

# Photophoresis of Heated Large Spherical Aerosol Particles

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**Abstract**—A theoretical description of the steady-state motion of a large solid aerosol spherical particle moving in a gas under the effect of an intense electromagnetic radiation is given in the Stokes approximation. In considering this motion, it is assumed that the mean temperature of the particle surface may differ substantially from the temperature of the gaseous medium surrounding the particle. In solving gas-dynamics equations, analytic expressions for the photophoretic force and velocity were obtained with allowance for the temperature dependences of the gas density, viscosity, and thermal conductivity.

## INTRODUCTION

The phenomenon of photophoresis in gases consists in the motion of aerosol particles in the field of electromagnetic radiation under the effect of a radiometric force. Photophoresis may play a substantial role in atmospheric processes [1–3], industrial gas cleaning from aerosol particles, creation of facilities intended for a selective separation of particles according to their dimensions, and so on. The photophoresis mechanism can be briefly described as follows. Owing to the interaction of electromagnetic radiation with a particle, a thermal energy of space density  $q_p$  is deposited within the particle, whereby it is heated nonuniformly. Colliding with the particle surface molecules of the gas surrounding the particle are reflected with a higher velocity from its heated than from its colder side. As a result, the particle acquires an uncompensated momentum directed from the heated to the colder side of the particle. Depending on the dimensions of the particle and on the optical properties of its material, either the illuminated or the dark side of the particle may prove to be hotter. Therefore, there may occur either a positive (particle motion in the direction of the propagation of radiation) or a negative photophoresis. Moreover, particle motion in a gas in the direction transverse with respect to the propagation of electromagnetic radiation may arise if the radiation flux is nonuniform over the cross section.

In the studies devoted to the theory of photophoresis and published thus far, this phenomenon was analyzed at small relative temperature gradients [4–7]—specifically, under the condition  $(T_{pS} - T_{g\infty})/T_{g\infty} \ll 1$ , where  $T_{pS}$  is the mean temperature of the particle surface and  $T_{g\infty}$  is the temperature of the gas medium at large distances from the particle surface. At large relative temperature gradients such that  $(T_{pS} - T_{g\infty})/T_{g\infty} \approx O(1)$ , this phenomenon has not yet received adequate

study. Hereafter, the indices  $g$  and  $p$  refer to the gas and particle, respectively, and we use the index  $S$  to label physics quantities taken at the mean temperature of the particle surface and the index  $\infty$  to label physics quantities characterizing the gaseous medium in an unperturbed flow.

If the mean temperature of the particle surface differs substantially from the temperature of the surrounding gaseous medium, then we run into a serious problem. In solving gas-dynamics equations, it is necessary to take into account the temperature dependence of molecular-transport coefficients (viscosity and thermal conductivity) for the gaseous medium and its density—that is, the set of gas-dynamics equations becomes substantially nonlinear. In view of this, there are a few articles in the literature that are devoted to studying particle motion in the case of a high temperature gradient [8–10]. It should be noted that, in [9], solutions of differential equations that describe the velocity and pressure fields were sought in the form of power series by the order-reduction method. As a result, ultimate expressions proved to be quite cumbersome and difficult in applications. In the present study, we solve directly the gas-dynamics equations in the form of generalized power series, and this makes it possible to represent expressions for the photophoretic force and velocity in a compact form and to simplify substantially numerical calculations for purposes of practical applications.

## FORMULATION OF THE PROBLEM

Let us consider a nonuniformly heated solid spherical aerosol particle of radius  $R$  suspended in a gas of temperature  $T_g$ , density  $\rho_g$ , thermal conductivity  $\lambda_g$ , and viscosity  $\mu_g$ . By a heated particle, we mean a particle for which the mean surface temperature differs

markedly from the temperature of the gaseous medium at a large distance from the particle surface. As was indicated above, the molecular-transport coefficients cannot be treated in this case as constants. In describing the properties of the gaseous medium (viscosity and thermal conductivity), we assume here that they are power-law functions of temperature [1]; that is,

$$\mu_g = \mu_{g\infty} \left( \frac{T_g}{T_{g\infty}} \right)^\beta, \\ \lambda_g = \lambda_{g\infty} \left( \frac{T_g}{T_{g\infty}} \right)^\alpha, \quad \lambda_p = \lambda_{p0} \left( \frac{T_g}{T_{g\infty}} \right)^\gamma,$$

where

$$\mu_{g\infty} = \mu_g(T_{g\infty}), \quad \lambda_{g\infty} = \lambda_g(T_{g\infty}), \\ \lambda_{p0} = \lambda_p(T_{g\infty}), \\ 0.5 \leq \alpha, \beta \leq 1, \quad -1 \leq \gamma \leq 1.$$

In particular, we have  $\alpha = 0.81$  and  $\beta = 0.72$  for air and  $\alpha = 0.77$  and  $\beta = 0.69$  for nitrogen in the temperature range from 300 to 900 K; for a copper particle,  $\gamma = -0.01$  up to the melting temperature. Also, we set  $T_{g\infty} = 273$ . The relative error of the above formulas (comparison with experimental data) does not exceed 5% [11].

Nonuniform heating of a particle is due to the absorption of the incident electromagnetic radiation. The degree of nonuniformity depends on the optical constants of the particle material and on the diffraction parameter [12]. Interacting with the surface heated nonuniformly, the gas begins moving along the surface in the direction of an increase in the temperature. This phenomenon is referred to as thermal sliding. It causes the appearance of a photophoretic force and the force of medium viscous resistance. As soon as the former becomes equal in magnitude to the latter, the particle in question begins moving at a constant velocity. This velocity is referred to as a photophoretic velocity ( $\mathbf{U}_{ph}$ ).

The particle moves at low Péclet and Reynolds numbers, and one assumes that the particle is uniform in composition and large. In what is concerned with the latter, it should be noted that the Knudsen number  $\text{Kn} = \lambda/R$ , where  $\lambda$  is the mean free path of molecules of a gaseous mixture, is applied to classify aerosol particles according to dimensions. Particles are referred to as large ones if  $\text{Kn} \leq 0.01$ , moderately large ones if  $0.01 \leq \text{Kn} \leq 0.3$ , and small ones if  $\text{Kn} \gg 1$ . In tackling the problem at hand, we apply the hydrodynamics method—that is, we solve fluid-dynamics equations supplemented with respective boundary conditions.

It is convenient to describe the motion of a particle in the system of spherical coordinates  $r$ ,  $\theta$ , and  $\varphi$  comoving with the center of mass of the aerosol particle being considered. The  $OZ$  axis is aligned with the direction of propagation of a uniform flux of radiation with intensity  $I_0$ . In this case, the spatial density of internal heat sources has a standard form [12]; that is,

$$q_p(\mathbf{r}) = 2\pi\chi k_0 I_0 B(\mathbf{r}), \quad (1)$$

where

$$B(r, \theta, \varphi) = \frac{1}{2\pi} \int_0^{2\pi} \frac{|E(r, \theta, \varphi)|^2}{E_0^2} d\varphi = B\left(r, \theta, \varphi = \frac{\pi}{4}\right)$$

is the dimensionless electromagnetic-energy source function in the case of unpolarized incident radiation,  $E(r, \theta, \varphi)$  is the local electromagnetic-field strength within the particle,  $E_0$  is the amplitude of the field strength in the incident wave,  $k_0 = 2\pi/\lambda_0$  is the wave number,  $\lambda_0$  is the wavelength, and  $m(\lambda_0) = n + i\chi$  is the complex refraction index of the particle material for a given radiation wave. In order to calculate the dimensionless source function  $B(\mathbf{r})$ , one usually employs the solution of the Mie problem for the internal field (see, for example, [4]). Since the reference frame used moves together with the center of mass of the aerosol particle, the problem reduces to analyzing the motion of a plane-parallel flow around the particle, the gas velocity at infinity being the sign-reversed photophoretic velocity (that is,  $\mathbf{U}_\infty = -\mathbf{U}_{ph}$ ).

Within the assumptions formulated above, the fluid-dynamics equations, the heat equation, and the boundary conditions (in the system of spherical coordinates) have the form [15, 16]

$$\frac{\partial}{\partial x_i} P_g = \frac{\partial}{\partial x_j} \left\{ \mu_g \left[ \frac{\partial U_i^g}{\partial x_j} + \frac{\partial U_j^g}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial U_k^g}{\partial x_k} \right] \right\}, \quad (2)$$

$$\frac{\partial}{\partial x_k} (\rho_g U_k^g) = 0,$$

$$\text{div}(\lambda_g \nabla T_g) = 0, \quad n_g = \frac{P_g}{kT_g}, \quad (3)$$

$$\text{div}(\lambda_p \nabla T_p) = -q_p. \quad (4)$$

where  $x_k$  are Cartesian coordinates;  $q_p$  is the density of heat sources, which are nonuniformly distributed over the particle volume and which are responsible for the heating of the particle;  $\rho_g = n_g m_g$ ;  $\rho_g$ ,  $m_g$ , and  $n_g$  are, respectively, the density, mass, and concentration of gaseous-medium molecules;  $k$  is the Boltzmann constant; and

$$r = R, \quad T_g = T_p,$$

$$\lambda_g \frac{\partial T_g}{\partial r} = \lambda_p \frac{\partial T_p}{\partial r} + \sigma_0 \sigma_1 (T_p^4 - T_{g\infty}^4), \quad (5)$$

$$U_r^g = 0, \quad U_\theta^g = K_{1S} \frac{v_g}{RT_g} \frac{\partial T_g}{\partial \theta},$$

$$r \rightarrow \infty, \quad U_r^g = U_\infty \cos \theta, \quad U_\theta^g = -U_\infty \sin \theta, \quad (6)$$

and

$$r \rightarrow p, \quad T_p \neq \infty. \quad (7)$$

Here,  $U_r^g$  and  $U_\theta^g$  are the components of the gas mass velocity  $\mathbf{U}_g$  and  $K_{1S}$  is the thermal-sliding coefficient,

which is found by methods of the kinetic theory of gases. In the case where the accommodation coefficients for the tangential momentum and energy ( $\alpha_\tau$  and  $\alpha_E$ , respectively) are equal to unity, the gas-kinetics coefficient is  $K_{IS} = 1.152$  (see, for example, [17]); further,  $\sigma_0$  is the Stefan–Boltzmann constant, while  $\sigma_1$  is the integrated degree of blackness [18].

In the boundary conditions in (5) at the surface of the aerosol particle, we have taken into account the equality of the temperatures, the continuity of the heat fluxes, the impenetrability condition for the normal component of the mass velocity, and thermal sliding for its tangential component. The boundary conditions in (6) are valid at large distances from the particle ( $r \rightarrow \infty$ ). The finiteness of physics quantities characterizing the particle for  $r \rightarrow 0$  is taken into account in (7).

We now reduce Eqs. (2)–(4) and the boundary conditions in (5)–(7) to a dimensionless form by introducing dimensionless coordinates, velocity, and temperature as follows:  $y_k = x_k/R$ ,  $t = T/T_{g\infty}$ ,  $\mathbf{V}_g = \mathbf{U}_g/U_\infty$ , and  $U_\infty = |\mathbf{U}_g|$ .

At small Reynolds numbers, the incoming flow has only a perturbing effect; therefore, we can seek a solution of fluid-dynamics equations in the form

$$\mathbf{V}_g = \mathbf{V}_g^{(1)} + \varepsilon \mathbf{V}_g^{(2)} + \dots, \quad P_g = P_g^{(0)} + \varepsilon P_g^{(1)} + \dots \quad (8)$$

$$(\varepsilon = Re_\infty = (\rho_{g\infty} U_\infty R) / \mu_{g\infty} \ll 1).$$

The form of the boundary conditions indicates that it is natural to seek expressions for the mass-velocity components  $V_r^\varepsilon$  and  $V_\theta^\varepsilon$  in the form of expansions in Legendre and Gegenbauer polynomials [16]. It is well known [16] that, in order to determine the resulting force that acts on the particle, it is sufficient to determine the first terms of these expansions.

## TEMPERATURE FIELDS OUTSIDE AND WITHIN A PARTICLE

In determining the photophoretic force and velocity, we restrict ourselves to corrections of the first order of smallness. In order to find them, it is necessary to know the temperature fields outside and within the particle being considered. For this, we must solve Eqs. (3) and (4). Solving these equations by the method of separation of variables, we obtain the temperatures  $t_g$  and  $t_p$  in the form

$$\begin{aligned} t_g(y, \theta) &= t_{g0}(y) + \varepsilon t_{g1}(y, \theta), \\ t_p(y, \theta) &= t_{p0}(y) + \varepsilon t_{p1}(y, \theta), \end{aligned} \quad (9)$$

where

$$t_{g0}(y) = \left(1 + \frac{\Gamma_0}{y}\right)^{1/(1+\alpha)},$$

$$t_{p0}(y) = \left(B_0 + \frac{H_0}{y} - \frac{1}{y} \int_1^y \Psi_0 dy + \int_1^y \frac{\Psi_0}{y} dy\right)^{1/(1+\gamma)},$$

$$t_{g1}(y, \theta) = \frac{\cos \theta \Gamma}{t_{g0}^\alpha y^2},$$

$$t_{p1}(y, \theta) = \frac{\cos \theta}{t_{p0}^\alpha} \left[ B_1 y + \frac{H_1}{y^2} + \frac{1}{3} \left( y \int_1^y \frac{\Psi_1}{y^2} dy - \frac{1}{y^2} \int_1^y \Psi_1 y dy \right) \right],$$

$$H_0 = \frac{R^2(1+\gamma)}{3\lambda_{p0} T_{g\infty}} J_0, \quad H_1 = \frac{R^2}{3\lambda_{p0} T_{g\infty}} J,$$

$$J_0 = \frac{1}{V} \int_V q_p dV, \quad J = \frac{1}{V} \int_V q_p z dV, \quad V = \frac{4}{3} \pi R^3,$$

Here,  $\int_V q_p z dV$  is the dipole moment of the heat-source density [1, 4, 6, 7, 9, 12, 14],

$$\begin{aligned} \Psi_0 &= -\frac{R^2(1+\gamma)}{2\lambda_{p0} T_{g\infty}} y^2 \int_{-1}^{+1} q_p(r, \theta) dx, \\ \Psi_1 &= -\frac{3R^2}{2\lambda_{p0} T_{g\infty}} y^2 \int_{-1}^{+1} q_p(r, \theta) x dx, \end{aligned}$$

$$x = \cos \theta, \quad z = r \cos \theta,$$

where  $y = x/r$  is the dimensionless radial coordinate.

The constants of integration that appear in the expressions for the temperature fields are determined from the boundary conditions at the particle surface. In particular, the resulting expression for the coefficient  $\Gamma$  is

$$\frac{\Gamma}{t_{gS}^\alpha} = \frac{R}{\lambda_{pS} T_{g\infty}} J. \quad (10)$$

where  $\delta = 1 + 2 \frac{\lambda_{gS}}{\lambda_{pS}} + 4 \frac{\sigma_0 \sigma_1 R}{\lambda_{pS}} T_{g\infty}^3 t_{pS}^3$ ,  $\lambda_{pS} = \lambda_{p0} t_{pS}^\alpha$ ,

$\lambda_{gS} = \lambda_{g\infty} t_{gS}^\alpha$ ,  $t_{pS} = t_{p0}(y=1)$ , and  $t_{gS} = t_{g0}(y=1)$ .

The mean value of the particle-surface temperature,  $T_{pS}$ , is determined by solving the set of equations

$$\begin{cases} T_{pS} = T_{gS} \\ \frac{I^{(S)}}{1+\alpha} t_{gS} = \frac{R^2}{2\lambda_{gS} T_{g\infty}} J_0 - \sigma_0 \sigma_1 \frac{R T_{g\infty}^3}{\lambda_{gS}} \left[ \left( \frac{T_{pS}}{T_{g\infty}} \right)^4 - 1 \right]. \end{cases} \quad (11)$$

where  $T_{pS} = t_{pS} T_{g\infty}$ ,  $T_{gS} = t_{gS} T_{g\infty}$ , and  $I^{(S)} = \Gamma_0 / (1 + \Gamma_0)$ .

If the strong inequality  $\lambda_g \ll \lambda_p$  holds (which takes place for the majority of gases), then the particle thermal conductivity is much greater than the gas thermal conductivity, in which case we can disregard the dependence on the angle  $\theta$  in the dynamic-viscosity coefficient for the particle–gas system (we assume a weak angular anisotropy of temperature distribution).

Taking this into account, we can assume that viscosity depends only on the temperature  $t_{g0}(r)$ —that is,  $\mu_g(t_g(r, \theta)) \approx \mu_g(t_{g0}(r))$ . This assumption makes it possible to separate the hydrodynamic and thermal parts of the problem, their coupling being due to the boundary conditions.

Substituting expressions (9) into the temperature dependence of the dynamic-viscosity coefficient, we arrive at

$$\mu_g(y, \theta) = \mu_{g\infty} t_{g0}^\beta(y). \quad (12)$$

In the following, expression (12) obtained for the dynamic-viscosity coefficient is used to determine the velocity and pressure fields in the vicinity of a heated aerosol particle spherical in shape.

#### SOLVING THE HYDRODYNAMIC PROBLEM: DERIVATION OF EXPRESSIONS FOR THE VELOCITY AND PRESSURE FIELDS

An investigation of the Navier–Stokes equation linearized in velocity and written in terms of spherical coordinates revealed that, if the particle thermal conductivity is much greater than the gas thermal conductivity (weak angular asymmetry of the temperature distribution), then this equation ultimately reduces to a nonhomogeneous third-order differential equation with an isolated singular point. A solution of this equation can be sought in the form of generalized power series.

Taking into account the boundary conditions in (5)–(7), we seek expressions for the mass-velocity components of the zero-order approximation (8) in the form

$$\begin{aligned} U_r^g(y, \theta) &= U_\infty \cos \theta G(y), \\ U_\theta^g(y, \theta) &= -U_\infty g(y) \sin \theta. \end{aligned} \quad (13)$$

where  $G(y)$  and  $g(y)$  are arbitrary functions depending on the coordinate  $y$ .

From the continuity equation (2) and the equation of state (3), we find that the functions  $G(y)$  and  $g(y)$  are related by the equation

$$\begin{aligned} g(y) &= \frac{1}{2} y \frac{dG(y)}{dy} + \left(1 + \frac{1}{2(1+\alpha)} l\right) G(y), \\ l = l(y) &= \frac{\Gamma}{y + \Gamma_0}. \end{aligned} \quad (14)$$

Substituting expressions (13) and (14) into the Navier–Stokes equation (2) linearized in velocity, taking into account expression (12), and separating variables after some algebra, we ultimately find that the function  $G(y)$  satisfies the nonhomogeneous third-order differential equation

$$\begin{aligned} y^4 \frac{d^3 G}{dy^3} + y^3 (4 + \gamma_1 l) \frac{d^2 G}{dy^2} - y^2 (4 + \gamma_2 l - \gamma_3 l^2) \frac{dG}{dy} \\ - y(2 - l) \gamma_3 l^2 G = -\frac{6A_2}{t_{g0}^\beta}. \end{aligned} \quad (15)$$

where

$$\begin{aligned} \gamma_1 = \frac{1 - \beta}{1 + \alpha}, \quad \gamma_2 = 2 \frac{1 + \beta}{1 + \alpha}, \quad \gamma_3 = \frac{2 + 2\alpha - \beta}{(1 + \alpha)^2}, \\ A_2 = \text{const.} \end{aligned} \quad (16)$$

First, we seek solutions of the respective homogeneous equation; that is,

$$\begin{aligned} \gamma \frac{d^3 G}{dy^3} + y^3 (4 + \gamma_1 l) \frac{d^2 G}{dy^2} - y^2 (4 + \gamma_2 l - \gamma_3 l^2) \frac{dG}{dy} \\ - y(2 - l) \gamma_3 l^2 G = 0. \end{aligned} \quad (17)$$

For Eq. (17), the point  $y = 0$  is a regular singular point [19–21]. This follows from Eq. (17) upon going over to the new variable  $l(y)$ . Therefore, we seek its solution in the form of a generalized power series [19–21]; that is,

$$G(y) = y^\rho \sum_{n=0}^{\infty} C_n l^n, \quad C_0 \neq 0. \quad (18)$$

Substituting the series in (18) into Eq. (17), we arrive at the equation  $\rho(\rho + 3)(\rho - 2) = 0$ ; its roots are  $\rho_1 = -3$ ,  $\rho_2 = 2$ , and  $\rho_3 = 0$ . We note that the difference of the roots is an integer in magnitude. According to the general theory of solutions of differential equations in the form of generalized power series (Frobenius method), the solutions other than the first one ( $\rho_1 = -3$  in our case) develop an additional term involving a logarithm multiplied by the first solution [19–21]. The recursion relations for the respective coefficients are derived by the method of indeterminate coefficients.

The solution corresponding to the root largest in magnitude has the form

$$G_1(y) = \frac{1}{y^3} \sum_{n=0}^{\infty} C_{1,n} l^n. \quad (19)$$

We do not present here the solution corresponding to the root  $\rho_2 = 2$ , since it does not satisfy the boundary conditions in (16) (finiteness of the solution for  $y \rightarrow \infty$ ).

The third solution that satisfies Eq. (17) and which is linearly independent with the solution  $G_1$  (it is proportional to the root  $\rho_3 = 0$ ) is sought in the form

$$G_3(y) = \sum_{n=0}^{\infty} C_{3,n} l^n + \omega_3 \ln(y) G_1(y). \quad (20)$$

The right-hand side of the nonhomogeneous equation in (15) suggests that a particular solution of this equation must be sought in the form

$$\begin{aligned} \tilde{G}(y) &= A_2 G_2(y), \\ G_2(y) &= \frac{1}{y} \sum_{n=0}^{\infty} C_{2,n} l^n + \omega_2 \ln(y) G_1(y). \end{aligned} \quad (21)$$

The coefficients  $C_{1,n}$  ( $n \geq 1$ ),  $C_{2,n}$  ( $n \geq 3$ ), and  $C_{3,n}$  ( $n \geq 4$ ) can be found by the method of indeterminate coefficients. The respective recursion relations have the form

$$\begin{aligned} C_{1,n} &= \frac{1}{n(n+3)(n+5)} \{ [(n-1)(3n^2+13n+8) \\ &+ \gamma_1(n+2)(n+3) + \gamma_2(n+2)] C_{1,n-1} \\ &- [(n-1)(n-2)(3n+5) + 2\gamma_1(n^2-4) + \gamma_2(n-2) \\ &+ \gamma_3(n+3)] C_{1,n-2} + (n-2)[(n-1)(n-3) \\ &+ \gamma_1(n-3) + \gamma_3] C_{1,n-3} \}, \end{aligned}$$

$$\begin{aligned} C_{2,n} &= \frac{1}{(n+1)(n+3)(n-2)} \\ &\times \left\{ [(n-1)(3n^2+n-6) + \gamma_1 n(n+1) + n\gamma_2] C_{3,n-1} \right. \\ &- [\gamma_3(n+1) + (n-1)(n-2)(3n-1) \\ &+ 2\gamma_1 n(n-2) + \gamma_2(n-2)] C_{2,n-2} \\ &+ (n-2)[(n-1)(n-3) + \gamma_3 + \gamma_1(n-3)] C_{2,n-3} \\ &+ \frac{\omega_2}{\Gamma_0^2} \sum_{k=0}^{n-2} (n-k-1) \Delta_k \\ &\left. - 6 \frac{(-\gamma_4)(1-\gamma_4) \dots (n-1-\gamma_4)}{n!} \right\}, \end{aligned}$$

$$\begin{aligned} C_{3,n} &= \frac{1}{n(n+2)(n-3)} \left\{ (n-1) \right. \\ &\times [3n^2 - 5n - 4 + \gamma_1 n + \gamma_2] C_{3,n-1} \\ &- [(n-1)(n-2)(3n-4) + 2\gamma_1(n-1)(n-2) \\ &+ \gamma_2(n-2) + n\gamma_3] C_{3,n-2} \\ &+ (n-2)[(n-1)(n-3) + \gamma_1(n-3) + \gamma_3] C_{3,n-3} \\ &\left. + \frac{\omega_3}{2\Gamma_0^3} \sum_{k=0}^{n-2} (n-k-2) \Delta_k \right\}, \end{aligned}$$

$$\begin{aligned} \Delta_k &= (3k^2 + 16k + 15) C_{1,k} - ((k-1)(6k+13) \\ &+ \gamma_1(2k+5) + \gamma_2) C_{1,k-1} \\ &+ (3(k-1)(k-2) + 2\gamma_1(k-2) + \gamma_3) C_{1,k-2}. \end{aligned}$$

In calculating the coefficients  $C_{1,n}$ ,  $C_{2,n}$ , and  $C_{3,n}$ , it is necessary to consider that  $C_{1,0} = 1$ ,  $C_{2,0} = 1$ ,  $C_{3,1} = 0$ ,  $C_{2,2} = 1$ ,  $C_{2,1} = -\frac{1}{8}(2\gamma_1 + \gamma_2 + 6\gamma_4)$ ,

$$\begin{aligned} \gamma_4 &= \frac{\beta}{1+\alpha}, \quad \frac{\omega_3}{2\Gamma_0^3} = -\frac{\gamma_3}{60}(10 + 3\gamma_1 + \gamma_2), \\ C_{3,1} &= 0, \quad C_{3,2} = \frac{1}{4\gamma_3}, \quad C_{3,3} = 1, \quad C_{3,0} = 1, \\ \frac{\omega_2}{\Gamma_0^2} &= \frac{1}{15} \left[ \frac{1}{4} (2\gamma_1 + \gamma_2 + 6\gamma_4)(4 - 3\gamma_1 + \gamma_2) \right. \\ &\left. + 3\gamma_3 + 3\gamma_4(\gamma_4 - 1) \right], \end{aligned}$$

for  $n < 0$ ,  $C_{1,n}$ ,  $C_{2,n}$ , and  $C_{3,n}$  are equal to zero.

Thus, the general solution satisfying Eq. (15) and the boundary conditions in (16) has the form

$$G(y) = A_1 G_1(y) + A_2 G_2(y) + G_3(y), \quad (22)$$

and we can represent the expressions for the mass-velocity components and for the pressure as

$$\begin{aligned} U_r^{\xi} &= U_{\infty} \cos\theta (A_1 G_1 + A_2 G_2 + G_3), \\ U_{\theta}^{\xi} &= -U_{\infty} \sin\theta (A_1 G_4 + A_2 G_5 + G_6), \\ P_g &= P_{g\infty} + \frac{\mu_{g\infty} U_{\infty} \beta}{R} \left\{ \frac{y^2 d^3 G}{2 y^3} + y \left[ 3 + \frac{\beta-1}{2} y f \right] \frac{d^2 G}{dy^2} \right. \\ &- \left[ 2 - y^2 f^I - \frac{\beta}{2} y^2 f^2 + (\beta-2) y f \right] \frac{dG}{dy} \\ &\left. + 2 \left[ y^2 f^{II} + y f^I (4 + y \beta f) - \frac{2}{3} f \right] G \right\}. \end{aligned} \quad (23)$$

where

$$\begin{aligned} f &= -\frac{l}{y(1+\alpha)}, \\ G_k &= \left( 1 + \frac{1}{2(1+\alpha)} \right) C_{k-3} + \frac{1}{2} y G_{k-3}^I \\ &(k = 4, 5, 6). \end{aligned}$$

while  $f^I$ ,  $f^{II}$ ,  $G_1^I$ ,  $G_2^I$ , and  $G_3^I$  are the first and second derivatives of the corresponding functions with respect to  $y$ .

The constants of integration  $A_1$  and  $A_2$  are determined from the boundary conditions in (5) at the surface of the aerosol particle.

#### DETERMINATION OF THE PHOTOPHORETIC FORCE AND VELOCITY AND ANALYSIS OF THE RESULTS

Thus, we have derived expressions for temperature fields outside and within an aerosol particle in the first

approximation in  $\varepsilon$ , along with the distribution of the velocity and pressure in its vicinity. The resulting strength that acts on the particle is determined by integrating the stress tensor over the aerosol-particle surface and is given by [15, 16]

$$F_z = \int_{(S)} (-P_g \cos\theta + \sigma_{rr} \cos\theta + \sigma_{r\theta} \sin\theta) r^2 \sin\theta d\theta d\varphi \Big|_{r=R}. \quad (24)$$

Here,  $\sigma_{rr}$  and  $\sigma_{r\theta}$  are the stress-tensor components, which have the form

$$\sigma_{rr} = \mu_g \left( 2 \frac{\partial U_r^g}{\partial y} - \frac{2}{3} \operatorname{div} U_g \right),$$

$$\sigma_{r\theta} = \mu_g \left( \frac{\partial U_\theta^g}{\partial y} + \frac{1}{y} \frac{\partial U_r^g}{\partial \theta} - \frac{U_\theta^g}{y} \right).$$

With allowance for the above expressions, we obtain

$$\mathbf{F} = \mathbf{F}_\mu + \varepsilon \mathbf{F}_{\text{ph}}, \quad (25)$$

where

$$\mathbf{F}_\mu = 6\pi R \mu_{g\infty} U_\infty f_\mu \mathbf{n}_z,$$

$$\mathbf{F}_{\text{ph}} = -6\pi R \mu_{g\infty} f_{\text{ph}} \mathbf{J} \mathbf{n}_z,$$

$\mathbf{n}_z$  is the unit vector aligned with the  $OZ$  axis.

The coefficients  $f_\mu$  and  $f_{\text{ph}}$  can be estimated as

$$f_\mu = \frac{2N_2}{3N_1}, \quad f_{\text{ph}} = \frac{4}{3} K_{TS} \frac{\nu_{gS}}{\lambda_{pS} \delta T_{g\infty} f_{gS} N_1} G_1. \quad (26)$$

where

$$N_1(y) = G_1(y)G_2^1(y) - G_2(y)G_1^1(y),$$

$$N_2(y) = G_1(y)G_3^1(y) - G_3(y)G_1^1(y)G_2^1G_3^1$$

are the first derivatives of the corresponding functions with respect to  $y$ .

Equating the resulting force  $\mathbf{F}$  to zero, we obtain the photophoretic velocity  $\mathbf{U}_{\text{ph}}$  ( $\mathbf{U}_{\text{ph}} = -\mathbf{U}_\infty$ ) for a heated large solid particle spherical in shape. Specifically, we have

$$\mathbf{U}_{\text{ph}} = -h_{\text{ph}} \mathbf{J} \mathbf{n}_z, \quad (27)$$

where  $h_{\text{ph}} = f_{\text{ph}}/f_\mu$

In estimating the coefficients  $f_\mu$ ,  $f_{\text{ph}}$ , and  $h_{\text{ph}}$ , it is necessary to consider that the index  $s$  labels those values of physics quantities that are taken at the mean relative particle-surface temperature  $T_{pS}$ , which is determined by Eq. (11); the functions  $G_1(y)$ ,  $G_1^1(y)$ ,  $G_2(y)$ ,  $G_2^1(y)$ ,  $G_3(y)$ ,  $G_3^1(y)$ ,  $N_1(y)$ , and  $N_2(y)$  are taken at  $y$ .

The formulas derived above can also be used at small relative temperature gradients in the vicinity of the particle. In the case where the heating of the particle surface is small—that is, the particle-surface temperature differs only slightly from the surrounding-medium temperature far from the particle ( $\Gamma_0 \rightarrow 0$ )—one can disregard the temperature dependence of

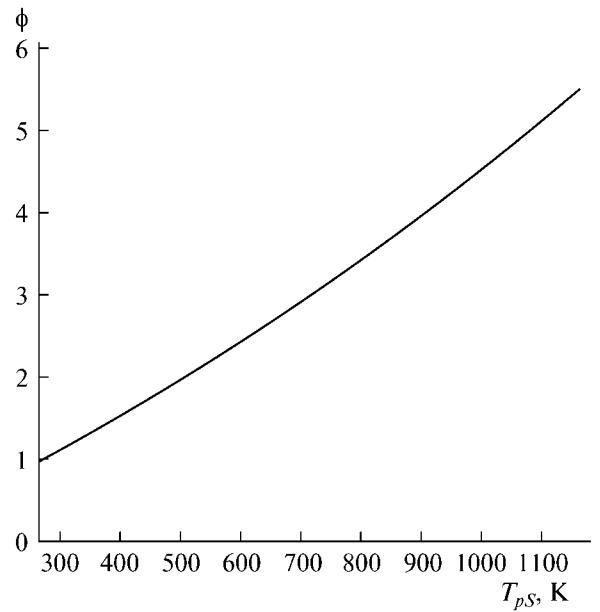


Fig. 1. Dependence of the function  $\phi$  on the mean particle-surface temperature  $T_{pS}$ .

molecular-transport coefficients (viscosity and thermal conductivity), in which case ( $y = 1$ ) we have  $G_1^1 = 1$ ,  $G_1^1 = -3$ ,  $G_2 = 1$ ,  $G_1 = -1$ ,  $G_3 = 1$ ,  $G_3^1 = 0$ ,  $N_1 = 2$ , and  $N_2 = 3$ . The expressions for the photophoretic force and velocity then coincide with the respective results from [3].

If the spatial distribution of the heat-source density is known, then expressions (25)–(27) can be used to take into account the effect of heating of the particle surface on the photophoretic force and velocity at arbitrary temperature gradients between the particle surface and the region far from it with allowance for the power-law form of the temperature dependence of the viscosity and thermal conductivity of the surrounding gaseous medium. These formulas have the most general character.

Numerical estimates of the effect that the heating of the aerosol-particle surface exerts on the photophoresis process are of interest. In Figs. 1 and 2, curves 1 and 2 represent the values of

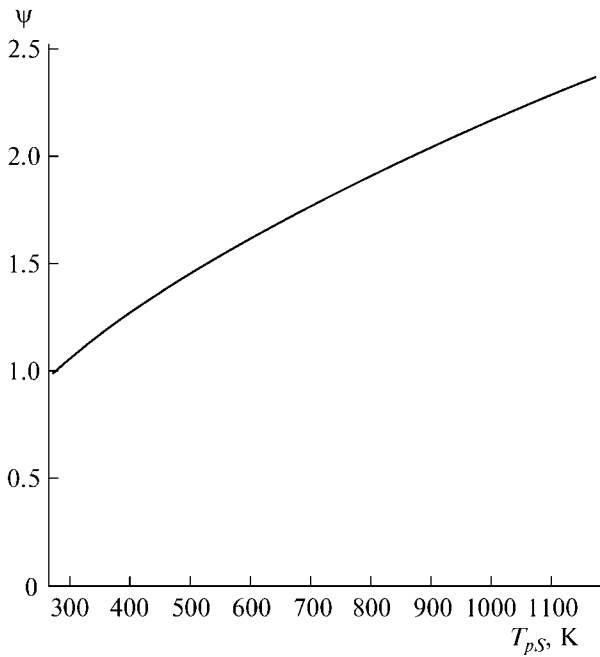
$$\phi = \frac{f_{\text{ph}}}{f_{\text{ph}}|_{T_{pS}=273\text{ K}}}, \quad \psi = \frac{h_{\text{ph}}}{h_{\text{ph}}|_{T_{pS}=273\text{ K}}}$$

$$f_{\text{ph}}|_{T_{pS}=273\text{ K}} = 9.34 \times 10^{-11},$$

$$h_{\text{ph}}|_{T_{pS}=273\text{ K}} = 9.34 \times 10^{-11}$$

versus  $T_{pS}$  for large copper particles of radius  $R = 25 \mu\text{m}$  moving in air under normal conditions.

From Eqs. (25)–(27), one can see that the magnitude and direction of the photophoretic force and velocity are determined by the magnitude and direc-



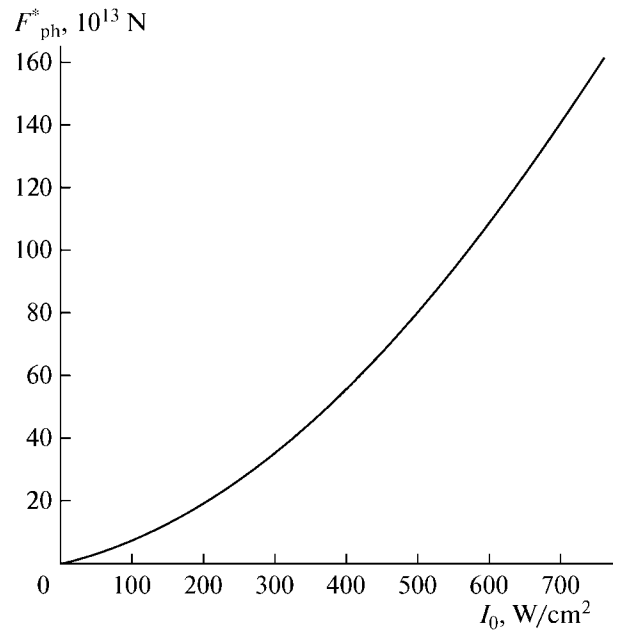
**Fig. 2.** Dependence of the function  $\phi$  on the mean particle-surface temperature  $T_{pS}$ .

tion of the dipole moment of the heat-source density,  $\int_V q_p z dV \mathbf{n}_z$ . In those cases where the dipole moment is negative (a greater part of thermal energy is deposited in that part of the particle which faces the radiation source), the particle moves along the direction of propagation of incident radiation. If the dipole moment is positive (a greater part of thermal energy is deposited in the dark region of the particle), the particle moves against the direction of propagation of incident radiation. In order to calculate the respective integral, it is necessary to know  $q_p$ , which is determined by solving the electrodynamic problem [1, 6, 14]. Numerical methods that make it possible to find the dipole moment of the heat-source density have been developed to date—for example, the listing of a code that can be used for this is presented in [22].

By way of example, we consider an extremely simple case where radiation absorption by the particle has a blackbody character. In this case, the absorption occurs in a thin layer of thickness  $\delta R \ll R$  adjacent to the heated part of the particle surface. The density of heat sources within the layer of thickness  $\delta R$  is then given by

$$q_p = \begin{cases} -\frac{I_0}{\delta R} \cos \theta, & \frac{\pi}{2} \leq \theta < \pi, \quad R - \delta R \leq r \leq R, \\ 0, & 0 \leq \theta \leq \frac{\pi}{2}, \end{cases} \quad (28)$$

where  $I_0$  is the intensity of incident radiation.



**Fig. 3.** Photophoretic force  $F_{ph}^*$  as a function of the incident-radiation intensity  $I_0$ .

In this case, the integrals are readily calculable. Specifically, we have

$$\int_V q_p dV = \pi R^2 I_0, \quad \int_V q_p z dV = -\frac{2}{3} \pi R^3 I_0,$$

The photophoretic force and velocity for large blackbody particles spherical in shape are then obtained in the form

$$\begin{aligned} \mathbf{F}_{ph}^* &= 3\pi R \mu_{g\infty} f_{ph} I_0 \mathbf{n}_z, \\ \mathbf{U}_{ph}^* &= \frac{\hbar_{ph}}{2} I_0 \mathbf{n}_z \left( \hbar_{ph} = \frac{f_{ph}}{f_\mu} \right). \end{aligned} \quad (29)$$

The mean particle-surface temperature  $T_{pS}$  is related to the intensity of incident radiation by the equation

$$\begin{cases} T_{pS} = T_{gS}, \\ \frac{I^{(S)} \lambda_{gS} T_{pS} t_{gS}}{1 + \alpha \lambda_{pS}} = \frac{R}{4\lambda_{pS}} I_0 \\ -\sigma_0 \sigma_1 \frac{RT_{g\infty}^4}{\lambda_{pS}} \left[ \left( \frac{T_{pS}}{T_{g\infty}} \right)^4 - 1 \right]. \end{cases} \quad (30)$$

In Figs. 3 and 4, the curves represent  $\phi^* = F_{ph}^*$  and  $\psi^* = U_{ph}^*$  versus  $I_0$  for large copper particles of radius  $R = 25 \mu\text{m}$  moving in air under normal conditions.

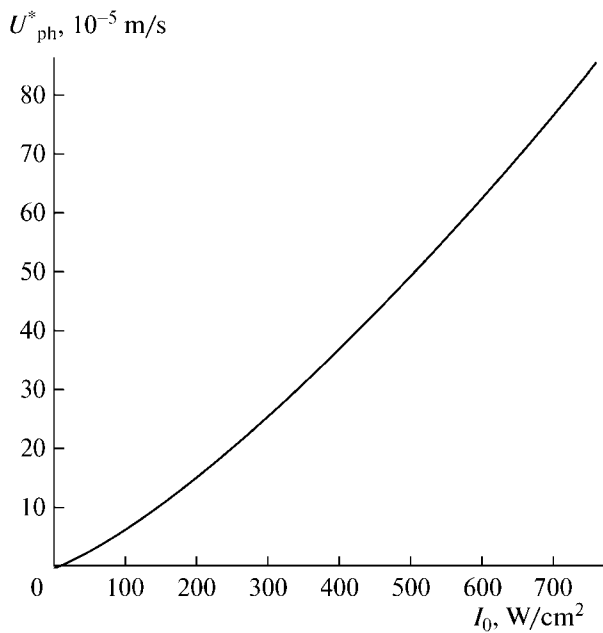


Fig. 4. Photophoretic velocity  $U_{ph}^*$  as a function of the incident-radiation intensity  $I_0$ .

### CONCLUSIONS

From the graphs presented in this article, one can see that, as the intensity of incident radiation grows, the photophoretic force and velocity increase nonlinearly, which is associated with the temperature dependence of molecular-transport coefficients and density. In the case of small temperature gradients, the dependence in question is linear, which agrees with the known results from [2, 4, 6, 12].

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