient form:

$$\omega \frac{d^2 N^{(s)}_{ПРИ}}{d\omega d\Omega} = \frac{e^2}{4\pi^2} \frac{P^{(s)^2}\theta^2}{(\theta^2 + \gamma^{-2} - \chi'_0)^2} R^{(s)}_{ПРИ}, \quad (1a)$$

$$R^{(s)}_{ПРИ} = \left(1 - \xi^{(s)} / \sqrt{\xi^{(s)^2} + \varepsilon}\right)^2 \frac{1 + e^{-2b^{(s)}\rho^{(s)}\Delta^{(s)}} - 2e^{-b^{(s)}\rho^{(s)}\Delta^{(s)}} \cos\left(b^{(s)}\left(\sigma^{(s)} + \left(\xi^{(s)} - \sqrt{\xi^{(s)^2} + \varepsilon}\right)/\varepsilon\right)\right)}{\left(\sigma^{(s)} + \left(\xi^{(s)} - \sqrt{\xi^{(s)^2} + \varepsilon}\right)/\varepsilon\right)^2 + \rho^{(s)^2}\Delta^{(s)^2}},$$

(1b)

where

$$\Delta^{(s)} = (\varepsilon + 1)/2\varepsilon - (1-\varepsilon)\xi^{(s)} / 2\varepsilon\sqrt{\xi^{(s)^2} + \varepsilon} - \kappa^{(s)} / \sqrt{\xi^{(s)^2} + \varepsilon}$$

$$\sigma^{(s)} = \left(\theta^2 / |\chi'_0| + \gamma^{-2} / |\chi'_0| + 1\right) / \nu^{(s)}, \nu^{(s)} = |\chi'_g| C^{(s)} / |\chi'_0|, \; b^{(s)} = \omega |\chi'_g| C^{(s)} L / 2\gamma_0. \quad (2)$$

Having integrated (1) on frequency function $\eta^{(s)}(\omega)$, let us write the following expression, describing PXR angular density:

$$\frac{dN^{(s)}_{PXR}}{d\Omega} = \frac{e^2 |\chi'_g| C^{(s)}}{8\pi^2 \sin^2\theta_B} \frac{P^{(s)^2}\theta^2}{(\theta^2 + \gamma^{-2} - \chi'_0)^2} \int_{-\infty}^{+\infty} R^{(s)}_{PXR} d\eta^{(s)}(\omega). \quad (3)$$

The expression, describing PXR spectrum (1b) in case of a thin target $\left(b^{(s)}\rho^{(s)} << 1\right)$, when the absorption coefficient $\rho^{(s)}$ can be neglected, takes on the following form:

$$R^{(s)}_{PXR} = 4\left(1 - \frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2} + \varepsilon}}\right)^2 \sin^2\left(\frac{b^{(s)}}{2}\left(\sigma^{(s)} + \left(\xi^{(s)} - \sqrt{\xi^{(s)^2} + \varepsilon}\right)/\varepsilon\right)\right)\left(\sigma^{(s)} + \left(\xi^{(s)} - \sqrt{\xi^{(s)^2} + \varepsilon}\right)/\varepsilon\right)^{-2}.$$

(4)

For brighter manifestation of the dynamic effects, we shall consider a crystal plate of such a thickness when the electron path length in the plate $L/\sin(\delta - \theta_B)$ exceeds manifold the ray wave extinction length in the crystal $L^{(s)}_{ext} = 1/\omega|\chi'_g|C^{(s)}$ i.e. $b^{(s)} >> 1$. As under the condition $b^{(s)} >> 1$ the PXR spectrum peak is very narrow, to integrate (8) a well-known approximation $\sin^2(Tx)/x^2 \to \pi T\delta(x)$ can be used. The formula for PXR angular distribution, resulting from (3) takes on the following form:

$$\frac{dN^{(s)din}_{PXR}}{d\Omega} = \frac{e^2 \omega_B \chi'^2_g C^{(s)^2} P^{(s)^2}}{4\pi \sin^2\theta_B} \cdot \frac{\varepsilon^2\theta^2}{\chi'^2_g C^{(s)^2} + \varepsilon(\theta^2 + \gamma^{-2} - \chi'_0)^2} \cdot \frac{L}{\sin(\delta - \theta_B)}. \quad (5)$$

Let us write the following well-known expression for PXR angular density for kinematic approximation (for instance, see [5,14])

$$\frac{dN^{(s)kin}_{PXR}}{d\Omega} = \frac{e^2 \omega_B \chi'^2_g C^{(s)^2} P^{(s)^2}}{4\pi \sin^2\theta_B} \cdot \frac{\theta^2}{(\theta^2 + \gamma^{-2} - \chi'_0)^2} \Gamma(\varepsilon), \; \Gamma(\varepsilon) = \frac{\varepsilon}{\omega_B \chi''_0}\left(1 - e^{-\frac{\omega_B \chi''_0 L}{\varepsilon \sin(\delta - \theta_B)}}\right). \quad (6)$$

One can see that the asymmetry coefficient ε which is contained in the kinematic expression, is only present in the geometrical factor $\Gamma(\varepsilon)$. In case of a thin non-absorptive crystal the geometrical factor is the electron path in the crystal plate

$$\Gamma(\varepsilon)=\frac{L}{\sin(\delta-\theta_B)}. \tag{7}$$

Hence the kinematic expression for PXR angular density takes on the following form:

$$\frac{dN_{PXR}^{(s)kin}}{d\Omega}=\frac{e^2\omega_B\chi_g'^2 C^{(s)2}P^{(s)2}}{4\pi\sin^2\theta_B}\cdot\frac{\theta^2}{(\theta^2+\gamma^{-2}-\chi_0')^2}\frac{L}{\sin(\delta-\theta_B)}. \tag{8}$$

It is important to underline that in the dynamic formula in contrast to the kinematic one, the asymmetry (angle δ) is present not only in the expression for electron path in the plate, but also in the PXR angular distribution expression, which can be the basis for the comparison of kinematic and dynamic formulas for the case of a thin crystal.

3. Contrastive analysis of kinematic and dynamic formulas for a thin crystal

For further analysis it is convenient to present the expressions (5) and (8) in the following way:

$$\frac{dN_{PXR}^{(s)din}}{d\Omega}=\frac{e^2\omega_B|\chi_g'|C^{(s)}P^{(s)2}}{4\pi\sin^2\theta_B}\cdot\frac{L}{\sin(\delta-\theta_B)}F^{din}, \tag{9a}$$

$$F^{din}=\nu^{(s)}\varepsilon^2\frac{\theta^2}{|\chi_0'|}\left(\nu^{(s)2}+\varepsilon\left(\frac{\theta^2}{|\chi_0'|}+\frac{1}{\gamma^2|\chi_0'|}+1\right)^2\right)^{-1}, \tag{9b}$$

$$\frac{dN_{PXR}^{(s)kin}}{d\Omega}=\frac{e^2\omega_B|\chi_g'|C^{(s)}P^{(s)2}}{4\pi\sin^2\theta_B}\cdot\frac{L}{\sin(\delta-\theta_B)}F^{kin}, \tag{10a}$$

$$F^{kin}=\nu^{(s)}\frac{\theta^2}{|\chi_0'|}\left(\frac{\theta^2}{|\chi_0'|}+\frac{1}{\gamma^2|\chi_0'|}+1\right)^{-2}. \tag{10b}$$

Functions (9b) and (10b) describe PXR angular density. It should be noted that in contrast to the kinematic approach, the dynamic one takes into account the influence of asymmetry on PXR angular part.

3.1 Symmetric reflection

At first let us consider a case of symmetric reflection ($\varepsilon=1$). In case of low energy of the radiating particle $\gamma\le 1/\sqrt{|\chi_0'|}\approx\omega/\omega_p$ (ω_p -plasma frequency) we can use the approximate equality $F_{din}\approx F_{kin}$ for all the observation angles θ. For relatively high energy $\gamma>>\omega/\omega_p$ for weak reflections (e.g. $\nu^{(s)}\approx 0{,}3$) $F_{din}\approx F_{kin}$. In case of strong reflections (e.g. $\nu^{(s)}\approx 0{,}9$) angular dependences, plotted on the basis of kinematic and dynamic formulas differ, which can be judged by the curves in figure 1. In this case the kinematic formulas will result in the error in photon absolute yield, therefore the results of rather accurate experiments on PXR absolute yield should be compared with the dynamic formula (5).

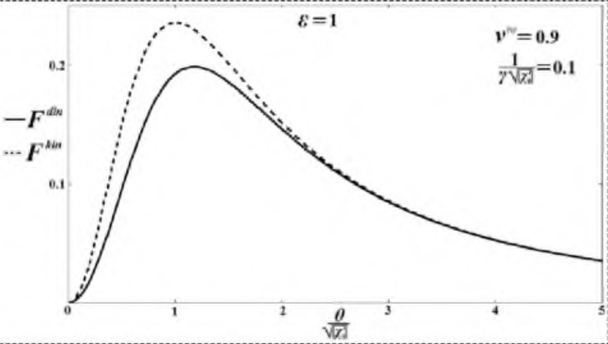

Figure 1. The comparison of PXR angular densities calculated on the basis of dynamic and kinematic formulas. Under the given circumstances the kinematic formula results in an error.

3.2 Asymmetric reflection

When the asymmetry growth which is caused by the decrease (when $\varepsilon > 1$) or increase (when $\varepsilon > 1$) of the electron incidence angle $\delta - \theta_B$ (see in figure 2 in [13]) the kinematic formula error increases, which is demonstrated by the curves represented in figure 2.

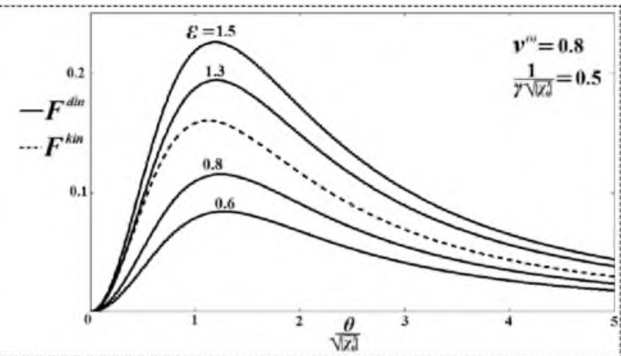

Figure 2. PXR angular density for different asymmetry. It demonstrates an increase of kinematic formula error under the growth of asymmetry.

In case of a strong asymmetry ($\varepsilon >> 1$), when the electron incidence angle to the plate surface ($\delta - \theta_B$) is relatively small, the expression $F^{din} / F^{kin} = \varepsilon$ follows from (9b) и (10b) both when relativistic electron energy is less or more than ω / ω_p.

Thus, in case of a strong asymmetry the PXR angular density is ε times more than it was predicted by the kinematic formula (see in figure 3).

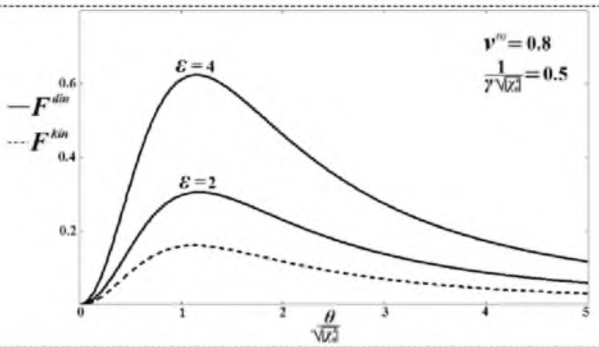

Figure 3. Under the strong asymmetry PXR angular density is ε times more than it was predicted by the kinematic formula PXR angular density for different asymmetry.

4. Contrastive analysis of kinematic and dynamic formulas for a thick crystal

For further analysis it is convenient to present the expressions (1) and (6) in the following way:

$$\frac{dN_{PXR}^{(s)din}}{d\Omega}=\frac{e^2\omega_B|\chi'_{\mathbf{g}}|C^{(s)}P^{(s)2}}{4\pi\sin^2\theta_B}F_{abs}^{din} \tag{11a}$$

$$F_{abs}^{din}=\nu^{(s)}\frac{\theta^2}{|\chi'_0|}\left(\frac{\theta^2}{|\chi'_0|}+\frac{1}{\gamma^2|\chi'_0|}+1\right)^{-2}\frac{L_{ext}}{2\pi}\int_{-\infty}^{+\infty}R_{PXR}^{(s)}d\eta^{(s)}(\omega) \tag{11b}$$

$$R_{PXR}^{(s)}=\left(1-\frac{\xi^{(s)}}{\sqrt{\xi^{(s)2}+\varepsilon}}\right)^2\frac{1+e^{-L_f\mu^{(s)}}-2e^{-\frac{L_f\mu^{(s)}}{2}}\cos\left(\frac{L_{e-}}{2L_{ext}}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)2}+\varepsilon}}{\varepsilon}\right)\right)}{\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)2}+\varepsilon}}{\varepsilon}\right)^2+\frac{L_{ext}^2\mu^{(s)2}}{\varepsilon^2}} \tag{11c}$$

$$\frac{dN_{PXR}^{(s)kin}}{d\Omega}=\frac{e^2\omega_B|\chi'_{\mathbf{g}}|C^{(s)}P^{(s)2}}{4\pi\sin^2\theta_B}F_{abs}^{kin}, \tag{12a}$$

$$F_{abs}^{kin}=\nu^{(s)}\frac{\theta^2}{|\chi'_0|}\left(\frac{\theta^2}{|\chi'_0|}+\frac{1}{\gamma^2|\chi'_0|}+1\right)^{-2}\frac{\varepsilon}{\mu_0}\left(1-e^{-L_f\mu_0}\right), \tag{12b}$$

where

$$\mu^{(s)}=\mu_0\cdot\left(\frac{\varepsilon+1}{2}-\frac{1-\varepsilon}{2}\frac{\xi^{(s)}}{\sqrt{\xi^{(s)2}+\varepsilon}}-\frac{\varepsilon\kappa^{(s)}}{\sqrt{\xi^{(s)2}+\varepsilon}}\right),\ \mu_0=\omega_B\chi''_0 .$$

$\mu^{(s)}$ is the effective coefficient of photon absorption in crystals, dependent on the asymmetry parameter ε. The parameter $\kappa^{(s)}$ determines the degree of manifestation of the abnormal photo absorption low (the Borrmann effect) [15] in x-ray waves crossing the crystal. This effect lies in the formation of a standing wave by incident and diffracted waves whose antinodes are situated in the middle of the space between neighboring atomic planes, where the electron density and consequently the photo absorption are minimal. Herewith two waves are formed in the crystal, one of which absorbed abnormal strong and other absorbed abnormal weak. The expression (11) describes the PXR branch which is absorbed anomalously weak.

As well in the case of free x-ray waves, the imaginary part value proximity in the appropriate decomposition coefficients of dielectric susceptibility of the crystal in the Fourier series on the reciprocal lattice vectors ($\kappa^{(s)}\approx 1$) is a necessary condition for the manifestation of this effect in PXR. It necessary to remind that this parameter depends on the selection of the system of diffracted atomic planes in the crystal, radiation frequency and its polarization.

As the solution of the equation

$$\sigma^{(s)}+\left(\xi^{(s)}-\sqrt{\xi^{(s)2}+\varepsilon}\right)/\varepsilon=0, \tag{13}$$

determines frequency ω_*, in whose vicinity the spectrum of PXR photons radiated under fixed observation angle is concentrated,

$$\xi^{(s)}(\omega_*)=(1-\varepsilon\sigma^{(s)2})/2\sigma^{(s)}, \tag{14}$$

than the effective absorption coefficient also depends on the observation angle θ. Thus, the account of absorption in the dynamic theory may cause the deformation of PXR angular density (see in figure 4).

Once again it should be noted that in the kinematic formula only the geometry of process depends on the reflection asymmetry, while the absorption coefficient μ_0 corresponds to a normal one in amorphous medium. In the dynamic formula (11) the effective absorption coefficient $\mu^{(s)}$ depends not only on the selection of the system of diffracted crystal atomic planes (parameter $\kappa^{(s)}$), but also on the reflection asymmetry and the observation angle.

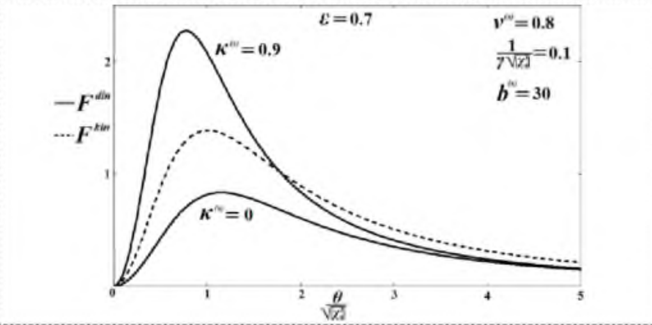

Figure 4. PXR angular density at the expense of different effective absorption coefficient for different observation angles θ.

5. Interpretation of the results of the experiment

In the work [14] the results of the experimental research of spectral angular distribution of parametric x-ray radiation (PXR) by electrons with the energy of 855 MeV are published. We would underline the high quality of the accelerator beam (small lateral dimensions and divergence) and that of the detecting equipment which allowed the authors [14] to obtain x-ray photon angular distribution in absolute values of high accuracy and high angular resolution. We compared the published experimental results with the angular distribution calculated by us based on the well-known kinematic formula (see e.g. [5, 14]) and then with the dynamic formula which we obtained [10]. As one can see in figure 7 the curve plotted on the basis of the kinematic formula (dot line 1), does not correspond to the results of the experiment. The curve plotted on the basis of our dynamic formula, which takes into account photon reflection from the system of atomic planes (111) asymmetrically situated with respect to the target surface (solid curve 2), completely coincides with the data of the experiment. It is important to note that the experiment conditions also correspond to conditions of a significant manifestation of anomalously low photo absorption (Borrmann effect) in PXR in the case of asymmetric refection [11]. To illustrate the presence of this effect the curve calculated by our dynamic formula without taking into account the Borrmann effect is plotted in figure 5 (solid curve3).

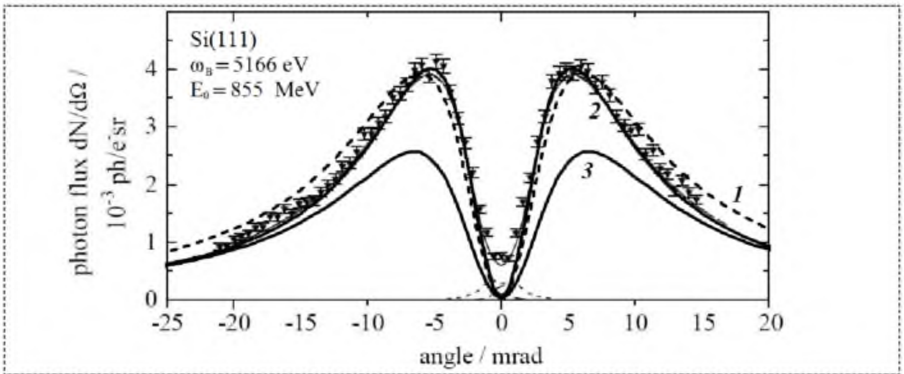

Figure 5. Dynamic Borrmann effect manifestation in PXR.

Conclusion

In the present work the contrastive analysis the kinematic and dynamic formulas for PXR was made for the cases of symmetric and asymmetric reflection of coulomb field of a relativistic electron. It was shown for a thin crystal that in the case of symmetric reflection in the region of radiating particle energy $\gamma >> \omega / \omega_p$ kinematic formula results in error even for thin non-absorptive crystal, and for the case $\gamma < \omega / \omega_p$ both the kinematic and dynamic formulas give the same result. It was shown that for the increase in reflection asymmetry the error resulting by kinematic formula for PXR increases and for strong asymmetry ($\varepsilon >> 1$) the angular density calculated by kinematic formula turn out in ε times less than the density calculated by means of dynamic formula.

It is shown that in the case of thick absorbing crystal the absorption coefficient in dynamic theory, in contrast to kinematic formula, depends on reflection asymmetry in the direction of photon radiation, which can cause the deformation of angular distribution of PXR. In the present work in the network of the developed theory it is done the interpretation of the results obtained in the experiment on registration of PXR on Mainz electron microtron MAMI and it was shown that the effect of abnormal photo absorption (Borrmann effect) was considerably manifested in that experiment.

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