Method for optimizing the shell of open pit mines based on parallel computing

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Abstract. One of the main stages in design of mineral development process is solution of finding optimal open pit problem. It is necessary to take into account the spatial distribution of mineral components and the allowable slope angles of the sides during solving this problem. The basis for performing calculations to optimize the extraction of reserves is a digital block model of the mineral deposit. The larger and more accurate block model of the ore deposit requires more computationally complex calculation process. In this regard, using of parallel computing in this area is very relevant. The importance of solving this problem determined by the need to have an accurate model of the open pit at various stages of its development. This model depends from current cost of useful components and the costs of their extraction. This task formulated as the Ultimate Pit Problem (UPIT). Existing approaches for solving this problem does not use modern high-performance computing approaches. This complicates analysis of large models using existing methods. The long data processing time makes it difficult to increase the accuracy of the constructed quarry models, which in turn negatively affects decision-making on the field development plan and entails lost profit. In this paper analysed the methods of computer modelling open pits from the position of parallelizing computations during profit optimization and proposes a mathematical formulation of the problem of searching for open pit limits based on block extraction order graph.

1. Introduction

As a rule, the interpretation of information about the occurrence and patterns of distribution of useful components in the field is carried out on a limited amount of geological exploration data. In this regard, the use of modern computer modeling methods and tools is becoming a prerequisite for processing the source data for making economically and technologically sound decisions.

One of the main stages of designing the development of mineral resources is the solution of the problem of finding the boundaries of a quarry. When finding the boundaries of the quarry, it is necessary to take into account the spatial distribution of mineral components and accepted stable or technologically permissible slope angles of the sides. The basis for performing calculations to optimize the extraction of reserves is a digital block model of the field. It is only natural that the larger and more accurate the block model of the ore deposit is, the more computationally complex is the calculation process. In this regard, the use of modern means of parallel computing in this area is very relevant. The importance of solving this problem is determined by the need to have an accurate model of the mine pit shape at various stages of its development and the current cost of useful components,

taking into account the costs of their extraction. This model is the basis for the subsequent design stages, such as the design of the transport network, the design of blasting, etc.

In this paper, this problem is considered in the following formulation: there is a cost block model of the field, it is necessary to find the form of a quarry, i.e. determine the set of blocks of the cost model at which the amount of profit is maximum. This task in international literature is formulated as the UPIT (Ultimate Pit Problem) task - the task of finding the optimal career shape.

Existing systems for modeling quarries of ore deposits do not imply the use of modern high-performance computing tools [1–5], which complicates the analysis of volumetric models using such systems. The long data processing time makes it difficult to increase the accuracy of the constructed quarry models, which in turn negatively affects the decision-making on the field development plan and entails lost profit. In this regard, the use of modern means of high-performance computing in the field of solving the problems of optimizing the shape of opencast mining pits is relevant.

The following authors contributed to the development of automated mathematical methods for determining the boundaries of quarries based on the block model of the field: Lerchs H, Grossman I F, Whittle D J, Denby B, Schofield D, Hochbaum D S, Ramazan S, Muir D C W, Khalokakaie R, Elahi Zeyni E [6-14].

It should be noted that the basis of the research is two main approaches to solving this problem: algorithms based on the floating cone method [6–7] and algorithms that are a modification and development of the Lerch-Grossman algorithm [10–12].

Algorithms based on various modifications of the floating cone method are characterized by simplicity of implementation, however, they do not always find a qualitative solution and, besides, they have great computational complexity and therefore are not effective when working with large volumes of data. Algorithms based on various modifications of the Lerch-Grossman algorithm are more accurate, but also have a low data processing speed.

It should be noted that at present there is no such mathematical model that would allow the implementation of effective parallel algorithms for finding the boundaries of a mine pit. In particular, existing models do not allow efficient use of distributed memory systems and make it difficult to take into account complex restrictions on the slope angles of open pit walls. Thus, the task of developing new methods and parallel algorithms for finding the boundaries of quarries of ore deposits is relevant.

2. Theoretical Foundations of the Ultimate Pit Problem

2.1. *A mathematical model for finding the boundaries of ore quarries*

To solve the problem of searching for open pit boundaries, a block model of mineral deposits is used. Each block of this model is characterized by a number (weight) showing the net profit obtained during its production, taking into account the percentage of useful elements, the cost of its production and the market value of useful components.

Fig. 1 shows an example of a cross section of a block model; the optimal open pit shape in this section is indicated by a bold line.

-4	-4	-4	-4	-4	8	12	12	0	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4
	-4	-4	-4	-4	0	12	12	8	-4	-4	-4	-4	-4	-4	-4	-4	-4	
		-4	-4	-4	-4	8	12	12	0	-4	-4	-4	-4	-4	-4	-4		
			-4	-4	-4	0	12	12	8	-4	-4	-4	-4	-4	-4			
				-4	-4	-4	8	12	12	0	-4	-4	-4	-4	-4			
					-4	-4	0	12	12	8	-4	-4	-4	-4				

Figure 1. An example of a cross section of a block model of a field

Dark blocks with a positive weight value are blocks that contain useful elements and it is advantageous to mine them, light blocks with a negative weight value are waste rock, which the enterprise only spends mining.

The task of determining the boundaries of the quarry (the shell of the quarry at the end of the life of the mining enterprise) is to find many recoverable three-dimensional blocks of ore and rock in order to maximize profits if there are restrictions associated with the stability of slopes of the sides.

We give a mathematical statement of the problem described above. Let there is a block model of the field (1):

$$P = [p_1,\ p_2, \ldots, p_i, \ldots, p_n] \tag{1}$$

Each element of which is characterized by the number (weight) p_i, $i \in [1, n]$, n is the total number of model blocks showing the net profit obtained during its extraction, taking into account the percentage of useful elements, the cost of its production and the market value of useful components.

Let's say:

A = the set of ordered pairs of blocks (i, j) such that block j is adjacent to block i and in order to extract block i, first you need to extract block j;

c_i = cost of extracting block i;

r_i = income from extracting block i;

p_i = profit from extracting block i, $p_i = r_i - c_i$;

$$y_i = \begin{cases} 1, if\ block\ i\ exctracted; \\ 0, if\ block\ i\ does\ not\ exctracted \end{cases} \tag{2}$$

The income from the extracting of block r_i is calculated by interpolating the values of the content of useful components obtained by taking vertical and inclined drilling samples and their current market value c_i. These values are determined for each block of the field model during mine surveying. Values c_i, r_i and p_i for each blocks are source data for this problem. The size of one block depend on the accuracy of the model, is ranges can be from $6m^3$ to $30m^3$. Restriction on quarry geometry (allowable slope angles) depends on the type of rock in every point of model. This restriction also provides as a source data for solving problem.

Then the task of finding the optimal career form, the profit from the development of which will be maximum, can be represented in the form of problem (3):

$$z = \sum_{i=1}^{n} p_i y_i\ , z \to max, y_i \le y_j, (i,j) \in A, x_i \in \{0,1\} \tag{3}$$

The restriction $y_i \le y_j$ shows that before removing block i, it is necessary to remove block j and reflects the restriction on the angles of inclination of the sides of the quarry. As a result of solving problem (3), we obtain the optimal quarry shape from the standpoint of the profit criterion.

The restrictions on the angles of inclination of the open pit walls at each point of the field in such a model are taken into account as the maximum height difference when moving to an adjacent block. This approach complicates the modeling of the complex form of quarries and leads to additional costs of computing resources when implementing algorithms that solve the problem.

A block model of a field can be represented in the form of a directed graph, if we introduce the concept of restricting the order of extraction of blocks. These restrictions require the fulfillment of the condition that when the current block is extracted, the overlying blocks that must be removed before the block in question directly affect it. The relationship between the blocks is specified explicitly as a transitive type relationship, that is, if you need to extract block B to extract block A, and to extract block B, you need to extract block C, then to extract block A you also need to extract block C. In Fig. 2 shows an example of a graph of the extraction sequence for the block model shown in Fig. 1.

When presenting the field model in this form, to solve the problem of optimizing the form of the quarry, it is necessary to find a sub graph in the column whose sum of weights will be maximum. In the case of representing the field model in the form of a graph of the block extraction order, the problem of finding the quarry shape can be formulated as follows.

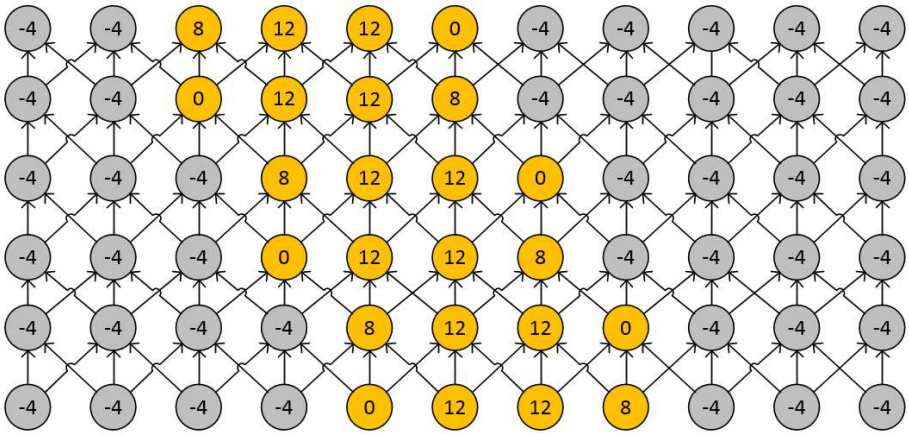


Figure 2. Presentation of the block model as a graph of the block extraction order

Let there be a weighted directed graph (4)

$$G = \{V, A\} \tag{4}$$

Where V=$\{v_i\}$, i∈[1,n] is the set of vertices, the number of vertices is equal to the number of elements of the block model, each vertex is characterized by weight (5)

$$v_i = p_i \tag{5}$$

A – a set of arcs. The directional arc from the vertex v_i to the vertex v_j means that in order to extract the vertex v_i, you must first extract the vertex v_j. Then the task of finding the optimal career shape is reduced to finding such a sub graph Y⊂V whose sum of the weights of the vertices is maximal:

$$z = \sum_{i=1}^{n} v_i x_i, z \to max, x_i \le x_j, (i,j) \in A, x_i \in \{0,1\} \tag{6}$$

Where x_i is the binary variable (7), taking the value 1 if the vertex i is in the desired subgraph, and 0 otherwise.

$$x_i = \begin{cases} 1, if\ vertex\ i\ included\ in\ Y \\ 0, if\ vertex\ i\ does\ not\ included\ in\ Y \end{cases} \tag{7}$$

The restriction $x_i \le x_j, (i,j) \in A$ can be rewritten taking into account the storage of the graph G in the form of the adjacency matrix (8):

$$a_{ij} x_i (x_j - 1) = 0 \tag{8}$$

Where a_{ij} is the element (i, j) of the adjacency matrix of the graph G, i.e.

$$a_{ij} = \begin{cases} 1, (i,j) \in A \\ 0, (i,j) \notin A \end{cases} \tag{9}$$

Equations (8) and (9) show that if vertex i included to solution (extracted, x_i = 1) and there is an arc from vertex i to vertex j (a_{ij} = 1) then vertex j also have to be included to solution (extracted), that means x_j have to be equal to 1.

Taking into account expression (8), problem (6) can be reformulated in the following form (10):

$$z = \sum_{i=1}^{n} v_i x_i, z \to max, \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_j (x_j - 1) = 0, x_i \in \{0,1\} \tag{10}$$

The solution of problem (10) allows us to find the optimal quarry shape for the field model presented in the form of a graph.

Presentation of the field data in the form of a graph of the extraction order allows taking into account the most complex restrictions on the shape and slope angles of the sides of the quarries, taking into account the differences in flow ability and fracturing of rocks at different points of the model. Moreover, this graph is calculated once before the main calculations for modeling the quarry shape, which does not significantly affect the total calculation time. In addition, such limitations of the search model for the optimal career shape allow you to build parallel computing procedures and achieve a reduction in the time it takes to solve a problem using a computer.

2.2. The method of finding the optimal boundaries of the quarries based on the targeted selection of many useful fragments of the quarry

One of the possible approaches to solving the problem of searching for optimal quarry boundaries on a field model presented in the form of a graph of block extraction order is the construction of an evolutionary random search method [15, 16].

Genetic optimization methods are based on modeling the processes of mutation, crossing, and natural selection. Mutation and natural selection - two driving factors of evolution in living nature are successfully modeled and show good results in solving technical optimization problems.

To implement the genetic method of random search, first of all, it is necessary to develop a format for representing chromosomes. In this paper, it is proposed to use a format for representing chromosomes that excludes the possibility of forming an open-pit mine of irregular shape during mutation and crossing procedures. Due to this, it is possible to significantly reduce the amount of computation and reduce the time to solve the problem.

In problem (10) with restrictions (8) and (9), the set of binary variables x_i can be taken as a chromosome, because their values are in the range from 0 to 1 and they completely determine one of the possible solutions to the problem, one form of a career, which is determined by one individual. If we take a set of such individuals, construct genetic operators and iteratively apply them to a given set of individuals, we can find the optimal solution to problem (10).

Consider the main steps of the method for solving the optimization problem based on the evolutionary random search method.

Step 1. Preparing the model. The first step in the method is to prepare the model. It consists in calculating the graph of the order of extraction of the elements of the initial block model of the field. This graph for one set of source data is calculated once, after which it can be saved and reused.

Step 2. Generation of the initial population. The second stage of the method is the generation of the initial population. It consists in generating a given number of random chromosomes (individuals). The number of individuals in a population is determined based on the amount of input data; the larger the initial model, the larger the population size should be. A large population increases diversity and reduces the likelihood of getting into a local extremum. However, the population size and the amount of data sent with a multi-population approach directly affects the amount of memory that is required for the computational procedure, so some balance must be observed. It was established experimentally that a sufficient population to solve the problem is about 100-300 individuals.

Step 3. Assessment of the fitness of chromosomes in a population. To assess the fitness of chromosomes in a population, it is necessary for each chromosome to calculate the value of the fitness function. This function should be chosen so that the higher its value, the higher the quality of the chromosome, the closer it is to the best possible solution.

Functionality (11) is used as a fitness function for assessing the quality of a quarry shape.

$$Z(X) = \sum_{i=1}^{n} v_i x_i \quad (11)$$

Where $X = \{x_1, x_2, ..., x_n\}$ is the chromosome and indicates whether the ith block of the quarry is included in the solution, $V = \{v_1, v_2, ..., v_n\}$ is the value of the weights of the graph of the order of the selection of quarry blocks, vi is the value profit obtained by extracting the next fragment of the quarry, taking into account the percentage of useful components. The value of the fitness function is

calculated as the sum of the weights of all the blocks entering the quarry, corresponding to the form represented by chromosome X.

The search for a solution stops when its implementation ceases to lead to an improvement in the maximum value of the fitness function in the population. The onset of this moment is determined by checking conditions (12):

$$|z_k - z_{k-1}| < \varepsilon \quad (12)$$

Where k ∈ [1, q] is the era of the genetic algorithm, ε is a constant and is selected based on the required calculation accuracy.

Step 4. Selection. The selection or selection of chromosomes consists in the selection (according to the values of the fitness function calculated at the previous stage) of those chromosomes that will participate in the creation of the next population (in the crossing procedure). This corresponds to the principle of natural selection in nature, according to which only the fit individuals survive and give offspring. The result of the selection is a set of chromosomes - the parent pool, which are involved in crossbreeding. The size of the parent pool is equal to the size of the population.

In the genetic method of random search for quarry boundaries, considered in this paper, selection by the roulette method is used.

Step 5. Crossing. After selection of chromosomes in the parent pool, the stage of crossing and production of a new population begins. Crossing occurs in two stages. First, individuals from the parent pool are divided into pairs. At the second stage of crossing, for each pair of chromosomes, a crossing point is determined - a natural number (or several with multi-point crossing), less than the length of the chromosome, indicating which genes parents should exchange. In the developed method of searching for optimal boundaries, multipoint crossing is used, because this type of crossing brings the greatest diversity in the population in this implementation, which reduces the likelihood of falling into a local extremum when searching for the optimal value.

Step 6. Mutation. The last step in the formation of a new population is mutation. It consists in the arbitrary change of a randomly selected gene in the chromosomes. The probability of mutation is set rather small (0.1 - 0.2), because in nature, this phenomenon rarely occurs.

The proposed method allows us to solve the problem of optimizing the shape of quarries from the position of the criterion of maximizing profits based on the targeted selection of many useful fragments of the quarry. In addition, this method has a large resource of parallelism and allows you to implement parallel computational procedures to reduce simulation time.

3. Parallel open pit mining algorithms

To expand the search area, reduce the likelihood of premature convergence and reduce the calculation time, a parallel version of the genetic algorithm was developed - a hierarchical parallel genetic algorithm with two levels of parallelism.

The first level of parallelism is organized through the use of the island model of the multipopulation parallel genetic algorithm. Here, acceleration is achieved by isolating several initial populations that develop independently and periodically exchange the best genetic material. This exchange is carried out through the mechanism of migration of individuals between populations. This approach reduces the likelihood of premature degeneration of populations, increase their diversity and accelerate the convergence of the search algorithm.

The second level of the hierarchy is organized by applying for each subpopulation a single-population model of a parallel genetic algorithm of the “Master-Subordinate” type. It lies in the fact that within the framework of one population, the fitness function of each individual is calculated in a separate stream, which ultimately leads to an acceleration of the algorithm. Moreover, one thread is the main "guardian" of the population and is responsible for the work of genetic operators, and a number of subordinate flows only calculate the fitness function.

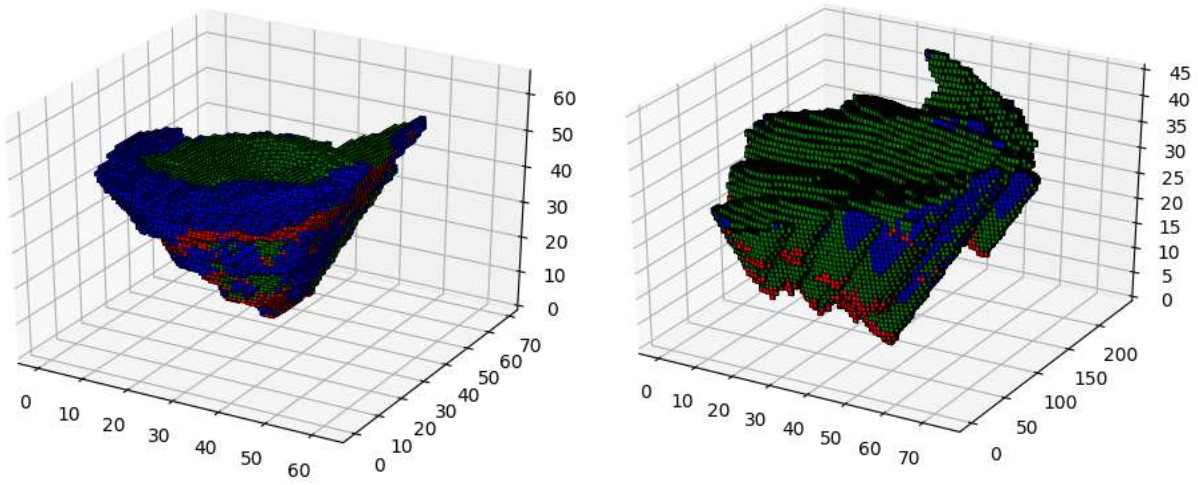
The main advantages of the proposed method are to provide a new principle for solving the problem of quarrying optimization, which allows working directly with a three-dimensional model of

the field, which significantly increases the adequacy of the resulting model. In addition, the possibilities of flexible scaling of the computing process can reduce the computation time of the model almost linearly with an increase in the number of computing resources.

4. Computational Experiments

Block data from the MineLib library [17] were used as initial data for conducting computational experiments. This library contains 11 field models ranging in volume from 1,000 to 2,000,000 blocks. For these models, quarry forms were calculated using the floating cone algorithm, the genetic algorithm and the Lerch-Grossman algorithm, a comparative analysis of the results was carried out, three-dimensional visualizations of the initial model data and calculation results were constructed.

In Fig. 5 (a) shows an example of the search result for the optimal quarry shape using a parallel genetic algorithm for the initial data of the "p4hd" model. Green color shows blocks that are included in the solution, red are blocks that have positive profit but do not included in the final solution, blue are blocks that have negative profit and not included in final solution, in Fig. 5 (b) - results for the initial data of the "mclaughlin_limit" model.



a b

Figure 5. The result of the calculation of data a) of the "p4hd" model, b) of the "mclaughlin_limit" model

Table 1 shows the results of modeling the shape of quarries for various field models using the Lerch-Grossman algorithm and a parallel genetic algorithm.

Table 1. Protocol of the procedure for checking the reliability of the results obtained using the genetic algorithm

Model	Blocks count	Relations count	Lerch-Grossman alg.	Parallel genetic alg.	Absolute error	Relative error
newman1	1,060	3,922	26,086,899	26,083,482	3,417	0.000131
zuck_small	9,400	145,640	1,422,726,898	1,422,610,234	116,664	0.000082
Kd	14,153	219,778	652,195,037	652,096,556	98,481	0.000151
zuck_medium	29,277	1,271,207	1,075,124,490	1,074,938,493	185,997	0.000173
p4hd	40,947	738,609	293,373,256	293,346,559	26,697	0.000091
marvin	53,271	650,631	1,415,655,436	1,415,448,750	206,686	0.000146
w23	74,260	764,786	510,973,998	510,935,675	38,323	0.000075
zuck_large	96,821	1,053,105	122,220,280	122,200,480	19,800	0.000162

sm2	99,014	96,642	2,743,603,730	2,743,271,754	331,976	0.000121
mclaughlin_limit	112,687	3,035,483	1,495,726,474	1,495,579,893	146,581	0.000098
mclaughlin	2,140,342	73,143,770	1,495,886,962	1,495,758,316	128,646	0.000086

The experimental results show that the profit values for various quarry models obtained using the genetic algorithm do not significantly differ from the reference values. This allows us to conclude that the developed algorithm gives reliable results.

The "Nezhegol" supercomputer of the Belgorod State National Research University was used as a technical platform for conducting computational experiments to study the efficiency and scaling of the developed parallel algorithms for searching the boundaries of quarries of ore deposits. For calculations, two cluster nodes were used, with 2 processors with a frequency of 2.4 GHz and 8 cores in each (32 physical computing cores).

Table 2 shows the results of measurements of the simulation time for the "zuck_medium" model using a parallel genetic algorithm using a different number of computational processes, acceleration values and efficiency.

Table 2. Dependence of simulation time, acceleration, and efficiency on the number of processes involved for a parallel genetic algorithm

Cores	Time, sec.	Speedup	Efficiency
1	111 415	1,00	1,00
4	40 342	2,76	0,69
8	20 823	5,35	0,67
12	14 045	7,93	0,66
16	9 915	11,24	0,70
20	7 894	14,11	0,71
24	6 108	18,24	0,76
28	4 934	22,58	0,81
32	4 258	26,17	0,82

The experimental results showed that the proposed approach reduces the solution time by 26 times, while achieving an efficiency level of 0.82.

5. Conclusion

The proposed approaches to finding the optimal quarry shape increase the efficiency of computer modeling of the process of developing open pit open pit mines from a position of reducing the calculation time. The developed cloud information system for the analysis and visualization of data on deposits allows, by reducing the calculation time, to conduct multiple modeling of the quarry shape with different input data and parameters, which in turn makes it possible to operate with more diverse data when planning the development of mineral deposits and in updating the current development plan. Prospects for further development of the topic are associated with the development of parallel methods for searching for career boundaries in the following important areas:

- Development and implementation of field analysis methods for specialized accelerators;
- Development of methods for optimizing the shape of quarries for non-uniform field models presented as octree.

In addition, research can be carried out in the direction of improving the developed methods by additional optimization of the calculation process during data exchange between nodes of a computer system.

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