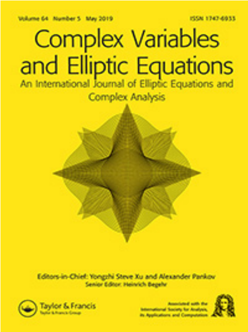

Complex Variables and Elliptic Equations
An International Journal

ISSN: 1747-6933 (Print) 1747-6941 (Online) Journal homepage: https://www.tandfonline.com/loi/gcov20

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To cite this article: H. Begehr, A. M. Nakhushev & A. P. Soldatov (2019) Honoring A.V. Bitsadze's service to science (on his 100th birthday), Complex Variables and Elliptic Equations, 64:5, 721-735, DOI: 10.1080/17476933.2018.1551891

To link to this article: https://doi.org/10.1080/17476933.2018.1551891

Published online: 20 Feb 2019.

COMPLEX VARIABLES AND ELLIPTIC EQUATIONS
2019, VOL. 64, NO. 5, 721–735
https://doi.org/10.1080/17476933.2018.1551891

EDITORIAL

Honoring A.V. Bitsadze's service to science (on his 100th birthday)

Andrei Vasilievich Bitsadze was born on the 9th (12th) of May 1916 in Tchkhruveti in the Chiatura district of the Georgian SSR. Andrei Vasilievich started working quite early in his life. After graduating from the Chiatura Pedagogical Institute, he worked, from the age of 16 already, as a teacher of Mathematics and Physics in the half-empty high schools of his native district. In 1940, he graduated with distinction from the University of Tbilisi and enrolled in the Graduate School of the Tbilisi Mathematical Institute (Academy of Sciences GSSR) where he worked till 1948. It was during this period and under the guidance of Nikolay Muskhelishvili that Andrei Vasilievich carried out his first scientific research on the tangent derivatives of a simple layer potential as well as on the Theory of Elasticity. In particular, he came up with an exact solution of the generalized problem of Hertz on local deformations upon compression of two flat elastic bodies.

It was at this very period that Andrei Vasilievich started dealing with elliptic systems. In the case of such systems with the Laplace operator in the principal part and with real analytical low-order coefficients, he extended at that time the known result of I.N. Vekua. It was at this point in time, it seems, that the foundations were laid for the results on elliptic system on the plane with piecewise constant coefficients that made his name widely known in the scientific world. In 1945, Andrei Vasilievich defended his Master's Thesis in the V.A. Steklov Institute of Mathematics (USSR Academy of Sciences). It was in this Institute where he foremost continued his research on elliptic systems

$$a_0 \frac{\partial^2 u}{\partial x^2} + a_1 \frac{\partial^2 u}{\partial x \partial y} + a_2 \frac{\partial^2 u}{\partial y^2} = 0 \qquad (\mathcal{L})$$

with constant matrix coefficients $a_j \in \mathbb{R}^{l \times l}$. Here, in particular, he looked at the now famous example of the 2×2-elliptic system with coefficients

$$a_0 = -1, \quad a_2 = 1, \quad a_1 = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} \in \mathbb{R}^{2 \times 2},$$

for which the homogeneous Dirichlet problem on the unit circle admits an infinite number of linearly independent solutions. Andrei Vasilievich's talk at the Meeting of the Moscow Mathematical Society, where he mentioned this example, dropped a 'bombshell' as, it was previously tacitly assumed that, as with the case of a single equation, the Dirichlet problem for elliptic systems has a universal character in the sense of its Fredholmness. Later on, Andrei Vasilievich introduced a class of systems that he called weakly connected, for which the Dirichlet problem is a Fredholm one. We describe this class immediately below.

Following M.V. Keldish, we will call the collection of vectors $x_1, \ldots, x_k \in \mathbb{R}^l$, that satisfy the equations

$$p(\nu)x_j + p'(\nu)x_{j-1} + \frac{1}{2}p''(\nu)x_{j-2} = 0, \quad 2 \le j \le k,$$

a chain of eigen- and associated vectors of the quadratic pencil $p(z) = a_0 + a_1 z + a_2 z^2$, corresponding to the eigenvalue ν on the upper half-plane of the characteristic equation $\det p(z) = 0$. In this setup, the elliptic system $\mathcal{L}$ is weakly connected, if there exists in $\mathbb{C}^l$ a basis consisting of such chains.

Using these chains of eigen- and associated vectors of the aforementioned formula, Andrei Vasilievich derived a representation of the general solution u of the elliptic system with the help of analytic functions. Let the set $\sigma(L)$ consist of points $\nu_1, \ldots, \nu_s$ on the upper half plane of the characteristic equation and let the columns of the matrix $b_j \in \mathbb{C}^{l\times l_j}$ be composed of chains corresponding to ν_j. In this case, there exist l_j—vector functions $\psi_j(\zeta)$, which are analytic with respect to the complex variable $\zeta = x + \nu_j y$, such that

$$u(x, y) = \sum_{j=1}^{s} \operatorname{Re} b_j \sum_{k=0}^{l_j-1} \frac{y^k}{k!} \Delta_j^k \psi_i^{(k)}(x + \nu_j y),$$

where Δ_j stands for a matrix whose elements are all zero except for those on the first diagonal above the main diagonal and which are equal to 1.

By means of this representation, Andrei Vasilievich established the Fredholmness of the Dirichlet problem for weakly coupled elliptic systems of the Hölder class where he reduced the problem to an equivalent system of singular integral equations on Γ.

It was Y.B. Lopatynsky (Ukrainian Mathematical Journal, 1953) who first derived sufficient conditions for reducing the boundary value problem of general type into regular integral equations. More exactly he described the method of reducing the boundary-value problem on bounded convex domains to a system of regular integral equations by means of potentials that he suggested. Earlier it was Z. Shapiro who had applied a similar condition to a system with constant coefficients in a three-dimensional domain. At the present time, this condition is known as the Shapiro-Lopatynsky (or 'complementarity') condition.

The question, then, is: how does this condition relate to the concept of weak connectedness? Using methods of linear algebra only, one can show (A. Soldatov, 2010) that the following statements are equivalent:

(a) Elliptic system (L) is weakly connected;
(b) Complementarity condition is satisfied;
(c) The matrix trinomial $p(t) = a_0 + ta_1 + t^2 a_2$ satisfies the condition

$$\det \int_{\mathbb{R}} p^{-1}(t)\,\mathrm{d}t \neq 0.$$

The mentioned Bitsadze example stimulated introducing various classes for which the Dirichlet problem has Fredholm property. By definition, we call an elliptic system strongly elliptic (M.I. Vishik) if the matrix $p(t)$ is positive definite for all $t \in \mathbb{R}$. We call an elliptic system ($\mathcal{L}$) a strenuously elliptic system (A. Soldatov, 2001) if there exists a non-negative definite block matrix $A = (a_{ij})_1^2$, the elements $a_{ij} \in \mathbb{R}^{l\times l}$ of which are related to the coefficients a_j by means of the relations: $a_0 = a_{11}$, $a_1 = a_{12} + a_{21}$, and $a_2 = a_{22}$.

In virtue of condition (c), these systems are knowingly strongly elliptic. Strenuously elliptic systems are characterized by the fact that the Dirichlet problem is uniquely solvable for these systems. For example, the Lamé system of plane anisotropic elasticity is a strenuously elliptic system. A more narrow class is represented by systems such as the Somiliano strongly elliptic systems, which are defined via the condition of a positively definite matrix A.

The results of Andrei Vasilievich on elliptic systems could have sufficed for a Doctoral Thesis. These results later led to his monograph 'Boundary-value problems for elliptic equations of the 2nd order' which was published during the period of his life he had spent in Novosibirsk. However, his second scientific advisor M.A. Lavrentyev continued gas dynamics and in view of the increasing supersonic speeds attained by flying objects in the mid-40s, the research attracted a lot of attention. At this point, Andrei Vasilievich established new methods which opened the door to significant advancements in the theory of equations of mixed and composed types and obtained outstanding results in the conception and investigation of qualitatively new boundary value problems, many of which bear to date his name. Andrei Vasilievich Bitsadze was the first one to prove the theorem of unique solvability of the general mixed problem for the Lavrentyev–Bitsadze equation:

$$(\operatorname{sgn} y)u_{xx} + u_{yy} = 0.$$

Despite the fact that this equation might look a thought one, in reality, it was proposed by M.A. Lavrentyev and Andrei Vasilievich for the description of gas flow, the adiabatic curve of which has a corner point.

In the western literature, the general mixed problem is often known/cited as the Moravets problem. The unique solvability of this problem shows the non-correctness of the Dirichlet problem in the mixed domain, the hyperbolic part of which is convex with respect to characteristics. Later on, however, it turned out that allowing a singularity of an arbitrarily small order of the solution at the boundary point on the line of change of type, preserves the well-poseness of the Dirichlet problem. For the weak solutions of the Tricomi equations, this was proved in the 70s by K. Moravets, for classic solutions of the Laverntyev–Bitsadze equation this fact is also valid.

In 1951, after an outstanding defense of the Doctoral Thesis, Andrei Vasilievich was appointed to the position of senior scientific collaborator at the Steklov Mathematical Institute. At the end of 50s, he was appointed to work in the People's Republic of China, where he investigated and published in several cases, the problems of equations of mixed type and their applications, as well as supervised several students.

In 1958, Andrei Vasilievich, in virtue of his outstanding contribution to Mathematics, was elected corresponding member of the USSR Academy of Sciences. The Novosibirsk era of Andrei Vasilievich's life started in 1959. Upon initiative of M.A. Lavrentyev, S.L. Sobolev and S.A. Christianovich, a Novosibrisk academic city was organized at that time and Andrei Vasilievich chaired the Department of General Theory of Functions of the Mathematics Institute of the Novosibirsk Division of the USSR Academy of Sciences as well as the Chair of Function Theory of the Novosibirsk State University. Here, Andrei Vasilievich continued his research on elliptic equations and equations of mixed type, founding the famous scientific school for these two areas.

In particular, he first derived theorems on existence and dimensionality of the solution space of the oblique derivative problem for harmonic functions in a three-dimensional domain. Very important research of Andrei Vasilievich (together with A.M. Nakhuskev) is dedicated to the problem of searching for multidimensional analogues of the Tricomi problem in mixed domains, where the diversity of changes of type of the equation is manifested either in the space or time form of the oriented surface. Further on, Andrei Vasilievich had to deal with qualitatively new problems both initial and boundary value problems for equations of mixed type on the plane, where the line of change of type is at the same time one of order degeneracy.

Andrei Vasilievich came up with a totally unexpected effect in the theory of hyperbolic systems. It is well known that in the case of a single hyperbolic equation of second order, the Goursat problem is well-posed. Andrei Vasilievich discovered that for linear hyperbolic systems this fact does not hold even in the case of simple roots of the characteristic equation.

In 1971, Andrei Vasilievich was invited to the Steklov Mathematical Institute (USSR Academy of Sciences) in Moscow where he led the recently founded Department of Partial Differential Equations. It was at that year that Andrei Vasilievich was elected full member of the Georgian SSR Academy of Sciences. At the same time and by decision of the Central Committee of the Soviet Communist Party, he successfully led, from 1979 to 1983, the Vekua Institute of Applied Mathematics of the Tbilisi University.

Starting from 1979, Andrei Vasilievich did research on the construction of wide classes of solutions of quasi-linear partial differential equations, encompassing equations such as the one for the gravitational field (Einstein equation), the Heisenberg's Ferromagnetism Theory equation, the Lorentz covariant equations. The solutions Andrei Vasilievich came up with were published, be it in form of monographs or handbooks on the solutions of the aforementioned equations, both as well in the Soviet Union as abroad. At the same time he taught the course on Partial Differential Equations at the Moscow State University.

The work of Andrei Vasilievich on non-local boundary value problems is also well known. The impetus to research in that area was offered by the formulation (together with Samarskii) of the non-local problem for the Laplace equation, where the Dirichlet condition on one part of the boundary gets combined with the values of the solution inside of the domain.

Andrei Vasilievich Bitsadze was an oustanding organizer of Russian science and education. He led several scientific projects and teams. His students are numerous. In particular his Ph.D. (Doctor NAUK) students are: Salakhitdinov, M.S. (Novosibirsk, 1965), Nguen-Tkhia Son (Novosibirsk, 1966], Tovmasyan, N.E. (Novosibirsk, 1966), Tersenov, S.A. (Novosibirsk, 1966), Meredov, M.M. (Novosibirsk, 1968), Prilepko, A.I. (Novosibirsk, 1968), Dzhuraev, T.Dz. (Novosibirsk, 1969), Didenko, V.I. (Novosibirsk, 1970), Nakhushev, A.M. (Novosibirsk, 1971), Yanushauskas, A.I, (Novosibirsk, 1973), Kalmenov, T.Sh. (Tashkent, 1982), Soldatov, A.P. (Moscow, 1982), Kharibegashvili, S.S. (Tbilisi, 1986). Several pioneering results are attributed to him and this in many areas of modern Mathematics and its applications, such as: theory of functions and functional analysis, differential equations and mathematical physics, numerical analysis and mathematical modeling. One will encounter in diverse areas of the theory of partial differential equations the Bitsadze system, the Lavrentyev–Bitsadze equation, the Bitsadze extremum principle, the Bitsadze–Samarksii problem. The name of Andrei Vasilievich Bitsadze, enjoys international reputation. Many of his monographs and handbooks were published and translated abroad in English, German, Chinese, Polish and other languages. Andrei Vasilievich Bitsadze was deputy Editor of the Siberian Mathematical Journal, member of the Office of Division of Mathematics (USSR Academy of Sciences), member of the National Committee of Mathematicians. His contribution was highly appreciated by the State, with the following decorations bestowed to him: the Order of Lenin (1971), the Order of the October revolution (1985), two Orders of the Labour Red Sign (1966, 1975).

Andrei Vasilievich Bitsadze lived a bright civil and scientific life (5.9.1916–4.6.1994). A man of principles, justly, at times emotional, always at the forefront, was treated by the scientific community as an authority in moral questions and served as example for the youth. His adherence to principles and insight when it came to evaluating people, were the causes of several failures when it came to him being elected a full member of the (USSR) Academy of Sciences. Having said that, taking into account his personality and outstanding scientific contribution, he would have deserved, even long time ago, his being elected. Among Andrei Vasilievich Bitsadze's students one will find over 13 Doctors of Science and 30 Candidates of Science; however, the number of people whom he helped and among whom he left a good memory, is considerably higher.

A.V.Bitsadze's Publications

1941

Tangent derivative of the single layer potential. In the book of N.I. Muskhelishvili "Singular integral equations". Moscow, 1946. (Russian).

1942

On local deformation under compression of elastic bodies, Soobshch. Akad. Nauk Gruz. SSR 3, 419–424 (1942). (Russian) Zbl. 0063.00374.

1943

On the general representation of solutions of elliptic differential equations, Soobshch. Akad. Nauk Gruz. SSR 4, 613–622 (1943). (Georgian, Russian summary) Zbl 0063.00375.

1944

Boundary value problems for systems of linear differential equations of elliptic type, Soobshch. Akad. Nauk Gruz. SSR. 5. No. 8 (1944). (Russian).

General representation of solutions of elliptic systems of differential equations and some of their applications. Ph.D. thesis. Akad. Sci. Gruz. SSR. 1944. (Russian).

1946

On some applications of the general representation of solutions of elliptic differential equations, Soobshch. Akad. Nauk Gruz. SSR. 7, No. 6 (1946). (Russian).

1947

Problems of oscillation of uniformly compressed thin elastic plate. Proceedings of Tbilisi State University, 30(1947) (Russian).

1948
General representation of solutions of the system of differential equations of the second order of elliptic type. In the book of I.N. Vekua "New methods for solving elliptic equations". Mocow-Liningrad, Nauka, 1948. §28, 29, 30 (Russian).
On the uniqueness of the solution of the Dirichlet problem for elliptic partial differential equations, Usp. Mat. Nauk 3, No. 6(28), 211–212 (1948). (Russian) Zbl 0041.21703.
On the so-called areal-monogenic functions, Dokl. Akad. Nauk SSSR, n. Ser. 59, 1385–1388 (1948). (Russian) Zbl 0032.34504.

1950
About one system of functions, Usp. Mat. Nauk 5, No. 4(38), 154–155 (1950). (Russian) Zbl 0041.03303.
On the uniqueness of the solution of the general boundary value problem for mixed - type equation, Soobshch. Akad. Nauk Gruz. SSR 11, 205–211 (1950). (Russian) Zbl 0041.21704.
On the problem of mixed type equations (with M.A. Lavrentiev), Dokl. Akad. Nauk SSSR, n. Ser. 70, 373–376 (1950). (Russian) Zbl 0040.20001.
On some mixed type problems, Dokl. Akad. Nauk SSSR, n. Ser. 70, 561–564 (1950). (Russian) Zbl 0040.20002.
On the problem of mixed type equations, Abstract of the thesis for degree of doctor of Science, Steklov Mathematical Institute of SSSR Acad. Sci. 1950. (Russian).

1951
On a general problem of mixed type, Dokl. Akad. Nauk SSSR, n. Ser. 78, 621–624 (1951). (Russian) Zbl 0042.33204.

1953
On the problem of mixed type equations, Tr. Mat. Inst. Steklova 41, 59 pp. (1953). (Russian) Zbl 0052.09703.
On an equation of mixed type, Usp. Mat. Nauk 8, No. 1(53), 174–175 (1953). (Russian) Zbl 0050.09404.
A spatial analogue of the Cauchy integral and some of its applications, Izv. Akad. Nauk SSSR, Ser. Mat. 17, 525–538 (1953). (Russian) Zbl 0053.39203.
A spatial analogue of the Cauchy integral and some of its applications, Dokl. Akad. Nauk SSSR, n. Ser., 389–392 (1953). (Russian) Zbl 0053.07202.
Inversion of one system of singular integral equations, Dokl. Akad. Nauk SSSR, n. Ser. 93, 595-597 (1953); errata 94, 980 (1954). (Russian) Zbl 0052.33606.

1955
On two-dimensional Cauchy type integrals, Soobshch. Akad. Nauk Gruz. SSR 16, 177–184 (1955). (Russian) Zbl 0065.08904.
Equations of mixed type, Proceedings of the 3rd all-Union Congress of Mathematics., Beijing, 1955. (Chinese).

1956
On a Frankl problem, Dokl. Akad. Nauk SSSR 109, 1091–1094 (1956). (Russian) Zbl 0085.30805.
On the problem of mixed type equations in multidimensional domains, Dokl. Akad. Nauk SSSR 110, 901–902 (1956). (Russian) Zbl 0074.07802.

Linear partial differential equations of mixed type, Proceedings of the 3rd all-Union Congress of Mathematics, 3, 1956.

1957
On the uniqueness of the solution of Frankl's problem for Chaplyguin's equation, Dokl. Akad. Nauk SSSR 112, 375–376 (1957). (Russian) Zbl 0085.30901.
On elliptical systems of second order partial differential equations, Dokl. Akad. Nauk SSSR 112, 983–986 (1957). (Russian) Zbl 0078.09001
On an elementary method for solving some boundary value problems of the holomorphic function theory and related singular integral equations, Usp. Mat. Nauk 12, No. 5(77), 185–190 (1957); corrections 13, No. 2(80), 270 and No. 4(82), 232 (1958). (Russian) Zbl 0099.06101.
Monograph on mathematics (review), Nature 10 (1957).
Zum Problem der Gleichungen vom gemischen Typus (On the problem of equations of mixed type), Verlag der Wissenschaften, Berlin, 1957 (German).

1958
Some linear problems for linear partial differential equations, Beijing. 1958. (Chinese)
Ill-posedness of the Dirichlet problem for mixed-type equations in mixed domains, Dokl. Akad. Nauk SSSR 122, 167–170 (1958). (Russian) Zbl 0145.35403.
Linear partial differential equations of mixed type, Trudy tret'ego vsesojuzn. mat. S'ezda, Moskva, Ijun-Ijul' 1956, 3, 36–42 (1958). (Russian) Zbl 0088.07404.

1959
Equation of mixed type, Publishing house, Moscow, Acad. Nauk SSSR, 1959. 164 pp. (Russian). Zbl 0087.09403.

1961
On the theory of equation of mixed-composite type (with M.S. Salakhitdinov), Sib. Mat. Zh. 2, 7–19 (1961). (Russian) Zbl 0121.07901.
On the equation of mixed-composite type. Nekot. Probl. Mat. Mekh. (Some problems of Mathematics and Mechanics) 47–49 (1961). Akad. Nauk SSSR, Sibirian Branch. (Russian) Zbl 0144.34701.
On the theory of harmonic functions, Proc. Tbilisi State Univ., 84 (1961). (Russian).

1962
On a three-dimensional analogue of the Tricomi problem, Sib. Mat. Zh. 3, 642–644 (1962). (Russian) Zbl 0148.08401.
Mixed-type equation in three-dimensional domains, Sov. Math., Dokl. 3, 510–512 (1962); translation from Dokl. Akad. Nauk SSSR 143, 1017–1019 (1962). (English. Russian original) Zbl 0125.05501.

1963
On the problem with an oblique derivative for harmonic functions in three-dimensional domains. Materials for the joint Soviet-American Symposium on partial differential equations. Akad. Nauk. SSSR, Siberian branch. Novosibirsk, 1963.
A homogeneous problem for the slant derivative for harmonic functions in three-dimensional regions, Sov. Math., Dokl. 4, 156–159 (1963); translation from Dokl. Akad. Nauk SSSR 148, 749–752 (1963). (English. Russian original) Zbl 0127.05302.

1964

The problem of the oblique derivative with polynomial coefficients, Sov. Math., Dokl. 5, 1102–1104 (1964); translation from Dokl. Akad. Nauk SSSR 157, 1273–1275 (1964). (English. Russian original) Zbl 0196.41702.

A particular case of the problem of the slant derivative for harmonic functions in three-dimensional regions, Sov. Math., Dokl. 5, 477–478 (1964); translation from Dokl. Akad. Nauk SSSR 155, 730–731 (1964). (English. Russian original) Zbl 0196.41701.

A class of multidimensional singular integral equations, Sov. Math., Dokl. 5, 1616–1618 (1964); translation from Dokl. Akad. Nauk SSSR 159, 955–957 (1964). (English. Russian original) Zbl 0142.09504.

Equations of the mixed type. Translated by P. Zador. Oxford-London-New York-Paris, Pergamon Press, XIII, 160 pp., 1964. (English) Zbl 0111.29205.

1965

Normally solvable elliptic boundary value problems. Sov. Math., Dokl. 6, 1347–1349 (1965); translation from Dokl. Akad. Nauk SSSR 164, 1218–1220 (1965). (English. Russian original) Zbl 0146.34501.

1966

A criterion for the convergence of the gradients of a sequence of harmonic functions, Sov. Math., Dokl. 7, 708–709 (1966); translation from Dokl. Akad. Nauk SSSR 168, 733–734 (1966). (English. Russian original) Zbl 0177.37501.

Boundary value problems for second order elliptic equations. Nauka, Moscow, 1966. (Russian) Zbl 0148.08904.

1967

On the Schwarz Lemma, Tr. Tbilis. Mat. Inst. Razmadze 33, 15–20 (1967). (Russian) Zbl 0207.08001.

Lectures on the theory of analytical functions of a complex variable. Novosibirsk State University, 1967. (Russian).

1968

Boundary value problems for second order elliptic equations. Translated from the Russian by Scripta Technica, Ltd. Translation edited by M.J. Laird. North-Holland Series in Applied Mathematics and Mechanics. 5, North-Holland Publishing Company, Amsterdam, 211 pp., 1968). (English) Zbl 0167.09401.

1969

On some simple generalizations of linear elliptic boundary problems, (with A.A. Samarskii), Sov. Math., Dokl. 10, 398–400 (1969); translation from Dokl. Akad. Nauk SSSR 185, 739–740 (1969). (English. Russian original) Zbl 0187.35501.

On the theory of equations of mixed type. Ellipt. Differentialgl., Kolloquium Berlin 1969, 91–96 (1971). (German) Zbl 0188.41003.

Fundamentals of the theory of analytical functions of a complex variable. Nauka, Moscow, 1969, 239 pp. (Russian) Zbl 0183.33601.

1970
Partial differential equations. Proceedings of a symposium dedicated to the 60th birthday of the member of the Academy, Sergei L'vovic Sobolev. (Differencial'nye uravnenija s castnymi proizvodnymi. Trudy simpoziuma posvjascennogo 60-letiju akademika Sergeja L'vovica Soboleva.), Izdatel'stvo Nauka, Moscow, 251 pp., 1970. (Russian) Zbl 0331.00018.
On the theory of non-Fredholm elliptic boundary value problems. In: Partial differential equations. Proceedings of the Symposium dedicated to S.L. Sobolev. Nauka, Moscow. 1970. (Russian).
On the theory of a class of equations of mixed type. Nekot. Probl. Mat. Mekh., k Semidesyatiletiyu Akad. M.A.Lavrent'ev, 112–119 (1970). (Russian) Zbl 0218.35066.
On the theory of equations of mixed type, Differ. Uravn. 6, 3–6 (1970). (Russian) Zbl 0213.37301.

1971
Theory of equations of mixed type, elliptic differential equations, Vol II., Akademie-Verlag, Berlin, 1971. (German).
On the theory of quasilinear ordinary differential equations of first order. Proc. Steklov Inst. Math. 112, 94–104 (1971); translation from Trudy Mat. Inst. Steklov 112, 95–104 (1971). (English. Russian original) Zbl 0283.34002.
Sur la theorie des problemes aux limites elliptiques nonfredholmiens. (On the theory of non-Fredholm elliptic boundary value problems). Actes Congr. internat. Math. 1970, 2, 685–690 (1971). (French) Zbl 0231.35025.

1972
On the theory of non-Fredholm elliptic boundary value problems. Proceedings of the International Congres of Mathematics in Nice. Nauka, Moscow, 1972.
On a class of equations of mixed type, Beitr. Analysis 4, 39–45 (1972). (Russian) Zbl 0261.35060.
On a system of linear partial differential equations, Sov. Math., Dokl. 13, 766–769 (1972); translation from Dokl. Akad. Nauk SSSR 204, 1031–1033 (1972). (English. Russian original) Zbl 0261.35014.
Theory of degenerate hyperbolic equations in multidimensional domains, (with A.M. Nakhushev), Sov. Math., Dokl. 13, 807–810 (1972); translation from Dokl. Akad. Nauk SSSR 204, 1289–1291 (1972). (English. Russian original) Zbl 0259.35057.
Correct formulations of problems for equations of mixed type in multidimensional domains, (with A.M. Nakhushev), Sov. Math., Dokl. 13, 857–860 (1972); translation from Dokl. Akad. Nauk SSSR 205, 9–12 (1972). (English. Russian original) Zbl 0259.35058.
Fundamentals of the theory of analytical functions of a complex variable (Osnovy teorii analiticeskih funkcii kompleksnogo peremennogo), second amended edition. Nauka, Moscow, 264 pp., 1972. (Russian) Zbl 0259.30001.
On the theory of mixed type equations whose order degenerates along the line of type change. Continuun mechanics and related problems of analysis. (To Muskhelishvili's 80th anniversary). Meh. splosn. Sredy rodstv. Probl. Analiz., 47–52 (1972). (Russian) Zbl 0254.35089.

Lecture on equations of mathematical physics. Engineering Physics Institute, Moscow, 1972. (Russian).
Equations of mixed type. Differ. Equations 6 (1970), 1–4 (1972). (English) Zbl 0247.35085.

1973
Grundlagen der Theorie analytischer Funktionen (Foundations of the theory of analytic functions), ed. in German by Udo Pirl. Akademie-Verlag, Berlin, 186 pp., 1973. (German) Zbl 0259.30002.
Partial differential equations. Mathematical Encyclopedia, 1973. (Russian).
Boundary value problems. Greet Soviet Encyclopedia (3rd edition). 1973. (Russian).

1974
On the theory of mixed type equations in multidimensional domains, (with A.M. Nakhushev), Differ. Uravn. 10, 2184-2191 (1974). (Russian) Zbl 0295.35062.
Introductory course of mathematical analysis. Moscow engineering physics Institute, Moscow, 1974. (Russian).
On the theory of the Maxwell-Einstein equation, (with V.I. Pashkovskij), Sov. Math., Dokl. 15, 762–764 (1974); translation from Dokl. Akad. Nauk SSSR 216, 249–250 (1974). (English. Russian original) Zbl 0298.35052.
On an application of functional theoretical methods in the linearized Navier-Stokes boundary value problem, Ann. Akad. Sci. Fennicae, Ser. A, Mathematica, 571 (1974).

1975
On the application of methods of function theory in the linearized Navier-Stokes problem, 5. Tag. Probl. Meth. Math. Phys., Heft 2, Karl-Marx-Stadt, 1973, 256–262 (1975). (German) Zbl 0422.35066.
On the question of formulating the characteristic problem for second order hyperbolic systems, Sov. Math., Dokl. 16, 1062–1066 (1975); translation from Dokl. Akad. Nauk SSSR 223, 1289–1292 (1975). (English. Russian original) Zbl 0333.35050.
The influence of the minor terms on the correctness of the formulation of characteristic problems for hyperbolic systems of second order, Sov. Math., Dokl. 16, 1437–1440 (1975); translation from Dokl. Akad. Nauk SSSR 225, 31–34 (1975). (English. Russian original) Zbl 0331.35040.
On an equation of the gravitational field, Sov. Math., Dokl. 16, 693–696 (1975); translation from Dokl. Akad. Nauk SSSR 222, 765–768 (1975). (English. Russian original) Zbl 0328.35018.
On some classes of solution to the Maxwell-Einstein equation, (with V.I. Pashkovskii), Tr. Mat. Inst. Steklova 134, 26–30 (1975). (Russian) Zbl 0318.35024.
An approximate set of exercises for the course of equations of mathematical physics. Moscow Engineering Physics Institute, Moscow, 1975. (Russian).

1976
Equations of mathematical physics. (Uravneniya matematicheskoj fiziki). Nauka, Moscow, 295 pp., 1976. (Russian) Zbl 0499.35001.

On the theory of systems of partial differential equations, Proc. Steklov Inst. Math. 142, 69–79 (1979); translation from Tr. Mat. Inst. Steklov 142, 67–77 (1976). (English. Russian original) Zbl 0412.35001.
On the theory of a certain class of equations of mixed type, Am. Math. Soc., Translat., II. Ser. 104, 97–103 (1976). (English) Zbl 0327.35052.
On the current state of the theory of mixed-type equations, Beitr. Analysis 8, 59–65 (1976). (Russian) Zbl 0326.35060.
The theory of non-Fredholm elliptic boundary value problems, Am. Math. Soc., Translat., II. Ser. 105, 95–103 (1976). (English) Zbl 0364.35019.
On a class of nonlinear partial differential equations. Funct. theor. Meth. part. Differ. Equat., Proc. int. Symp., Darmstadt 1976, Lect. Notes Math. 561, 10–16 (1976). (English) Zbl 0339.35026.
Equations of mixed type in multidimensional regions, (with A.M. Nakhushev), Differ. Equations 10 (1974), 1689-1694 (1976). (English) Zbl 0321.35060.

1977
Collection of problems on the equations of mathematical physics (Sbornik zadach po uravneniyam matematicheskoj fisiki), (with D.F. Kalinichenko), Nauka, Moscow, 223 pp., 1977. (Russian) Zbl 0464.35079.
On a class of quasi-linear partial differential equations. Problems of mathematical physics and numerical mathematics, Work Collect., Moscow 1977, 63–70 (1977). (Russian) Zbl 0409.35002.
Theory of a class of nonlinear partial differential equations, Differ. Equations 13, 1388–1399 (1977). (English) Zbl 0394.35025.
On the Dirichlet and Neumann problems for nonlinear elliptic equations of second order, Sov. Math., Dokl. 18, 615–619 (1977); translation from Dokl. Akad. Nauk SSSR 234, 265–268 (1977). (English. Russian original) Zbl 0385.35029.
Theory of a class of nonlinear partial differential equations, Differ. Uravn. 13, 1993–2008 (1977). (Russian) Zbl 0384.35017.
On some classes of solutions of the Maxwell-Einstein equation, (with V.I. Pashkovskij), Proc. Steklov Inst. Math. 134 (1975), 31–35 (1977). (English) Zbl 0373.35015.
On some classes of exact solutions of the gravitational field equations, Sov. Math., Dokl. 18, 411–412 (1977); translation from Dokl. Akad. Nauk SSSR 233, 517–518 (1977). (English. Russian original) Zbl 0371.35037.
On some classes of exact solutions of the system of Maxwell-Einstein equations, Festakt 200. Wiederkehr des Geburtstages von Carl Friedrich Gauss, Berlin, 1977.
On the Tricomi problem for nonlinear equations of mixed type, Sov. Math., Dokl. 18 (1977), 999–1003 (1978); translation from Dokl. Akad. Nauk SSSR 235, 733–736 (1977). (English. Russian original) Zbl 0378.35053.

1978
Waves in the flow of a liquid of variable density, Differ. Equations 14, 750–754 (1978). (English) Zbl 0448.76025.
Waves in a liquid flow of variable density, Differ. Uravn. 14, 1053–1059 (1978). (Russian) Zbl 0403.76036.

On a boundary value problem for the Helmholtz equation, Sov. Math., Dokl. 19, 494–496 (1978); translation from Dokl. Akad. Nauk SSSR 239, 1273–1275 (1978). (English. Russian original) Zbl 0398.35032.

1979
On a system of nonlinear partial differential equations, Differ. Uravn. 15, 1267–1270 (1979). (Russian) Zbl 0418.35029.

1980
Equations of mathematical physics. Transl. from the Russian by V. M. Volosov and I. G. Volosova, Mir., Moscow, 318 pp., 1980. (English) Zbl 0499.35002.
On exact solutions of a version of the gravitational field equations, Sov. Math., Dokl. 22, 53–54 (1980); translation from Dokl. Akad. Nauk SSSR 253, 266–267 (1980). (English. Russian original) Zbl 0482.35056.
Some problems of the theory of differential equations. Work collection. (Nekotorye voprosy teorii obyknovennykh differentsial'nykh uravnenii. Sbornik trudov). Trudy Instituta Prikladnoj Matematiki im. I. N. Vekua. Vyp. 8. Tbilisskij Gosudarstvennyj Universitet. Tbilisi: Izdatel'stvo Tbilisskogo Universiteta, 170 pp., 1980). (Russian) Zbl 0476.00009.
A system of nonlinear partial differential equations, Differ. Equations 15, 903–905 (1980). (English) Zbl 0431.35028.

1981
Some classes of partial differential equations, (Nekotorye klassy uravnenij v chastnykh proizvodnykh), Nauka, Moscow, 448 pp., 1981. (Russian) Zbl 0511.35001.
Exact solutions of some variants of equations of a gravitational field, Tr. Mat. Inst. Steklova 157, 19–24 (1981). (Russian) Zbl 0493.35061.
On exact solutions of a class of systems of quasilinear partial differential equations, Sov. Math., Dokl. 23, 319–322 (1981); translation from Dokl. Akad. Nauk SSSR 257, 780–783 (1981). (English. Russian original) Zbl 0468.35018.

1982
Equations of mathematical physics. Textbook. (Uravneniya matematicheskoj fiziki. Uchebnoe posobie). 2nd ed., rev. and compl. Nauka, Moscow, Glavnaya Redaktsiya Fiziko-Matematicheskoj Literatury, 336 pp., 1982. (Russian) Zbl 0554.35001.
On the Cauchy problem for a class of nonlinear first-order partial differential equations, Sov. Math., Dokl. 26, 5–7 (1982); translation from Dokl. Akad. Nauk SSSR 265, 14–16 (1982). (English. Russian original) Zbl 0524.35022.
On a nonlinear equation of parabolic type, Sov. Math., Dokl. 25, 856–858 (1982); translation from Dokl. Akad. Nauk SSSR 264, 1293–1295 (1982). (English. Russian original) Zbl 0517.35045.
Exact solutions of certain nonlinear partial differential equations, Differ. Equations 17, 1100–1104 (1982); translation from Differ. Uravn. 17, 1774–1778 (1981). (English. Russian original) Zbl 0488.35066.

1983
On the theory of self-dual SU(3) gauge fields, Sov. Math., Dokl. 27, 523–525 (1983); translation from Dokl. Akad. Nauk SSSR 270, 21–23 (1983). (English. Russian original) Zbl 0542.35076.

A new class of exact solutions of Yang's equations for SU(2) gauge fields, Sov. Math., Dokl. 27, 396–399 (1983); translation from Dokl. Akad. Nauk SSSR 269, 781–784 (1983). (English. Russian original) Zbl 0542.35075.
Exact solutions of some variants of the equations of the gravitational field, Proc. Steklov Inst. Math. 157, 19–24 (1983). (English) Zbl 0518.35020.
A collection of problems on the equations of mathematical physics, (with D.F. Kalinichenko), Transl. from the Russian by V. M. Volosov and I. G. Volosova. Mir Publishers, Moscow, 334 pp., 1983. (English) Zbl 0508.35001.

1984
On the construction of exact solutions for some classes of nonlinear equations describing non-stationary processes, Current problems of mathematical physics and numerical mathematics, Collect. Artic., Moscow. 1984, 34–40 (1984). (Russian) Zbl 0594.35048.
On a class of exact solutions of Lorentz-covariant equations, Sov. Math., Dokl. 30, 65–66 (1984); translation from Dokl. Akad. Nauk SSSR 277, 274–276 (1984). (English. Russian original) Zbl 0592.35085.
On the theory of nonlocal boundary value problems, Sov. Math., Dokl. 30, 8–10 (1984); translation from Dokl. Akad. Nauk SSSR 277, 17–19 (1984). (English. Russian original) Zbl 0586.30036.
Foundations of the theory of analytical functions of a complex variable. Textbook. (Osnovy teorii analiticheskikh funktsij kompleksnogo peremennogo. Uchebnik). 3rd ed., compl., Nauka, Moscow, Glavnaya Redaktsiya Fiziko-Matematicheskoj Literatury, 320 pp., 1984. (Russian) Zbl 0583.30002.
Equations of mathematical physics (Rownania fizyki matematycznej), Warsaw, 1984. (Polish).
Some problems of dynamics of the Georgian Black seacoast, (with R. I. Sajaga, S.H. Topurca), Proc. Akad. Nauk. Georgian SSR, 113, No. 1 (1984). (Russian).

1985
On a class of conditionally solvable nonlocal boundary value problems for harmonic functions, Sov. Math., Dokl. 31, 91–94 (1985); translation from Dokl. Akad. Nauk SSSR 280, 521–524 (1985). (English. Russian original) Zbl 0607.30039.

1986
Singular integral equations of the first kind with Neumann kernels, Differ. Uravn. 22, No. 5, 823–828 (1986). (Russian) Zbl 0604.45003; Differ. Equations 22, 591–604 (1986). (English) Zbl 0614.45008.
On some integral equations of the first kind, Sov. Math., Dokl. 33, 270–272 (1986); translation from Dokl. Akad. Nauk SSSR 286, 1292–1295 (1986). (English. Russian original) Zbl 0604.45002.
Cauchy problem for harmonic functions, Differ. Equations 22, 8–14 (1986); translation from Differ. Uravn. 22, No. 1, 11–18 (1986). (English. Russian original) Zbl 0597.31001.

1987
On polyharmonic functions, Sov. Math., Dokl. 35, No. 3, 540–544 (1987); translation from Dokl. Akad. Nauk SSSR 294, No. 3, 521–525 (1987). (English. Russian original) Zbl 0666.31008.

Partial differential equations, (with V.S. Vinogradov, A.A. Dezin, V.A. Il'in), Proc. Steklov Inst. Math. 176, 263–300 (1988); translation from Tr. Mat. Inst. Steklova 176, 259–299 (1987). (English. Russian original) Zbl 0656.35002; (Russian) Zbl 0636.35003.
The multidimensional Hilbert transform, Sov. Math., Dokl. 35, 390–392 (1987); translation from Dokl. Akad. Nauk SSSR 293, 1039–1041 (1987). (English. Russian original) Zbl 0632.44004.

1988
Integral equations of the first kind with singular kernels generated by the Schwarz kernel, Sov. Math., Dokl. 38, No. 1, 188–194 (1989); translation from Dokl. Akad. Nauk SSSR 301, No. 6, 1289–1294 (1988). (English. Russian original) Zbl 0673.45002.
Some properties of polyharmonic functions, Differ. Equations 24, No. 5, 543–548 (1988); transl. from Differ. Uravn. 24, No. 5, 825–831 (1988). (English. Russian original) Zbl 0663.31007.
Some classes of partial differential equations, transl. from the Russian by H. Zahavi. Advanced Studies in Contemporary Mathematics. 4. New York etc.: Gordon and Breach Science Publishers. xi, 504 p. (1988). (English) Zbl 0749.35002.

1989
On integral equations of the linear theory of contact problems, Sov. Math., Dokl. 38, No. 3, 496–500 (1989); translation from Dokl. Akad. Nauk SSSR 303, No. 2, 265–270 (1988). (English. Russian original) Zbl 0674.45002.

1990
On the Neumann problem for harmonic functions, Sov. Math., Dokl. 41, No. 2, 193–195 (1990); translation from Dokl. Akad. Nauk SSSR 311, No. 1, 11–13 (1990). (English. Russian original) Zbl 0722.35032.

1991
Singular integral equations of the first kind, Proc. Steklov Inst. Math. 200, 49–59 (1993); translation from Tr. Mat. Inst. Steklova 200, 46–56 (1991). (English. Russian original) Zbl 0793.45004.

1992
On a hyperbolic system of first-order quasilinear equations, Russ. Acad. Sci., Dokl., Math. 46, No. 3, 454–457 (1993); translation from Dokl. Akad. Nauk, Ross. Akad. Nauk 327, No. 4-6, 423–427 (1992). (English. Russian original) Zbl 0791.35075.
Function-theoretic methods for singular integral equations, Complex Variables, Theory Appl. 19, No. 1–2, 1–13 (1992). (English) Zbl 0767.45001.

1993
Two-dimensional analogues of the inversion formulas of Hardy and Hilbert, Russ. Acad. Sci., Dokl., Math. 48, No. 3, 635–639 (1994); translation from Dokl. Akad. Nauk, Ross. Akad. Nauk 333, No. 6, 696–698 (1993). (English. Russian original) Zbl 0827.44001.

1994
On structural properties of solutions of hyperbolic systems of partial differential equations of the first order, Mat. Model. 6, No. 6, 22–31 (1994). (Russian. English summary) Zbl 1075.35535.

Partial differential equations. Series on Soviet and East European Mathematics. 2. Singapore: World Scientific. xiii, 227 pp. (1994). (English) Zbl 0956.35501.
On the theory of quasilinear partial differential equations, Differ. Equations 30, No. 5, 749–754 (1994); translation from Differ. Uravn. 30, No. 5, 814–820 (1994). (English. Russian original) Zbl 0847.35033.

1995
Integral equation of the first kind, Series on Soviet and East European Mathematics. 7. Singapore: World Scientific. vii, 265 pp, 1995. (English) Zbl 0844.45002.

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