

СИСТЕМНЫЙ АНАЛИЗ И УПРАВЛЕНИЕ SYSTEM ANALYSIS AND PROCESSING OF KNOWLEDGE

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Pihnastyi O.M.
Belousova Yu.O.

ABOUT THE STABILITY OF FLOW PARAMETERS
OF A PRODUCTION LINE

National Technical University «Kharkiv Polytechnic Institute» 79-2 Pushkinskaya, Kharkov, 61102, Ukraine

e-mail: pihnastyi@gmail.com, aizeku1996@gmail.com

Abstract

The model of the technological process for enterprises with the production method of production is investigated. The dynamics of macroparameters of the production process is considered: inter-operational stocks and the rate of processing of subjects of labour on technological operations along the technological route. Balance equations are presented for macroparameters of the technological process, which express differential dependencies between macroparameters. The linearization of the system of balance equations is performed and the equations of the perturbed state for macroparameters of the closed system of balance equations are written down. Criteria of stability for macroparameters of the production system relative to the normative unperturbed state are determined. Conditions of the stability of the production system functioning are presented. The interrelation between the magnitude of the tempo and the magnitude of the inter-operational reserve of basic products is shown, which ensures the stable functioning of the production process. A system of equations for the perturbed state of a production system for a special case of stability theory is considered (the case of one zero root of the characteristic equation of the system).

Keywords: stability of mass production processes functioning; production systems; flow production line; enterprise; business process; basic product; technological chain; distribution function.

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Пигнастый О.М.
Белюсова Ю.О.

ОБ УСТОЙЧИВОСТИ ПОТОКОВЫХ ПАРАМЕТРОВ
ПРОИЗВОДСТВЕННОЙ ЛИНИИ

Национальный Технический Университет «ХПИ», ул. Пушкинская, д. 79-2, г. Харьков, 61102, Украина

e-mail: pihnastyi@gmail.com, aizeku1996@gmail.com

Аннотация

Исследуется модель производственного процесса для предприятий с поточным методом организации производства. Рассматривается динамика макропараметров производственного процесса: межоперационных заделов и темпа обработки предметов труда на технологических операциях вдоль технологического маршрута. Представлены балансовые уравнения для макропараметров технологического процесса, которые выражают дифференциальные зависимости между макропараметрами. Выполнена линеаризация системы балансовых уравнений и записаны уравнения возмущенного состояния для макропараметров замкнутой системы балансовых уравнений. Определены критерии устойчивости для макропараметров производственной системы относительно нормативного невозмущенного состояния. Показана взаимосвязь величины темпа и величины межоперационных задела базовых продуктов, которая обеспечивает устойчивое

функционирование производственного процесса. Рассмотрена система уравнений возмущенного состояния производственной системы для особого случая теории устойчивости (случай одного нулевого корня характеристического уравнения системы).

Ключевые слова: устойчивость функционирования процессов массового производства; производственные системы; поточная производственная линия; предприятие; бизнес-процесс; базовый продукт; технологическая цепочка; функция распределения базовых продуктов.

INTRODUCTION

The process of uneven development of the economy, especially its individual parts, fluctuations in the volume of production and sales, the emergence of significant production declines can be characterized as a crisis situation, which should be considered as a general regularity inherent to the market economy. Even a stably developing enterprise in a certain period of time can turn out unable to follow the changes in the market situation and to get into the long-term economic crisis.

The enterprise is a complex system that implements a complete set of dynamic processes, which in theory and practice of organizational management are called business processes. Business process – an amount of different activities containing certain resources (tangible, financial, labor, time, etc.) at the input, products and services (tangible and intangible) – at the output (see Figure 1). The process itself is conversion, which adds cost [1-5].

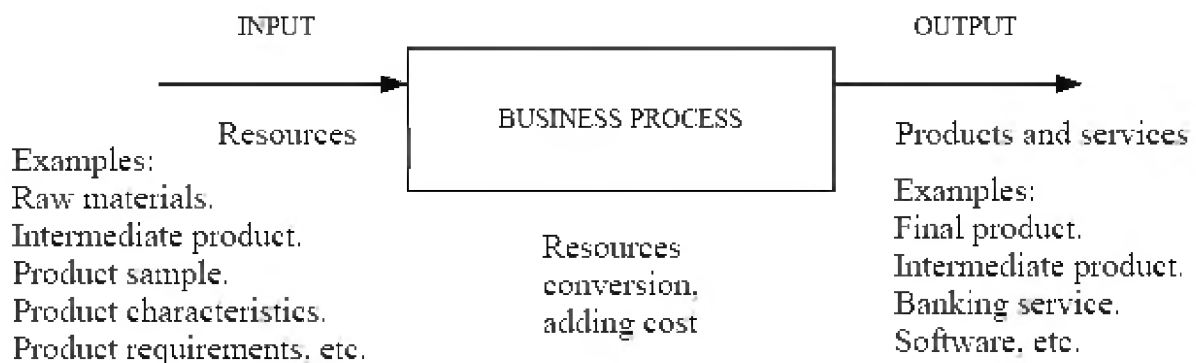


Fig. 1. The presentation of the business process, adopted in the standard ISO 9000

The enterprise as an open system builds its functioning in an essential relationship with the external environment [2, 3]. Therefore, part of the parameters of its business processes, for example, such as product sales volume, tax rates, energy tariffs, market prices, exchange rates, are formed in the external environment and can be interpreted as a demonstration of perturbing factors. Under the perturbing factors there're meant the forces that are not taken into account when describing business processes and, ultimately, affecting the production and output of products. It is known that the impact of these factors on the behavior of the system, which is under consideration, is ambiguous for different business processes: they make an essential impact on some of them, on the other – minor, because the perturbed state differs from the unperturbed not so much. They can act instantly, what comes down to a small change in the initial state of the system, and continuously, i.e. the compiled equations of the business process differ from the true ones by some small correction terms not included in the equations [6,7]. Since the perturbing factors are inevitable, and the presence of dynamic quantities among the parameters of business processes makes the enterprise functioning less manageable and predictable, and the possibility of achieving the goal is small, the task of modeling the stable functioning of processes acquires a very important theoretical and practical significance [7-11]. The stability issue gets the particular importance when the bigger accuracy and speed control are needed. K.Kempf points out that minor deviations in the parameters of the flow line for the production

semiconductor products can lead to large deviations in the flow parameters [12]. The experimental data indicate a deviation in the throughput of the production line from the standard value to 20%. The time of the perturbation progress was several weeks.

1. ANALYSIS AND RESEARCH OF PUBLICATIONS ON THE TOPIC OF THE ARTICLE

The stability research through the business processes parameters of value chains with mass output of products was carried out in the works [8,9,14]. At the same time, the concept of a basic product (BP) was introduced to construct a model of the production process. Basic product – an element of a large system, to which the value is transferred as it moves along the technological chain.

As all the input resources of a business process are basically expressed through costs, the production process will be characterized by the moments of the BP distribution function in terms of the rates of cost changes $\chi(t, S, \mu)$ in the phase space (S, μ) . Here S is the sum of the total costs spent by the enterprise on BP making in the current moment of time; μ – the rate of cost changes. Similar approaches to the systems description were suggested in works [1,16].

Let's write down the production process as a total derivative of the BP distribution function in terms of the rates of cost changes [16,17]:

$$\frac{d\chi(t, S, \mu)}{dt} = \frac{\partial\chi}{\partial t} + \frac{\partial\chi}{\partial S} \mu + \frac{\partial\chi}{\partial \mu} f(t, S) = \lambda_{equip} \{ \psi[\tilde{\mu} \rightarrow \mu] [\chi]_1 - \mu\chi \}, \quad (1)$$

where $f(t, S) = \frac{d\mu}{dt}$ – the function that determines the rate of cost changes of the base product (BP – base product) when it moves along the technological chain of production process [18]; λ_{equip} – equipment density along the technological chain of the production process; $\tilde{\mu} = \mu + \Delta\mu$ – rate of change in BP costs after the impact of technological equipment; $\Delta\mu$ – increase in costs caused by the impact on the BP of technological equipment; $\psi[\tilde{\mu} \rightarrow \mu]$ – the function describing the process of technological equipment impact on BP and determined by its parameters; $[\chi]_1 = \int_0^{\infty} d\mu \cdot \mu \cdot \chi(t, S, \mu)$ – BP motion rate along the technological chain (the first moment of the distribution function) [4,19].

Note that the right side of equation (1) determines the regularities of technological equipment operation during the realization of the production process.

Suppose that [19]

$$\frac{1}{K_v} = \frac{\xi}{\left[\frac{1}{\lambda_{equip}} \right]}$$

where $\frac{1}{\lambda_{equip}}$ characterizes the increase in the cost of BP on a unit of equipment along the entire chain

of technological operations; ξ – step on the variable S , defining its measure unit.

Let us consider the case of an established operating mode of an enterprise with a high density of technological equipment, i.e. $K_v \ll 1$. Multiplying the equation (1) respectively by 1, μ and integrating over the entire range μ , we obtain in zero approximation by the small parameter $K_v \ll 1$ a closed system of equations for describing the production process [14-16]

$$\begin{cases} \frac{\partial [\chi]_0}{\partial t} + \frac{\partial [\chi]_1}{\partial S} = 0; \\ \frac{\partial [\chi]_1}{\partial t} + \frac{\partial [\chi]_2}{\partial S} = f(t, S)[\chi]_0. \end{cases} \quad (2)$$

Getting through the following conditions [20]:

$$\int_0^{\infty} \psi[\mu \rightarrow \tilde{\mu}] d\tilde{\mu} = 1, J = \sum_{m=0}^{\infty} (K_v)^m J_m; \lambda_{equip} \{ \psi[\tilde{\mu} \rightarrow \mu][\chi]_1 - \mu\chi \} = 0; \quad (3)$$

$$\int_0^{\infty} \mu\psi[\mu \rightarrow \tilde{\mu}] d\tilde{\mu} = \mu_{\psi} = \frac{[\chi]_{1\psi}}{[\chi]_0}, \int_0^{\infty} \mu\psi[\mu \rightarrow \tilde{\mu}] d\tilde{\mu} = \int_0^{\infty} \mu \frac{\mu\chi}{[\chi]_1} d\tilde{\mu} = \frac{[\chi]_2}{[\chi]_1}, \quad (4)$$

where $J = \lambda_{equip} \{ \psi[\tilde{\mu} \rightarrow \mu][\chi]_1 - \mu\chi \}$, $[\chi]_0 = \int_0^{\infty} d\mu\chi(t, S, \mu)$ – BP reserves (zero moment of the distribution function); $[\chi]_2 = \int_0^{\infty} \mu^2 \chi d\mu$ – the second moment of the BP distribution function; $[\chi]_{1\psi}$ – the equipment operation rate, defined by its passport data.

The following solution satisfies the system of equations (2) for describing the production process [6-8,13,18]

$$[\chi]_0^* = [\chi]_0^*(t, S); [\chi]_1^* = [\chi]_1^*(t, S), \quad (5)$$

with initial $[\chi]_0^{init} = [\chi]_0(0, S); [\chi]_1^{init} = [\chi]_1(0, S) \quad (6)$

and boundary $[\chi]_0^{bound} = [\chi]_0(t, 0); [\chi]_1^{bound} = [\chi]_1(t, S_d) \quad (7)$

conditions, where S_d – the total average cost price of manufacturing the BP on the characteristic section of the production line.

The solution (5) of the equations system is called the production process plan and is a necessary condition for stability. This plan defines the balance between the BP movement along the technological chain and the required output from production.

The purpose of the article is to study the model of the stable functioning of the system's production process in the case of small perturbations affecting it.

2. MAIN MATERIAL

Let the parameters of the production process – tact and reserve – have random small perturbations $[y]_0, [y]_1$ relative to its unperturbed position $[\chi]_0^*, [\chi]_1^*$, determined by the conditions (5).

We expand these parameters in the neighborhood of the unperturbed position [21, 22]

$$[\chi]_0 = [\chi]_0^* + [y]_0; [\chi]_1 = [\chi]_1^* + [y]_1. \quad (8)$$

Let us linearize the system of equations by small perturbations in the neighborhood of the unperturbed state of the system parameters, expanding the function $f(t, S) \cdot [\chi]_0$ in the neighborhood $[\chi]_0^*, [\chi]_1^*$ [23]:

$$f(t, S)[\chi]_0 = f^*(t, S)[\chi]_0^* + b_1[y]_0 + b_2[y]_1 + \Delta(O^2), \quad (9)$$

where $\Delta(O^2)$ – members of a higher infinitesimal order relatively to perturbations of the production system parameters.

Let us represent the system of equations (2) in the form

$$\frac{\partial[\chi]_0^*}{\partial t} + \frac{\partial[\chi]_1^*}{\partial S} = 0; \quad (10)$$

$$\frac{\partial[\chi]_1^*}{\partial t} + \frac{\partial[\chi]_1^*}{\partial S} \frac{[\chi]_{1\psi}}{[\chi]_0^*} + \frac{[\chi]_1^*}{[\chi]_0^*} \frac{\partial[\chi]_{1\psi}}{\partial S} - \frac{[\chi]_1^*[\chi]_{1\psi}^*}{([\chi]_0^*)^2} \frac{\partial[\chi]_0^*}{\partial S} = f^*(t, S)[\chi]_0^*.$$

Taking into account the expressions (10), the system of equations of the production system state, linearized with respect to small perturbations $[y]_0, [y]_1$ parameters $[\chi]_0^*$ and $[\chi]_1^*$, write down like this:

$$\frac{\partial[y]_0}{\partial t} + \frac{\partial[y]_1}{\partial S} = 0; \quad (11)$$

$$\frac{\partial[y]_1}{\partial t} + \frac{\partial[y]_1}{\partial S} B_{\left(\frac{\partial[y]_1}{\partial S}\right)} + [y]_1 B_{(y_1)} + \frac{\partial[y]_0}{\partial S} B_{\left(\frac{\partial[y]_0}{\partial S}\right)} + [y]_0 B_{(y_0)} = 0,$$

где

$$B_{\left(\frac{\partial[y]_1}{\partial S}\right)} = \frac{[\chi]_{1\psi}}{[\chi]_0^*}, B_{\left(\frac{\partial[y]_0}{\partial S}\right)} = -\frac{[\chi]_1^*[\chi]_{1\psi}^*}{([\chi]_0^*)^2},$$

$$B_{(y_1)} = \frac{1}{[\chi]_0^*} \frac{\partial[\chi]_{1\psi}}{\partial S} - \frac{[\chi]_{1\psi}^*}{([\chi]_0^*)^2} \frac{\partial[\chi]_0^*}{\partial S} - b_2,$$

$$B_{(y_0)} = -\frac{\partial[\chi]_1^*}{\partial S} \frac{[\chi]_{1\psi}}{([\chi]_0^*)^2} - \frac{[\chi]_1^*}{([\chi]_0^*)^2} \frac{\partial[\chi]_{1\psi}}{\partial S} + 2 \frac{[\chi]_1^*[\chi]_{1\psi}^*}{([\chi]_0^*)^3} \frac{\partial[\chi]_1^*}{\partial S} - b \quad (12)$$

The period of the perturbation T_{pert} existence of production indicators in practice usually ranges from several days to several weeks (usually not more than a couple of weeks), while the period of change in coefficients $B_{\left(\frac{\partial[y]_1}{\partial S}\right)}, B_{\left(\frac{\partial[y]_0}{\partial S}\right)}, B_{(y_1)}, B_{(y_0)}$ is determined by the strategic management of the enterprise and ranges from several months to several years [24-26]. The last allows us to assume that the introduced coefficients do not depend explicitly on time during the period of the perturbation T_{pert} existence, since the changes in time $\Delta B_{\left(\frac{\partial[y]_1}{\partial S}\right)}, \Delta B_{\left(\frac{\partial[y]_0}{\partial S}\right)}, \Delta B_{(y_1)}, \Delta B_{(y_0)}$ of a value of the coefficients $B_{\left(\frac{\partial[y]_1}{\partial S}\right)}, B_{\left(\frac{\partial[y]_0}{\partial S}\right)}, B_{(y_1)}, B_{(y_0)}$ for the period of existence of the perturbation T_{pert} of

the production indicators detected by the dispatching service of the enterprise are much smaller than the values of the coefficients themselves:

$$\frac{B\left(\frac{\partial[y_1]}{\partial S}\right)}{T_{pert}} \gg \frac{\partial B\left(\frac{\partial[y_1]}{\partial S}\right)}{\partial t}, \frac{B\left(\frac{\partial[y_0]}{\partial S}\right)}{T_{pert}} \gg \frac{\partial B\left(\frac{\partial[y_0]}{\partial S}\right)}{\partial t}, \frac{B_{(y_1)}}{T_{pert}} \gg \frac{\partial B_{(y_1)}}{\partial t}, \frac{B_{(y_0)}}{T_{pert}} \gg \frac{\partial B_{(y_0)}}{\partial t}.$$

Thus, we will assume that the coefficients in the partial differential equations depend only on S. We expand the small perturbations $[y]_0, [y]_1$ of the parameters $[\chi]_0$ and $[\chi]_1$ in the Fourier series [23]:

$$\begin{aligned} [y]_0 &\approx \{y_0\}_0 + \sum_{j=1}^{\infty} \{y_0\}_j \cdot \sin[k_j \cdot S] + \sum_{j=1}^{\infty} [y_0]_j \cdot \cos[k_j \cdot S]; \\ [y]_1 &\approx \{y_1\}_0 + \sum_{j=1}^{\infty} \{y_1\}_j \cdot \sin[k_j \cdot S] + \sum_{j=1}^{\infty} [y_1]_j \cdot \cos[k_j \cdot S], \end{aligned} \quad (13)$$

where $k_j = \frac{2\pi j}{S_a}$, $\{y_0\}_0, \{y_0\}_j, [y_0]_j, \{y_1\}_0, \{y_1\}_j, [y_1]_j$ – coefficients $[y]_0, [y]_1$.

Substituting $[y]_0, [y]_1$ with their expansion into Fourier series (13), we obtain a new form of equations (11):

$$\begin{aligned} \frac{d\{y_0\}_0}{dt} + \sum_{j=1}^{\infty} \left(\frac{d\{y_0\}_j}{dt} - [y_1]_j k_j \right) \sin[k_j S] + \sum_{j=1}^{\infty} \left(\frac{d[y_0]_j}{dt} + \{y_1\}_j k_j \right) \cos[k_j S] = 0; \quad (14) \\ \left(\frac{d\{y_1\}_0}{dt} + B_{(y_1)} \{y_1\}_0 + B_{(y_0)} \{y_0\}_0 \right) + \sum_{j=1}^{\infty} \left(\frac{d\{y_1\}_j}{dt} - B_{\left(\frac{\partial[y_1]}{\partial S}\right)} [y_1]_j k_j + B_{(y_1)} \{y_1\}_j - \right. \\ \left. - \sum_{j=1}^{\infty} B_{\left(\frac{\partial[y_0]}{\partial S}\right)} [y_0]_j k_j + B_{(y_0)} \{y_0\}_j \right) \sin[k_j S] + \sum_{j=1}^{\infty} \left(\frac{d[y_1]_j}{dt} + B_{\left(\frac{\partial[y_1]}{\partial S}\right)} \{y_1\}_j k_j + \right. \\ \left. + \sum_{j=1}^{\infty} B_{(y_1)} [y_1]_j + B_{\left(\frac{\partial[y_0]}{\partial S}\right)} \{y_0\}_j k_j + B_{(y_0)} [y_0]_j \right) \cos[k_j S] = 0. \end{aligned} \quad (15)$$

Equation (15) must be zero for any values of S. Hence the system of equations for each of the harmonics has the form:

- for $j = 0$

$$\begin{cases} \frac{d\{y_0\}_0}{dt} = 0; \\ \frac{d\{y_1\}_0}{dt} + B_{(y_1)} \{y_1\}_0 + B_{(y_0)} \{y_0\}_0 = 0; \end{cases} \quad (16)$$

- for $j > 0$

$$\left\{ \begin{array}{l} \frac{d\{y_0\}_j}{dt} - [y_1]_j k_j = 0; \\ \frac{d[y_0]_j}{dt} + \{y_1\}_j k_j = 0; \\ \frac{d\{y_1\}_j}{dt} - \mathbf{B}_{\left(\frac{\partial[y]_h}{\partial s}\right)} [y_1]_j k_j + \mathbf{B}_{([y]_h)} \{y_1\}_j - \mathbf{B}_{\left(\frac{\partial[y]_b}{\partial s}\right)} [y_0]_j k_j + \mathbf{B}_{([y]_b)} \{y_0\}_j = 0; \\ \frac{d[y_1]_j}{dt} + \mathbf{B}_{\left(\frac{\partial[y]_h}{\partial s}\right)} \{y_1\}_j k_j + \mathbf{B}_{([y]_h)} [y_1]_j + \mathbf{B}_{\left(\frac{\partial[y]_b}{\partial s}\right)} \{y_0\}_j k_j + \mathbf{B}_{([y]_b)} [y_0]_j = 0; \end{array} \right. \quad (17)$$

with the corresponding characteristic equations:

- for $j = 0$

$$\left| \begin{array}{cc} (\mathcal{G}) & 0 \\ \mathbf{B}_{([y]_b)} & (\mathbf{B}_{([y]_h)} + \mathcal{G}) \end{array} \right| = \mathcal{G}(\mathbf{B}_{([y]_h)} + \mathcal{G}) = 0 \Rightarrow \mathcal{G}_{0_1} = 0; \mathcal{G}_{0_2} = -\mathbf{B}_{([y]_h)}; \quad (18)$$

- for $j > 0$; $k_j = \frac{2 \cdot \pi \cdot j}{S_d}$

$$\left| \begin{array}{cccc} (\mathcal{G}) & 0 & 0 & (-k_j) \\ 0 & (\mathcal{G}) & (k_j) & 0 \\ \mathbf{B}_{([y]_b)} & (-\mathbf{B}_{\left(\frac{\partial[y]_b}{\partial s}\right)} k_j) & (\mathcal{G} + \mathbf{B}_{([y]_h)}) & (-\mathbf{B}_{\left(\frac{\partial[y]_h}{\partial s}\right)} k_j) \\ \mathbf{B}_{\left(\frac{\partial[y]_b}{\partial s}\right)} k_j & \mathbf{B}_{([y]_b)} & \mathbf{B}_{\left(\frac{\partial[y]_h}{\partial s}\right)} k_j & (\mathcal{G} + \mathbf{B}_{([y]_h)}) \end{array} \right| = \quad (19)$$

$$= \left\{ \mathcal{G}(\mathcal{G} + \mathbf{B}_{([y]_h)}) + k_j \mathbf{B}_{\left(\frac{\partial[y]_b}{\partial s}\right)} k_j \right\}^2 + \left\{ \mathcal{G} \mathbf{B}_{\left(\frac{\partial[y]_h}{\partial s}\right)} k_j - k_j \mathbf{B}_{([y]_b)} \right\}^2 = 0$$

Whence we obtain the quadratic equations:

$$\mathcal{G}^2 + \mathcal{G} \left(\mathbf{B}_{([y]_h)} \pm i \mathbf{B}_{\left(\frac{\partial[y]_h}{\partial s}\right)} k_j \right) + \left(k_j^2 \mathbf{B}_{\left(\frac{\partial[y]_b}{\partial s}\right)} \mp i k_j \mathbf{B}_{([y]_b)} \right) = 0, \quad (20)$$

which give the values of the characteristic equation roots:

$$g_{j,2} = -\frac{\left(B_{([y]_h)} + iB_{\left(\frac{\partial[y]_h}{\partial s}\right)} k_j \right)}{2} \pm \sqrt{\frac{\left(B_{([y]_h)} + iB_{\left(\frac{\partial[y]_h}{\partial s}\right)} k_j \right)^2}{4} - \left(k_j^2 B_{\left(\frac{\partial[y]_0}{\partial s}\right)} - k_j B_{([y]_0)} \right)}, \quad (21)$$

$$g_{j,3,4} = -\frac{\left(B_{([y]_h)} - iB_{\left(\frac{\partial[y]_h}{\partial s}\right)} k_j \right)}{2} \pm \sqrt{\frac{\left(B_{([y]_h)} - iB_{\left(\frac{\partial[y]_h}{\partial s}\right)} k_j \right)^2}{4} - \left(k_j^2 B_{\left(\frac{\partial[y]_0}{\partial s}\right)} + ik_j B_{([y]_0)} \right)}.$$

If the roots of equation (21) have a negative real part ($j > 0$), then the production process is stable with respect to the j th harmonic of the Fourier expansion of the perturbation of the production processes description parameters [21,22].

Let us study the system of equations of the production system state (16), linearized with respect to small perturbations $[y]_0$, $[y]_1$ for the case $j = 0$:

$$\begin{cases} \frac{d\{y_0\}_0}{dt} = 0; \\ \frac{d\{y_1\}_0}{dt} + B_{([y]_h)}\{y_1\}_0 + B_{([y]_0)}\{y_0\}_0 = 0; \end{cases} \Rightarrow \begin{cases} \{y_0\}_0 = c_{\{y_0\}_0} = const; \\ \frac{d\{y_1\}_0}{dt} = -B_{([y]_h)}\{y_1\}_0 + A_{\{y_1\}_0}(c_{\{y_0\}_0}). \end{cases} \quad (22)$$

The system of equations (22) includes the characteristic equation (20) with one zero root $g_{0_1} = 0$. Such systems in the stability theory belong to the critical cases of the motion stability study, and the case of one zero root $g_{0_1} = 0$ – to a special case.

The system of equations of the production system state with respect to small perturbations $[y]_0$, $[y]_1$ (22) for the case $j = 0$ has a solution

$$\{y_0\}_0 = c_{\{y_0\}_0} = const, \quad (23)$$

$$\{y_1\}_0 = \delta_{\{y_1\}_0} + \{\tilde{y}_1\}_0(c_{\{y_0\}_0}), \quad \delta_{\{y_1\}_0} = e^{-B_{([y]_h)}t}, \quad \{\tilde{y}_1\}_0(c_{\{y_0\}_0}) = const.$$

The constant $\{\tilde{y}_1\}_0(c_{\{y_0\}_0})$ is defined from equality

$$-B_{([y]_h)} \cdot \{\tilde{y}_1\}_0(c_{\{y_0\}_0}) + A_{\{y_1\}_0}(c_{\{y_0\}_0}) = 0. \quad (24)$$

The trivial solution $\{y_0\}_0 = c_{\{y_0\}_0} = 0$, $\{y_1\}_0 = 0$ is contained in the solution family of the considered system of equations and corresponds to a zero value of the constant $c_{\{y_0\}_0} = 0$ (in this case the system of equations of the production system state with respect to small perturbations $[y]_0$, $[y]_1$ admits an integral – a family of invariant surfaces, on each of which there is a singular point $\{y_0\}_0 = c_{\{y_0\}_0}$, $\{y_1\}_0 = \delta_{\{y_1\}_0} + \{\tilde{y}_1\}_0(c_{\{y_0\}_0})$ [21,22]). The investigated unperturbed steady state of the considered production system (defined by the necessary conditions of the system stability (5)) corresponds to the trivial solution. In the same way other steady states of the considered system correspond to the solution $\{y_0\}_0 = c_{\{y_0\}_0} = const$, $\{y_1\}_0 = \delta_{\{y_1\}_0} + \{\tilde{y}_1\}_0(c_{\{y_0\}_0})$.

Thus, in the special case, the investigated unperturbed state belongs to the family of steady states, which is defined by the system of equations (22). In this case, the unperturbed state is always stable. Then the stability will not be asymptotic. However, any perturbed state sufficiently close to the unperturbed state, not tending to the unperturbed state with $t \rightarrow \infty$, still tends to one of the steady states of the above-mentioned family. In other words, if we use variables $\{y_0\}_0 = c_{\{y_0\}_0}, \delta_{\{y_1\}_0}$, then for any equations solution of the disturbed state for which the initial values $\{y_0\}_0|_0 = c_{\{y_0\}_0}, \delta_{\{y_1\}_0}|_0$ are sufficiently small, the equalities are correct

$$\lim_{t \rightarrow \infty} \{y_0\}_0 = c_{\{y_0\}_0}, \lim_{t \rightarrow \infty} \delta_{\{y_1\}_0} = 0,$$

where $c_{\{y_0\}_0}$ – a certain definite constant (depending on the particular perturbed state taken).

All the states of the family $\{y_0\}_0 = c_{\{y_0\}_0} = cons, \{y_1\}_0 = \delta_{\{y_1\}_0} + \{\tilde{y}_1\}_0(c_{\{y_0\}_0})$, sufficiently close to the unperturbed state, possess exactly the same properties as the unperturbed state. The last is a special case of more general Lyapunov theorem about the special case. Thus, the conditions of the production system stability with respect to small perturbations $[y]_0, [y]_1$ of the parameters $[\chi]_0$ and $[\chi]_1$ we will write down in the form of the characteristic equation roots real part negativity:

- for $j = 0$:

$$\operatorname{Re}[-B_{([y]_1)}] < 0; \quad (25)$$

for $j > 0, k_j = \frac{2 \cdot \pi \cdot j}{S_d}$:

$$\operatorname{Re} \left[-\frac{\left(B_{([y]_1)} + iB_{\left(\frac{\partial [y]_1}{\partial s}\right)} k_j \right)}{2} \pm \sqrt{\frac{\left(B_{([y]_1)} + iB_{\left(\frac{\partial [y]_1}{\partial s}\right)} k_j \right)^2}{4} - \left(k_j^2 B_{\left(\frac{\partial [y]_b}{\partial s}\right)} - i k_j B_{([y]_b)} \right)} \right] < 0; \quad (26)$$

$$\operatorname{Re} \left[-\frac{\left(B_{([y]_1)} - iB_{\left(\frac{\partial [y]_1}{\partial s}\right)} k_j \right)}{2} \pm \sqrt{\frac{\left(B_{([y]_1)} - iB_{\left(\frac{\partial [y]_1}{\partial s}\right)} k_j \right)^2}{4} - \left(k_j^2 B_{\left(\frac{\partial [y]_b}{\partial s}\right)} + i k_j B_{([y]_b)} \right)} \right] < 0. \quad (27)$$

Also note, that investigating the problem of the production process stability we assumed, that the equations of the production system state with respect to small perturbations $[y]_0, [y]_1$ of the parameters $[\chi]_0$ and $[\chi]_1$ are analytical in the considered area and the investigating unperturbed movement lies in this area.

CONCLUSION

Thus, the research of the production system stability with mass production was carried out in the article. For a closed system of equations of the production process parameters state, the equations of the

perturbed state of the system are obtained. A system of equations for the perturbed state of a production system for a special case of stability theory is considered (the case of one zero root of the characteristic equation of the system). For the equation of the perturbed state – equation (11), the conditions of the stability of the production system functioning are obtained – equations (25) – (27). Such approach allows us – through the coefficients $B_{\left(\frac{\partial[y]_1}{\partial S}\right)}$, $B_{\left(\frac{\partial[y]_0}{\partial S}\right)}$, $B_{([y]_1)}$, $B_{([y]_0)}$ – to determine the relationship between

the reserve, BP rate and parameters of the technological equipment, and therefore, to find the necessary equipment characteristics and necessary parameters of the technological process that ensure stable operation of the production.

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Pihnastyi Oleh Mikhailovich, Professor of Department of distributed information systems and cloud technologies, National Technical University "KhPI", Doctor of Engineering

Belousova Yuliia Olehovna, Engineer of Department of distributed information systems and cloud technologies, National Technical University "KhPI"

Пигнастый Олег Михайлович, профессор кафедры распределенных информационных систем и облачных технологий, Национальный технический университет "ХПИ", доктор технических наук

Белусова Юлия Олеговна, инженер кафедры распределенных информационных систем и облачных технологий, Национальный технический университет "ХПИ"