Photophoresis of Heated Moderately Large Spherical Aerosol Particles

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Abstract—Steady-state motion of a moderately large solid aerosol spherical particle electromagnetically irradiated in a gas is described theoretically in the Stokes approximation. In the consideration of the motion, it was supposed that the average temperature of the particle surface could differ considerably from the temperature of the gaseous medium surrounding the particle. In the process of the solution of the gas dynamic equations, analytical expressions for the photophoresis force and velocity were obtained, with allowance for the dependencies of density and viscosity of the gaseous medium and thermal conductivity on temperature.

INTRODUCTION

The phenomenon of photophoresis in a gas is the motion of aerosol particles in an electromagnetic radiation field under the action of the radiometric force. Photophoresis can play a significant part in atmospheric processes [1–3], in purifying industrial gases from aerosol particles, creating systems intended for the selective separation of particles by size, etc. The mechanism of photophoresis can be outlined as follows. When electromagnetic radiation interacts with the particle, heat energy is released inside it with a certain radiant density q_p. The energy inhomogeneously heats the particle. After collision with the particle surface, gas molecules that surround the particle are reflected from the heated side of it with a larger velocity than from the cold side. As a result, the particle acquires an uncompensated momentum directed from the hot side of the particle to the cold one. Depending on the particle's dimensions and the optical properties of its material, both the illuminated and shadow side of the particle may be warmer. Therefore, both the positive (particle motion along the direction of radiation), and negative photophoresis take place. In addition, if the radiation flux is inhomogeneous in the section, a particle can move in the gas transversely to the propagation direction of the electromagnetic radiation [4].

In works on the theory of photophoresis published to date, this phenomenon was studied at small relative temperature differences [4–7], i.e., when the inequality

$$(T_{pS} - T_{g\infty})/T_{g\infty} \ll 1$$

is valid. Here, T_{pS} is the mean temperature of the particle surface and $T_{g\infty}$ is the temperature of the gaseous medium far from it. For considerable relative differences of temperature, i.e., when $(T_{pS} - T_{g\infty})/T_{g\infty} \sim 0(1)$, this phenomenon has been little studied. Hereinafter, the indices g and p refer to the gas and particle, respectively. Index S denotes the values of physical parameters taken at the average temperature of the particle surface; index ∞, physical parameters characterizing the gaseous medium in a undisturbed flow.

If the average temperature of the particle surface significantly differs from the temperature of the ambient gaseous medium, we meet a serious problem. In solving gas dynamics equations, it is necessary to take into account the dependence of the coefficients of molecular transfer (viscosity and thermal conductivity) and the density of the gaseous medium on temperature, i.e., the system of gas dynamics equations becomes significantly nonlinear. In connection with this, there are few works studying particle motion at large differences in the temperature [8–10].

It should be noted that the solution of the differential equations describing the velocity and pressure fields in [9] was sought in the form of a power series by lowering the order. This led to rather awkward final expressions too complicated in practice. In this paper, the solution of the gas dynamic equations is represented in the form of generalized power series, which permits one to give the expressions for the photophoresis force and velocity in a concise form and to significantly simplify numerical calculations for practical applications.

1. FORMULATION OF THE PROBLEM

Let us consider a solid inhomogeneously heated spherical aerosol particle with a radius R suspended in a gas with temperature T_g, density ρ_g, thermal conductivity λ_g, and viscosity μ_g. By a heated particle we mean a particle whose average surface temperature significantly differs from the temperature of the gaseous medium far from the particle. In this case, as mentioned above, the coefficients of molecular transfer cannot be considered as constants. In the description of properties of the gaseous medium and of the particle, we consider the following form of their dependence on temperature [11]:

$$\mu_g = \mu_{g\infty} t_g^{\beta}, \lambda_g = \lambda_{g\infty} t_g^{\alpha}, \lambda_p = \lambda_{p0} t_p^{\gamma},$$

where $\mu_{g\infty} = \mu_g(T_{g\infty})$, $\lambda_{g\infty} = \lambda_g(T_{g\infty})$, $\lambda_{p0} = \lambda_p(T_{g\infty})$, and $t_h t_h = T_h/T_{g\infty}$ $(h = g, p)$, $0.5 \le \alpha, \beta \le 1, -1 \le \gamma \le 1$.

The inhomogeneous heating of the particle is caused by absorption of electromagnetic radiation. The degree of inhomogeneity depends on the optical constants of the particle material and on the diffraction parameter [12]. Interacting with the inhomogeneously heated surface, the gas begins to move along the surface in the direction of increasing temperature. This phenomenon is called the thermal creep of the gas. Thermal creep causes the appearance of the photophoretic force and the force of viscous resistance of the medium. When the photophoretic force becomes equal to the viscous resistance force, the particle begins to move uniformly. The velocity of uniform motion of the particle is called the photophoretic velocity ($\mathbf{U}_{ph}$).

In the theoretical description of the process of photophoretic motion of the particle, we assume that, due to smallness of the times of thermal and diffusion relaxation, the heat transfer process in the particle–gaseous medium system is quasi-stationary. The particle motion occurs at small Peclet and Reynolds numbers and the particle is assumed to be homogeneous in its composition and moderately large [13, 15, 16]. As for the latter, note that the aerosol particles are classified by size using the Knudsen criterion $\mathrm{Kn} = \lambda/R$, where λ is the average length of the free path of molecules of the gaseous mixture. Particles are called large as $\mathrm{Kn} \le 0.01$, moderately large as $0.01 \le \mathrm{Kn} \le 0.3$, and small as $\mathrm{Kn} \gg 1$. The problem is solved by the hydrodynamic method, i.e., equations of hydrodynamics with the corresponding boundary conditions are solved.

It is convenient to describe the particle motion in a spherical coordinate system r, θ, φ associated with the center of masses of the aerosol particle. The OZ axis is directed to the propagation of a homogeneous radiation flux with intensity I_0. In this case, the radiant density of inner heat sources has a standard form [12]

$$q_p(\mathbf{r}) = 2\pi\chi k_0 I_0 B(\mathbf{r}), \tag{1}$$

where

$$B(r, \theta, \varphi) = \frac{1}{2\pi}\int_0^{2\pi} \frac{|E(r, \theta, \varphi)|^2}{E_0^2} d\varphi = B\left(r, \theta, \varphi = \frac{\pi}{4}\right)$$

is the dimensionless source function of electromagnetic energy in the case of nonpolarized incident radiation, $E(r, \theta, \varphi)$ is the local electric field strength inside the particle, E_0 is the amplitude of the field strength in the incident wave, $k_0 = 2\pi/\lambda_0$ is the wavenumber, λ_0 is the wavelength, and $m(\lambda_0) = n + i\chi$ is the complex refractive index of the substance of the aerosol particle. Usually, the dimensionless source function $B(\mathbf{r})$ is calculated using the solution of the Mie problem for the internal field (e.g., [12]). Since the frame of reference is associated with the center of the moving aerosol particle, the problem is reduced to the analysis of a particle flowing by an infinite plane-parallel flow; the velocity of the gas at infinity is equal with the opposite sign to the photophoresis velocity, i.e., $\mathbf{U}_\infty = -\mathbf{U}_{ph}$.

In the context of the formulated assumptions, the equations of hydrodynamics, thermal conductivity, and boundary conditions (represented in the spherical coordinate system) have the form [17, 18]

$$\frac{\partial}{\partial x_i} P_g = \frac{\partial}{\partial x_j}\left\{\mu_g\left[\frac{\partial U_i^g}{\partial x_j} + \frac{\partial U_j^g}{\partial x_i} - \frac{2}{3}\delta_i^j \frac{\partial U_k^g}{\partial x_k}\right]\right\}, \quad \frac{\partial}{\partial x_k}(\rho_g U_k^g) = 0, \tag{2}$$

$$\mathrm{div}(\lambda_g \nabla T_g) = 0, \quad n_g = P_g/kT_g, \tag{3}$$

$$\mathrm{div}(\lambda_p \nabla T_p) = -q_p. \tag{4}$$

Here, x_k are the Cartesian coordinates, $\rho_g = n_g m_g$, ρ_g, m_g, and n_g are the density, mass, and concentration of molecules of the gaseous medium, and k is the Boltzmann constant.

At a large distance from the particle ($r \to \infty$), the boundary conditions are valid, and the finiteness of physical parameters characterizing the particle as $r \to 0$ is taken into account in the following conditions:

$$y \to \infty, \; U_r^g = U_\infty \cos\theta, \quad U_\theta^g = -U_\infty \sin\theta, \; P_g = P_{g\infty}, \tag{5}$$

$$y \to 0, \quad T_p \ne \infty, \tag{6}$$

where U_r^g and U_θ^g are the normal and tangent components of the mass velocity of the gas $\mathbf{U}_g$, $y = r/R$, and $U_\infty = |\mathbf{U}_\infty|$.

The following boundary conditions hold on the particle surface [13, 15, 16]:

$$y = 1, \quad -\lambda_g \frac{\partial T_g}{\partial y} + \lambda_p \frac{\partial T_p}{\partial y} = -C_q^{\mathrm{T}} \mathrm{Kn} \frac{\lambda_g}{\sin\theta}\frac{\partial}{\partial\theta} \times \left(\sin\theta \frac{\partial T_g}{\partial\theta}\right) - \sigma_0\sigma_1 R(T_p^4 - T_{g\infty}^4),$$

$$U_r^g = C_V^{\mathrm{T}}\mathrm{Kn}\frac{\nu_g}{RT_g}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T_g}{\partial\theta}\right),$$

$$T_g - T_p = K_T^{\mathrm{T}}\mathrm{Kn}\frac{\partial T_g}{\partial y}, \tag{7}$$

$$U_\theta^g = C_m\mathrm{Kn}\left(\frac{\partial U_\theta^g}{\partial y} + \frac{1}{y}\frac{\partial U_r^g}{\partial\theta} - \frac{U_\theta^g}{y}\right) + K_{TS}\frac{\nu_g}{RT_g}\left[\left(1 + \mathrm{Kn}(\beta'_{RT} + \beta_{RT}^{\mathrm{B}})\right)\frac{\partial T_g}{\partial\theta} + \mathrm{Kn}(\beta_{RT} - \beta_{RT}^{\mathrm{B}})\frac{\partial^2 T_g}{\partial y\partial\theta}\right],$$

where σ_0 is the Stefan–Boltzmann constant and σ_1 is the total emissivity [19].

In formulating boundary conditions for moderately large particles, the whole volume of gas is mentally divided into two parts—the Knudsen layer (a fraction of gas immediately adjacent to the particle surface with a thickness on the order of the free path) and the rest of the gas. The flow in the gas volume beyond the kinetic layer is described by the usual hydrodynamic equations; the boundary conditions for the hydrodynamic equations are formed in the Knudsen layer. To describe the gas motion in this layer, one should solve the kinetic equations, e.g., [15, 16]. In this paper (in boundary conditions (7) on the particle surface), all terms linear in the Knudsen number corrections to the velocity of a moderately large aerosol particle are taken into account. For the heat flux and normal component of the velocity U_r^g, we consider the radial heat flux discontinuity proportional to the coefficients C_q^{T} and C_V^{T}, respectively. The correction coefficient K_T^{T} takes into account the temperature jump on the particle surface and the last boundary condition shows that the velocity of gas creep along a spherical surface with a small curvature is composed of the thermal, isothermal, and Burnett slip, as well as of the slip arising due to the inhomogeneity of temperature along a curved surface. They are proportional to the coefficients K_{TS}, C_m, β_{RT}^{B}, β'_{RT}, and β_{RT}, respectively. The expressions for the coefficients K_{TS}, C_m, K_T^{T}, C_q^{T}, C_V^{T}, β'_{RT}, β_{RT}, and β_{RT}^{B} are found by methods of kinetic theory of gases and can be taken from [15, 16]. If the accommodation coefficients of the tangent momentum and energy are close to unity, they are equal to $K_{TS} = 1.152$, $C_m = 1.131$, $K_T^{\mathrm{T}} = 2.179$, $C_q^{\mathrm{T}} = 0.548$, $C_V^{\mathrm{T}} = 0.941$, $\beta'_{RT} = -0.405$, $\beta_{RT} = 3.731$, and $\beta_{RT}^{\mathrm{B}} = 3.651$, respectively.

Let us make Eqs. (2)–(4) and boundary conditions (5)–(7) dimensionless by introducing dimensionless coordinates, velocity and temperature, as follows: $y_k = x_k/R$, $t = T/T_{g\infty}$, and $\mathbf{V}_g = \mathbf{U}_g/U_\infty$.

When the numbers $\mathrm{Re}_\infty = (\rho_{g\infty}U_\infty R)/\mu_{g\infty} \ll 1$, the incoming flow has only a disturbing effect; therefore, the solution of the hydrodynamics equation should be sought in the form

$$\mathbf{V}_g = \mathbf{V}_g^{(1)} + \varepsilon\mathbf{V}_g^{(2)} + \ldots, \qquad P_g = P_g^{(0)} + \varepsilon P_g^{(1)} + \ldots + (\varepsilon = \mathrm{Re}_\infty). \tag{8}$$

The form of the boundary conditions points to the fact that the expressions for the components of mass velocity V_r^g and V_θ^g are sought as expansions in Legendre and Gegenbauer polynomials [17]. It is well-known [17] that it is sufficient to determine the first terms of these expansions for determining the total force acting on the particle.

2. TEMPERATURE FIELDS INSIDE AND OUTSIDE THE PARTICLE

Determining the photophoretic force and velocity, we restrict ourselves to corrections of the first order of infinitesimals. To find them, one should know temperature fields outside and inside the particle; for this purpose, it is necessary to solve Eqs. (3)–(4). Solving them by the method of separation of variables, we obtain the following expressions for t_g and t_p:

$$t_g(y,\theta) = t_{g0}(y) + \varepsilon t_{g1}(y,\theta), \qquad t_p(y,\theta) = t_{p0}(y) + \varepsilon t_{p1}(y,\theta), \tag{9}$$

where

$$t_{g0}(y) = \left(1 + \frac{\Gamma_0}{y}\right)^{\frac{1}{1+\alpha}},$$

$$t_{p0}(y) = \left(B_0 + \frac{H_0}{y} - \frac{1}{y}\int_1^y \psi_0 dy + \int_1^y \frac{\psi_0}{y}dy\right)^{\frac{1}{1+\gamma}},$$

$$t_{g1}(y,\theta) = \frac{\cos\theta}{t_{g0}^{\alpha}}\frac{\Gamma}{y^2},$$

$$t_{p1}(y,\theta) = \frac{\cos\theta}{t_{p0}^{\gamma}}\left[By + \frac{H_1}{y^2} + \frac{1}{3}\left(y\int_1^y \frac{\psi_1}{y^2}dy - \frac{1}{y^2}\int_1^y \psi_1 y dy\right)\right],$$

$$H_0 = \frac{R^2(1+\gamma)}{3\lambda_{p0}T_{g\infty}}J_0,\ H_1 = \frac{R^2}{3\lambda_{p0}T_{g\infty}}J,$$

$$J_0 = \frac{1}{V}\int_V q_p\,dV,\ J = \frac{1}{V}\int_V q_p\,z dV,\ V = \frac{4}{3}\pi R^3,$$

$\int_V q_p z dV$ is the dipole moment of density of heat sources [1, 3, 4–7, 12, 14, 23],

$$\psi_0 = -\frac{R^2(1+\gamma)}{2\lambda_{p0}T_{g\infty}} y^2 \int_{-1}^{+1} q_p(r,\theta)dx,$$

$$\psi_1 = -\frac{3R^2}{2\lambda_{p0}T_{g\infty}} y^2 \int_{-1}^{+1} q_p(r,\theta)\,x dx,$$

$$x = \cos\theta,\ z = r\cos\theta.$$

The integration constants entering in the expressions for temperature fields are determined from the boundary conditions on the particle surface. In particular, for the coefficient Γ, we have

$$\frac{\Gamma}{t_{gS}^{\alpha}} = \frac{R}{\lambda_{pS}T_{g\infty}\delta} J. \tag{10}$$

Here,

$$\delta = 1 + 2\frac{\lambda_{gS}}{\lambda_{pS}}(1 - C_q^T \mathrm{Kn}) + 4\frac{\sigma_0\sigma_1 R}{\lambda_{pS}} T_{g\infty}^3 t_{pS}^3 + K_T^{\mathrm{T}}\mathrm{Kn}\left(2 - \frac{\alpha\,\ell^{(S)}}{1+\alpha}\right)\left(1 + 4\frac{\sigma_0\sigma_1 R}{\lambda_{pS}} T_{g\infty}^3 t_{pS}^3\right),$$

$$\lambda_{pS} = \lambda_{p0} t_{pS}^{\gamma},\ \lambda_{gS} = \lambda_{g\infty} t_{gS}^{\alpha},\ t_{pS} = t_{p0}(y=1),$$

$$t_{gS} = t_{g0}(y=1).$$

The average temperature of the particle surface T_{pS} is determined from the solution of the following system of equations in which $T_{pS} = t_{pS}T_{g\infty}$, $T_{gS} = t_{gS}T_{g\infty}$, $\ell^{(S)} = \ell(y=1)$, $\ell = \dfrac{\Gamma_0}{y+\Gamma_0}$, and

$$\begin{cases} \left(1 + K_T^{\mathrm{T}}\mathrm{Kn}\dfrac{\ell^{(S)}}{1+\alpha}\right)T_{gS} = T_{pS}, \\ \dfrac{\ell^{(S)}}{1+\alpha} t_{gS} = \dfrac{R^2}{3\lambda_{gS}T_{g\infty}} J_0 - \sigma_0\sigma_1 \dfrac{R T_{g\infty}^3}{\lambda_{gS}}\left[\left(\dfrac{T_{pS}}{T_{g\infty}}\right)^4 - 1\right]. \end{cases} \tag{11}$$

If the inequality $\lambda_g \ll \lambda_p$ is valid (it holds for most gases), the coefficient of thermal conductivity of the particle is much larger than that of the gas; then, one can neglect the dependence on the angle θ in the coefficient of dynamic viscosity in the particle–gas system (a weak angular asymmetry of the temperature distribution is assumed). With allowance for this, one can consider that the viscosity is connected only with the temperature $t_{g0}(r)$, i.e., $\mu_g(t_g(r,\theta)) \approx \mu_g(t_{g0}(r))$. This assumption permits one to consider the hydrodynamic part separately from the thermal one; they are connected to each other via the boundary conditions.

Substituting (9) into the expression for the dynamic viscosity, we have

$$\mu_g = \mu_{g\infty} t_{g0}^{\beta}. \tag{12}$$

Hereinafter, (12) is used in finding the velocity and pressure fields in the vicinity of a heated spherical aerosol particle.

3. SOLUTION OF THE HYDRODYNAMIC PROBLEM. FINDING EXPRESSIONS FOR THE VELOCITY AND PRESSURE FIELDS

The study of the velocity linearized Navier–Stokes equation in the spherical coordinate system demonstrated the following: if the coefficient of thermal conductivity of the particle is supposed to be much larger in its value than that of the gas (a weak angular asymmetry of the temperature distribution), then this equation can be finally reduced to an inhomogeneous differential equation of the third order with an isolated singular point. The solution of this equation can be found in the form of a generalized power series.

Starting from boundary conditions (5)–(7), we seek the expressions for the zero approximation components of the mass velocity (8) in the form

$$\begin{aligned} U_r^g(y,\theta) &= U_\infty \cos\theta\, G(y), \\ U_\theta^g(y,\theta) &= -U_\infty \sin\theta\, g(y). \end{aligned} \tag{13}$$

Here, $G(y)$ and $g(y)$ are arbitrary functions of the radial coordinate $y = r/R$.

From the continuity equation (2) and equation of state (3), we find the connection between the functions $G(y)$ and $g(y)$:

$$g(y) = \frac{1}{2} y \frac{dG(y)}{dy} + \left(1 + \frac{1}{2(1+\alpha)}\ell(y)\right)G(y). \tag{14}$$

Substituting expressions (13) and (14) into the velocity-linearized Navier–Stokes equation (2), taking into account (12), and separating the variables, we finally obtain after the transformations the following inhomogeneous differential equation of the third order for determining the function $G(y)$

$$y^4\frac{d^3G}{dy^3} + y^3(4 + \gamma_1\ell)\frac{d^2G}{dy^2} - y^2(4 + \gamma_2\ell - \gamma_3\ell^2) \times \frac{dG}{dy} - y(2-\ell)\gamma_3\ell^2 G = -\frac{6A_2}{t_{g0}^{\beta}} \tag{15}$$

with the boundary conditions

$$\begin{aligned} G(y=1) &= 0,\quad G(y\to\infty) \to 1, \\ g(y=1) &= \mathrm{const},\quad g(y\to\infty) \to 1. \end{aligned} \tag{16}$$

Here,

$$\gamma_1 = \frac{1-\beta}{1+\alpha},\ \gamma_2 = 2\frac{1+\beta}{1+\alpha},$$
$$\gamma_3 = \frac{2+2\alpha-\beta}{(1+\alpha)^2},\ A_2 = \text{const}. \tag{17}$$

First, we find the solution of the associated homogeneous equation of (15), i.e.,

$$y^4\frac{d^3G}{dy^3} + y^3(4+\gamma_1\ell)\frac{d^2G}{dy^2} - y^2(4+\gamma_2\ell-\gamma_3\ell^2) \times \frac{dG}{dy} - y(2-\ell)\gamma_3\ell^2 G = 0. \tag{18}$$

The point $y = 0$ is a regular singular point for Eq. (17) [20–22]. Therefore, we seek for the solution in the form of a generalized power series [20–22]:

$$G(y) = y^{\rho}\sum_{n=0}^{\infty} C_n\ell^n, \quad C_0 \neq 0. \tag{19}$$

Substituting series (18) into (17) and equating coefficients at y^{ρ}, we obtain the determining equation $\rho(\rho+3)(\rho-2) = 0$ the roots of which are $\rho_1 = -3$, $\rho_2 = 2$, and $\rho_3 = 0$, respectively. Note that the difference of the roots (in absolute value) is an integer. Therefore, according to the general theory of solving differential equations in the form of generalized power series (the Frobenius method), an additional summand with a logarithm multiplied by the first solution appears in other solutions but the first one (in our case, $\rho_1 = -3$) [20–22]. The recurrent expressions for the corresponding coefficients are determined by the method of undetermined coefficients.

The largest of the roots (in absolute value) is in correspondence with the solution

$$G_1(y) = \frac{1}{y^3}\sum_{n=0}^{\infty} C_{1,n}\ell^n. \tag{20}$$

We do not present the solution that corresponds to the root $\rho_2 = 2$ because its does not satisfy boundary conditions (16) (the finiteness of the solution as $y \to \infty$).

The third solution of Eq. (17) which is linearly independent of the solution G_1 (proportional to the root $\rho_3 = 0$) is sought in the form

$$G_3(y) = \sum_{n=0}^{\infty} C_{3,n}\ell^n + \omega_3\ln(y)G_1(y). \tag{21}$$

The form of the right-hand side of the inhomogeneous equation (15) shows that its partial solution should be sought in the form

$$\tilde{G}(y) = A_2G_2(y),$$
$$G_2(y) = \frac{1}{y}\sum_{n=0}^{\infty} C_{2,n}\ell^n + \omega_2\ln(y)\,G_1(y). \tag{22}$$

The values of the coefficients $C_{1,n}(n \geq 1)$, $C_{2,n}(n \geq 3)$, and $C_{3,n}(n \geq 4)$ are found by the method of undetermined coefficients, and they can be determined by the corresponding recurrent relations:

$$C_{1,n} = \frac{1}{n(n+3)(n+5)}\Big\{\big[(n-1)(3n^2+13n+8) + \gamma_1(n+2)(n+3) + \gamma_2(n+2)\big] \times C_{1,n-1} - \big[(n-1)(n-2)(3n+5) + 2\gamma_1(n^2-4) + \gamma_2(n-2) + \gamma_3(n+3)\big]\,C_{1,n-2} + (n-2)\big[(n-1)(n-3) + \gamma_1(n-3) + \gamma_3\big]C_{1,n-3}\Big\},$$

$$C_{2,n} = \frac{1}{(n+1)(n+3)(n-2)} \times \Big\{\big[(n-1)(3n^2+n-6) + \gamma_1 n(n+1) + n\gamma_2\big]C_{2,n-1} - \big[\gamma_3(n+1) + (n-1)(n-2)(3n-1) + 2\gamma_1 n(n-2) + \gamma_2(n-2)\big]C_{2,n-2} + (n-2)\big[(n-1)(n-3) + \gamma_3 + \gamma_1(n-3)\big]C_{2,n-3} + \frac{\omega_2}{\Gamma_0^2}\sum_{k=0}^{n-2}(n-k-1)\Delta_k - 6\,\frac{(-\gamma_4)(1-\gamma_4)\ldots(n-1-\gamma_4)}{n!}\Big\},$$

$$C_{3,n} = \frac{1}{n(n+2)(n-3)}\Big\{(n-1) \times \big[3n^2 - 5n - 4 + \gamma_1 n + \gamma_2\big]C_{3,n-1} - \big[(n-1)(n-2)(3n-4) + 2\gamma_1(n-1)(n-2) + \gamma_2(n-2) + n\gamma_3\big]C_{3,n-2} + (n-2)\big[(n-1)(n-3) + \gamma_1(n-3) + \gamma_3\big] \times C_{3,n-3} + \frac{\omega_3}{2\Gamma_0^3}\sum_{k=0}^{n-3}(n-k-2)(n-k-1)\Delta_k\Big\},$$

$$\Delta_k = (3k^2+16k+15)C_{1,k} - \big((k-1)(6k+13) + \gamma_1(2k+5) + \gamma_2\big)C_{1,k-1} + \big(3(k-1)(k-2) + 2\gamma_1(k-2) + \gamma_3\big)C_{1,k-2}.$$

Calculating the coefficients $C_{1,n}$, $C_{2,n}$, and $C_{3,n}$ by the recurrent formulas, it is necessary to take into account that

$$C_{1,0} = 1,\ C_{2,0} = 1,\ C_{3,1} = 0, C_{2,2} = 1,$$

$$C_{2,1} = -\frac{1}{8}(2\gamma_1 + \gamma_2 + 6\gamma_4),\ \gamma_4 = \beta/(1+\alpha),$$

$$\frac{\omega_3}{2\Gamma_0^3} = -\frac{\gamma_3}{60}(10 + 3\gamma_1 + \gamma_2),\ C_{3,1} = 0,\ C_{3,2} = \frac{1}{4}\gamma_3,$$

$$C_{3,3} = 1,\ C_{3,0} = 1,$$

$$\frac{\omega_2}{\Gamma_0^2} = \frac{1}{15}\left[\frac{1}{4}(2\gamma_1 + \gamma_2 + 6\gamma_4)\right.$$
$$\left.\times (4 + 3\gamma_1 + \gamma_2) + 3\gamma_3 + 3\gamma_4(\gamma_4 - 1)\right];$$

$C_{1,n}$, $C_{2,n}$, and $C_{3,n}$ equal zero as $n < 0$.

Thus, the general solution of Eq. (15) satisfying the boundary conditions (16) has the form

$$G(y) = A_1G_1(y) + A_2G_2(y) + G_3(y) \quad (23)$$

and the expressions for the components of mass velocity and pressure are

$$U_r^g = U_\infty \cos\theta(A_1G_1 + A_2G_2 + G_3),$$
$$U_\theta^g = -U_\infty \sin\theta(A_1G_4 + A_2G_5 + G_6),$$

$$P_g = P_{g\infty} + \frac{\mu_{g\infty}U_\infty}{R}t_{g0}^{\beta} \times \left\{\frac{y^2}{2}\frac{d^3G}{dy^3}\right.$$
$$+ y\left[3 + \frac{\beta-1}{2}yf\right]\frac{d^2G}{dy^2} - \left[2 - y^2f^{\mathrm{I}} - \frac{\beta}{2}y^2f^2\right. \quad (24)$$
$$\left.+(\beta-2)yf\right]\frac{dG}{dy} + 2\left[y^2f^{\mathrm{II}} + yf^{\mathrm{I}}(4 + y\beta f) - \frac{2}{3}f\right]G\Big\}.$$

Here,

$$f = -\frac{\ell}{y(1+\alpha)},$$

$$G_k = \left(1 + \frac{\ell}{2(1+\alpha)}\right)G_{k-3} + \frac{1}{2}yG_{k-3}^{\mathrm{I}} \ (k = 4,\ 5,\ 6);$$

$$f^{\mathrm{I}},\ f^{\mathrm{II}}, G_1^{\mathrm{I}}, G_2^{\mathrm{I}}, G_3^{\mathrm{I}}$$

are the first and second derivatives of the corresponding functions with respect to y.

The integration constants A_1 and A_2 are determined from the boundary conditions on the aerosol particle surface.

4. DETERMINATION OF THE PHOTOPHORETIC FORCE AND VELOCITY. ANALYSIS OF THE OBTAINED RESULTS

Thus, we have obtained the expressions for the temperature fields outside and inside the aerosol particle in the first approximation with respect to ε, as well as the velocity and pressure distributions in its vicinity. The resulting force acting on the particle is determined by integration of the stress tensor over the surface of the aerosol particle and has the form [17, 18]

$$F_z = \int\limits_{(S)} (-P_g\cos\theta + \sigma_{rr}\cos\theta \quad (25)$$
$$- \sigma_{r\theta}\sin\theta)r^2\sin\theta d\theta d\varphi\Big|_{r=R}.$$

Here, σ_{rr}, $\sigma_{r\theta}$, U_r^g, and U_θ^g are the components of the stress tensor and the radial and tangent components of the mass velocity:

$$\sigma_{rr} = \mu_g\left(2\frac{\partial U_r^g}{\partial y} - \frac{2}{3}\mathrm{div}\,U_g\right),$$

$$\sigma_{r\theta} = \mu_g\left(\frac{\partial U_\theta^g}{\partial y} + \frac{1}{y}\frac{\partial U_r^g}{\partial\theta} - \frac{U_\theta^g}{y}\right).$$

With allowance for the expressions presented above, we obtain that the total force is composed of the force of viscous resistance of the medium $\mathbf{F}_\mu$ and of the photophoretic force $\mathbf{F}_{ph}$:

$$\mathbf{F} = \mathbf{F}_\mu + \varepsilon\mathbf{F}_{ph}, \quad (26)$$

where $\mathbf{F}_\mu = 6\pi R\mu_{g\infty}U_\infty f_\mu \mathbf{n}_z$, $\mathbf{F}_{ph} = -6\pi R\mu_{g\infty}f_{ph}Jn_z$, and $\mathbf{n}_z$ is the unit vector in the direction of the OZ axis.

The values of the coefficients f_μ and f_{ph} can be estimated by the expressions

$$f_\mu = \frac{2}{3}\frac{N_2 + C_m\mathrm{Kn}\,N_4}{N_1 + C_m\mathrm{Kn}\,N_3}, \quad (27)$$

$$f_{ph} = \frac{4}{3}K_{TS}\frac{\nu_{gS}}{\lambda_{pS}\delta T_{g\infty}}\frac{G_1}{N_1 + C_m\mathrm{Kn}\,N_3}$$
$$\times\left\{1 + \mathrm{Kn}\left[\beta'_{RT} + \beta_{RT}^{\mathrm{B}} - (\beta_{RT} - \beta_{RT}^{\mathrm{B}})\left(2 - \frac{\alpha\ell^{(S)}}{1+\alpha}\right)\right.\right.$$
$$\left.\left.+\ 2C_V^T\frac{G_4}{G_1}\left(1 - C_m\mathrm{Kn}\frac{G_4^{\mathrm{I}} + G_1 - G_4}{G_4}\right)\right]\right\}.$$

Setting the resulting force $\mathbf{F}$ equal to zero, we obtain the following expression for the photophoresis velocity $\mathbf{U}_{ph}(\mathbf{U}_{ph} = -\mathbf{U}_\infty)$ of a solid moderately large heated spherical particle:

$$\mathbf{U}_{ph} = -h_{ph}J\mathbf{n}_z, \quad (28)$$

where $h_{ph} = f_{ph}/f_\mu$.

When estimating the coefficients f_μ, f_{ph}, and h_{ph}, it is necessary to take into account that the index S denotes the physical parameters that are taken at the average relative temperature of the particle surface T_{pS} which is determined by formula (11); the functions

$$G_1(y),\ G_1^{\mathrm{I}}(y),\ G_1^{\mathrm{II}}(y),\ G_2(y),\ G_2^{\mathrm{I}}(y),\ G_2^{\mathrm{II}}(y),$$
$$G_3(y),\ G_3^{\mathrm{I}}(y),\ G_3^{\mathrm{II}}(y),\ N_1(y),$$
$$N_2(y),\ N_3(y),\ \text{and}\ N_4(y)$$

are taken at $y = 1$:

$$N_1(1) = G_1(1)G_2^{\mathrm{I}}(1) - G_2(1)G_1^{\mathrm{I}}(1),$$

$$N_2(1) = G_1(1)G_3^{\mathrm{I}}(1) - G_3(1)G_1^{\mathrm{I}}(1),$$

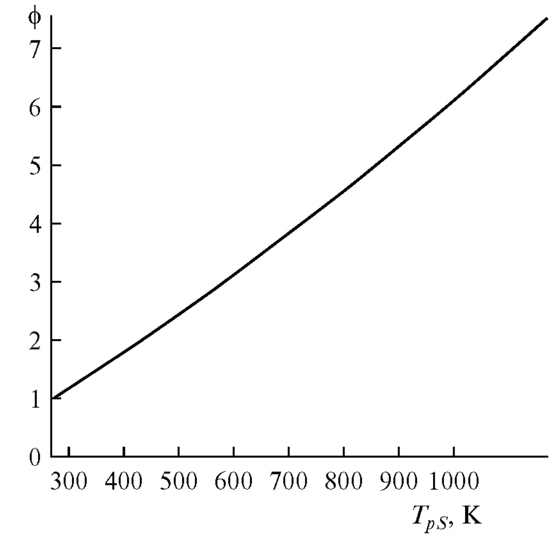

Fig. 1. The curve of the dependence of the function φ on the average temperature of the particle surface T_{pS}.

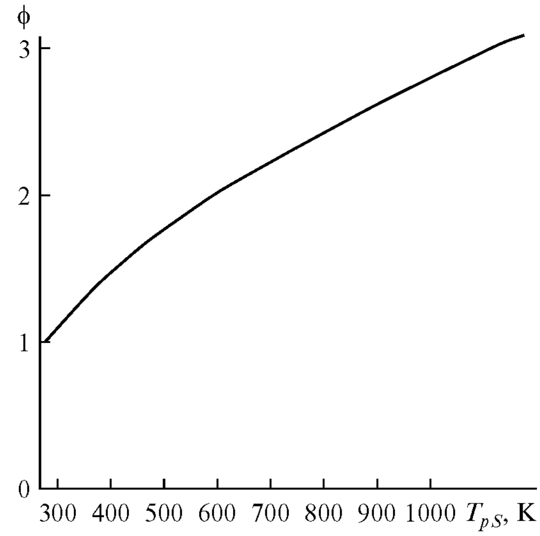

Fig. 2. The curve of the dependence of the function ψ on the average temperature of the particle surface T_{pS}.

$$N_3(1) = G_2(1)G_1^{\mathrm{II}}(1) - G_1(1)G_2^{\mathrm{II}}(1)$$
$$+\left(2+\frac{\ell^{(S)}}{1+\alpha}\right)\left(G_2(1)G_1^{\mathrm{I}}(1) - G_1(1)G_2^{\mathrm{I}}(1)\right),$$

$$N_4(1) = G_3(1)G_1^{\mathrm{II}}(1) - G_1(1)G_3^{\mathrm{II}}(1)$$
$$+\left(2+\frac{\ell^{(S)}}{1+\alpha}\right)\left(G_3(1)G_1^{\mathrm{I}}(1) - G_1(1)G_3^{\mathrm{I}}(1)\right).$$

The formulas obtained above can be also used at small relative differences of temperature in the vicinity of the particle. In the case where the value of heating of the particle surface is small, i.e., the average temperature of the surface insignificantly differs in its value from the temperature of the ambient medium far from the particle ($\Gamma_0 \to 0$), the dependence of coefficients of molecular transfer (viscosity and thermal conductivity) on temperature can be neglected and then ($y = 1$) we have

$$G_1 = 1,\ G_1^{\mathrm{I}} = -3,\ G_1^{\mathrm{II}} = 12,\ G_1^{\mathrm{III}} = -60,$$
$$G_2 = 1,\ G_2^{\mathrm{I}} = -1,\ G_2^{\mathrm{II}} = 2,\ G_3^{\mathrm{III}} = -6,$$
$$G_3 = 1,\ G_3^{\mathrm{I}} = 0,\ G_3^{\mathrm{II}} = 0,\ G_3^{\mathrm{III}} = 0,$$
$$N_1 = 2,\ N_2 = 3,\ N_3 = 6,\ \text{and}\ N_4 = 6.$$

In this case, the formulas for the photophoresis force and velocity coincide with the results of [23].

Numerical estimates of how the heating of the aerosol particle surface has an effect on the photophoresis are of interest. Figures 1 and 2 show the curves connecting the values

$$\varphi = f_{ph}/f_{ph}\big|_{T_{pS}=273\ \mathrm{K}},\ \psi = h_{ph}/h_{ph}\big|_{T_{pS}=273\ \mathrm{K}}$$

at

$$f_{ph}\big|_{T_{pS}=273\ \mathrm{K}} = 1.15\times10^{-10},\ h_{ph}\big|_{T_{pS}=273\ \mathrm{K}} = 1.38\times10^{-10}$$

with the values T_{pS} for moderately large particles of copper with a radius $R = 5$ μm moving in air in normal conditions.

To illustrate the dependence of $\mathbf{F}_{ph}$ and $\mathbf{U}_{ph}$ on the intensity of the incident radiation, we consider the simplest case where the particle absorbs the radiation incident on it as a black body. In this case, the absorption occurs in a thin layer with a thickness $\delta R \ll R$ adjacent to the heated part of the particle surface. Here, the density of heat sources inside the layer with a thickness δR is determined by the formula [14]

$$q_p = \begin{cases} -\dfrac{I_0}{\delta R}\cos\theta, & \dfrac{\pi}{2} \le \theta \le \pi,\ R - \delta R \le r \le R, \\ 0, & 0 \le \theta \le \dfrac{\pi}{2}. \end{cases} \tag{29}$$

In this case, the integrals are easy to calculate:

$$\int_V q_p dV = \pi R^2 I_0,\quad \int_V q_p z dV = -\frac{2}{3}\pi R^3 I_0,$$

and we obtain the following expressions for the photophoretic force and velocity of absolutely black moderately large spherical particles:

$$\mathbf{F}_{ph}^{*} = 3\pi R\mu_{g\infty} f_{ph} I_0 \mathbf{n}_z,$$
$$\mathbf{U}_{ph}^{*} = \frac{h_{ph}}{2} I_0 \mathbf{n}_z (h_{ph} = f_{ph}/f_{\mu}). \tag{30}$$

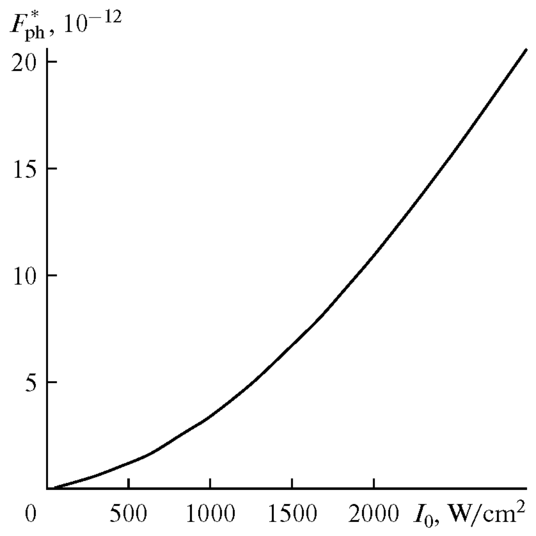

Fig. 3. The curve of the dependence of the photophoretic force F^*_{ph} on the intensity of the incident radiation I_0.

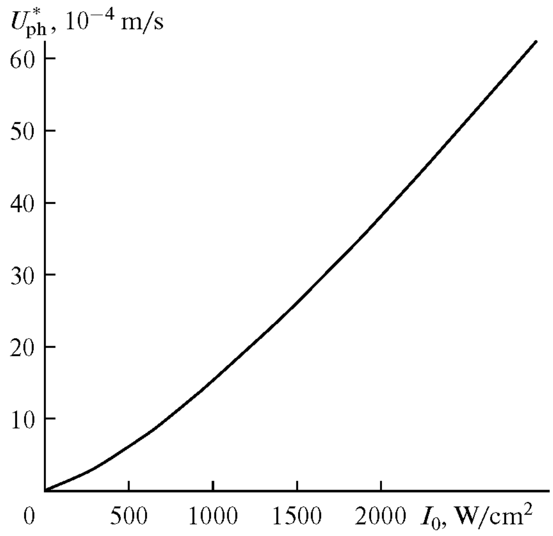

Fig. 4. The curve of the dependence of the photophoretic velocity U^*_{ph} on the intensity of the incident radiation I_0.

The average temperature of the particle surface T_{pS} is connected with the intensity of the incident radiation by the formula

$$\begin{cases} \left(1 + K_T^{\rm T}{\rm Kn}\dfrac{\ell^{(S)}}{1+\alpha}\right)T_{gS} = T_{pS} \\ \dfrac{\ell^{(S)}}{1+\alpha}\dfrac{\lambda_{gS}}{\lambda_{pS}}T_{gS} = \dfrac{R}{4\lambda_{gS}}I_0 - \sigma_0\sigma_1\dfrac{RT^4_{g\infty}}{\lambda_{gS}}\left[\left(\dfrac{T_{pS}}{T_{g\infty}}\right)^4 - 1\right]. \end{cases} \quad (31)$$

Figures 3 and 4 show the curves connecting the values F^*_{ph} and U^*_{ph} with those of I_0 for moderately large particles of copper with a radius $R = 5$ m moving in air in normal conditions.

CONCLUSIONS

Formulas (25)–(27) permit one to take into account the influence of heating of the particle surface on the value of the photophoretic force and velocity at arbitrary differences of temperature between the particle surface and a region far from it, with allowance for the power dependence of viscosity and thermal conductivity on temperature when the distribution of the density of heat sources over the volume is known. The formulas obtained are of the most general character.

It is seen from formulas (25)–(27) that the value and the direction of the photophoresis force and velocity are determined by the value and direction of the dipole moment of the heat source density $\int\limits_V q_p z dV \mathbf{n}_z$. In cases where the dipole moment is negative (when the most part of thermal energy is released in the region of the particle that faces the radiation source), the particle moves in the direction of the incident radiation. If the dipole moment is positive (the most part of thermal energy is released in the shadow region of the particle), the particle moves oppositely to the direction of radiation propagation. To calculate the integral, it is necessary to know the quantity q_p that is determined from the solution of the electrodynamics problem [1, 6, 14]. At present, there are numerical methods permitting one to find the value of the dipole moment of the density of heat sources; for example, the listing of the program is presented in [24].

It is seen from Figs. 1–4 that the photophoresis force and velocity increase nonlinearly with an increase in the radiation intensity, which is caused by the dependence of the coefficients of molecular transfer and density on temperature. In the case of small differences, a linear character of the dependence is observed, which coincides with the well-known results [2, 4, 6, 12].

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