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FIELDS, PARTICLES, AND NUCLEI

Incoherent Form Factor in Diffraction and Smith–Purcell Radiations

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It has been shown that polarization radiation of charged particle beams generally includes an incoherent form factor caused by a finite transverse size of a beam. Consequently, a widespread opinion that the form factor characterizes only coherent radiation of charge particle bunches is generally invalid. The reason for the existence of incoherent form factor is the interaction of charged particles with the target edge in the direction perpendicular to their trajectory. The incoherent form factor exists for diffraction radiation, Smith–Purcell radiation, and other types of polarization radiation in the case of transversely limited targets: transition radiation, parametric X-ray radiation, and Cherenkov radiation. It has been shown that the difference of the incoherent form factor from unity increases with a decrease in the ratio of the impact parameter to the transverse size of the bunch. Furthermore, it has been shown that the transverse part of the coherent form factor differs from unity to the same extent as the incoherent form factor.

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1. INTRODUCTION

The emission of electromagnetic waves by charged particles has numerous applications. This includes the emission of electromagnetic radiation in various ranges at compact accelerators, synchrotrons, and free-electron lasers and the diagnostics of beams of electrons, protons, and other charged particles at accelerators and colliders.

The spectral–angular distribution of radiation from a beam is usually represented in the form [1, 2]

$$\frac{d^2W_N(\mathbf{n},\omega)}{d\omega d\Omega} = \frac{d^2W_1(\mathbf{n},\omega)}{d\omega d\Omega}G(\mathbf{k}), \tag{1}$$

where

$$\frac{d^2W_1(\mathbf{n},\omega)}{d\omega d\Omega} = cr^2|\mathbf{E}_1(\mathbf{r},\omega)|^2 \tag{2}$$

is the energy emitted by one particle in the infinitesimal frequency range $d\omega$ into the infinitesimal solid angle interval $d\Omega$, $G(\mathbf{k})$ is the form factor describing the properties of the beam, ω is the radiation frequency, $\mathbf{k}$ is the wave vector, $\mathbf{n} = \mathbf{k}/k$ is the unit vector in the direction of the wave vector, c is the speed of light in vacuum, $\mathbf{r} = r\mathbf{n}$ is the position vector of the observation point, and $\mathbf{E}_1(\mathbf{r},\omega)$ is the Fourier transform of the radiation field from one particle. In many works, the form factor is written in the form [1, 3, 4]

$$G(\mathbf{k}) = N + N(N-1)G_{\text{coh}}(\mathbf{k}), \tag{3}$$

where N is the number of particles in the bunch and $G_{\text{coh}}(\mathbf{k})$ is the coherence factor or coherent form factor, which is the Fourier transform of the particle distribution function $f(\mathbf{r}')$ in the beam [5, 6]:

$$G_{\text{coh}}(\mathbf{k}) = \left|\int d^3r'\exp(-i\mathbf{k}\cdot\mathbf{r}')f(\mathbf{r}')\right|^2. \tag{4}$$

Since the form factor in Eq. (3) includes the sizes, profile, and internal structure of the beam, it was called the structure factor in old works [1]. Individually, the authors of [6] refer to $\sqrt{G_{\text{coh}}(\mathbf{k})}$ as the structure factor and to $G_{\text{coh}}(\mathbf{k})$ as the form factor.

The first term in Eq. (3) describes incoherent radiation when the intensities of radiation from individual particles are added. The second term originates from interference of fields and describes coherent radiation. Since the number of charged particles in a bunch at modern colliders reaches $N = 10^{11}$, the factor N^2 is in practice very large; consequently, the behavior of the coherence factor G_{coh} as a function of the angle and frequency is of key importance.

Formulas (3) and (4) are usually justified as follows. Each moving charged particle is described by a certain current density so that all points of its trajectory can be considered as sources of a radiation field. In view of the superposition principle, total radiation can be calculated as the sum of radiation fields from individual particles of the beam. Let the distribution of particles in the beam be described by the function $f(\mathbf{r}')$, where $\mathbf{r}'$ is the position vector of a particle. The radiation field from the nth particle with the position vector $\mathbf{r}'_n$ differs from the field $\mathbf{E}_1(\mathbf{r}, \omega)$ from the particle located at the origin of the coordinate system only in the phase incursion denoted as $\psi_n = \mathbf{k} \cdot \mathbf{r}'_n$. Thus, the total radiation field from N particles is given by the expression

$$\mathbf{E}(\mathbf{r}, \omega) = \mathbf{E}_1(\mathbf{r}, \omega) \sum_{n=1}^{N} \exp(-i\psi_n). \quad (5)$$

Since the spectral–angular distribution of radiation is proportional to the square of the intensity of the radiation field averaged over the position of particles in the beam, then

$$\frac{d^2W_N(\mathbf{n}, \omega)}{d\omega d\Omega} = \frac{d^2W_1(\mathbf{n}, \omega)}{d\omega d\Omega} \left\langle \sum_{n=1}^{N} |\exp(-i\psi_n)|^2 + \sum_{n=1}^{N} \sum_{\substack{m=1, \\ m \neq n}}^{N-1} \exp(-i\psi_n) \exp(i\psi_m) \right\rangle, \quad (6)$$

where the angle brackets stand for averaging over the positions of all particles in the bunch. Therefore,

$$\frac{d^2W_N(\mathbf{n}, \omega)}{d\omega d\Omega} = \frac{d^2W_1(\mathbf{n}, \omega)}{d\omega d\Omega} \times \left[N + N(N-1) \left| \int d^3r' \exp(-i\psi(\mathbf{r}')) f(\mathbf{r}') \right|^2 \right], \quad (7)$$

which coincides with Eqs. (1)–(4), where $\psi(\mathbf{r}') = \mathbf{k} \cdot \mathbf{r}'$.

The derivation and form of Eq. (7) are valid always when radiation originates directly from charged particles, e.g., for synchrotron and undulator radiation, as well as bremsstrahlung and related types of radiation.

However, in addition to the radiation from a charged particle, which is due to its accelerated motion, there is a polarization radiation mechanism. In this case, radiation is directly emitted by matter excited by the field of the particle. In this case, according to the approach by I.M. Frank and V.L. Ginzburg [7], the particle itself can be considered as uniformly moving but only if the energy loss of the particle can be neglected compared to its kinetic energy. The polarization mechanism is responsible for Cherenkov, transition, and parametric X-ray radiation, as well as diffraction radiation (when particles move near the edge of a target) and Smith–Purcell radiation (when particles move over the surface of a diffraction grating), which are directly associated with the existence of the edge of a target [8].

The authors of the overwhelming majority of works and monographs use Eq. (7) for all these types of radiation. Indeed, the consideration above Eq. (5) can be easily generalized to polarization radiation as follows. Instead of directly emitting charged particles of a beam, one can consider a surface on which polarization currents are induced by the dynamically varying field of fast charged particles. In this case, radiation from different points of this surface is summed similarly and Eq. (5) remains valid. We note that this consideration in the case of transition radiation from a finite-thickness plate infinite in the directions perpendicular to the trajectory of particles coincides with the results of the exact theory of transition radiation obtained directly from the solution of Maxwell's equations [1]. This is expectable because the derivation of Eqs. (5) and (7) is based only on the superposition principle, which is valid for systems of noninteracting particles within linear electrodynamics.

However, we show in this work that Eq. (7) is generally invalid for a whole class of phenomena caused by the proximity of the edge of a target in the case of polarization radiation from charged particle beams.

2. FORM FACTOR IN POLARIZATION RADIATION

As an example, we consider qualitatively the generation of diffraction radiation as a process of scattering of the Coulomb field of a moving charged particle. Describing the Coulomb field as a set of virtual photons with the momentum $\mathbf{q}$, we apply the energy and momentum conservation laws as in [9]. We note that the violation of the mass shell relation for virtual photons does not violate the energy and momentum conservation laws when virtual photons reach the mass shell by means of a scatterer (medium), in our case, in the process of transformation of the Coulomb field of charged particles in the field of the electromagnetic wave of radiation with the wave vector $\mathbf{k}$. Examples of application of this technique of qualitative estimates to problems of radiation from charged particles can be found, e.g., in well-known works [3, 10, 11].

We consider a particle moving near the edge of a target in the form of a screen (plane-parallel plate), cylindrical rod, etc. Let the particle move along the x axis in which the target is limited, the z axis be perpendicular to the target surface, and the properties of the target surface along the y axis be uniform, i.e., the projection of the momentum on this axis be conserved. Taking into account the momentum and energy conservation laws within the equivalent photon method, we have

$$q_y = k_y, \quad q = k. \quad (8)$$

We also write the dispersion relation following from the explicit form of the Coulomb field of charged particles (zero argument of the delta function of the Fourier transform of the Coulomb field of the particle moving at the velocity $\mathbf{v}$):

$$\omega = \mathbf{q} \cdot \mathbf{v}. \quad (9)$$

Since $q = \sqrt{q_x^2 + q_y^2 + q_z^2}$, we obtain

$$\mathbf{q} = \left(\frac{\omega}{c\beta}, k_y, -i\frac{\omega}{\gamma\beta c}\sqrt{1+\gamma^2\beta^2 n_y^2}\right), \quad (10)$$

where $\mathbf{n} = \mathbf{k}c/\omega$, $\beta = v/c$ is the dimensionless velocity of a charged particle emitting radiation, and $\gamma = 1/\sqrt{1-\beta^2}$ is the Lorentz factor of particles. We note that Eq. (10) is obtained under the assumption that the coordinate axes were such that the x axis coincides with the positive direction of the velocity of the particle (so that $\omega = q_x v$) and the assumption that radiation is detected only in the upper half-plane (the negative sign is chosen in front of the square root in q_z). An expression for the total radiation field is written in the form similar to Eq. (5) as the sum of fields of radiation from all points of the emitting target surface:

$$\mathbf{E}(\mathbf{r},\omega) = \mathbf{E}_1(\mathbf{r},\omega)\sum_{n=1}^{N}\exp(-i\mathbf{q}\cdot\mathbf{r}'_n). \quad (11)$$

We emphasize that the condition $q_y = k_y$ corresponding to the homogeneity (large sizes) of the target along the y axis (the y size should be noticeably larger than both the respective size of the bunch and the characteristic transverse size of the field of moving charged particles $\gamma\beta\lambda/2\pi$) is significant for the applicability of Eq. (11) because the momentum conservation along the y axis allows removing the axial symmetry of the problem; as a result, all particles of the bunch are equivalent as sources of radiation except for their distance to the target surface. The condition $q_y = k_y$ is not satisfied for targets with small y sizes; consequently, Eq. (11) is invalid and this case requires a separate consideration.

The calculation of the spectral–angular distribution of radiation yields the following expression similar to Eq. (6):

$$\frac{d^2W_N(\mathbf{n},\omega)}{d\omega d\Omega} = \frac{d^2W_1(\mathbf{n},\omega)}{d\omega d\Omega}\left\langle\sum_{n=1}^{N}|\exp(-i\mathbf{q}\cdot\mathbf{r}'_n)|^2 + \sum_{n=1}^{N}\sum_{\substack{m=1,\\ m\neq n}}^{N-1}\exp(-i\mathbf{q}\cdot\mathbf{r}'_n)\exp(i\mathbf{q}^*\cdot\mathbf{r}'_m)\right\rangle. \quad (12)$$

Here, the asterisk * stands for complex conjugation. A significant difference of Eq. (12) from Eq. (6) is a complex phase $\mathbf{q}\cdot\mathbf{r}'_n$; as a result, Eq. (12) instead of widely used Eq. (7) gives

$$\frac{d^2W_N(\mathbf{n},\omega)}{d\omega d\Omega} = \frac{d^2W_1(\mathbf{n},\omega)}{d\omega d\Omega}F, \quad (13)$$

where

$$F = NF_{\mathrm{inc}} + N(N-1)F_{\mathrm{coh}}. \quad (14)$$

Here,

$$F_{\mathrm{inc}} = \int d^3r'|\exp(-i\mathbf{q}\cdot\mathbf{r}')|^2 f(\mathbf{r}'), \quad (15)$$

$$F_{\mathrm{coh}} = \left|\int d^3r'\exp(-i\mathbf{q}\cdot\mathbf{r}')f(\mathbf{r}')\right|^2 \quad (16)$$

are the incoherent and coherent form factors, respectively. In Eqs. (15) and (16), the vector $\mathbf{q}$ is given by Eq. (10) in terms of the components of the wave vector $\mathbf{k}$ of the radiation field and the velocity of the charged particle $\mathbf{v}$, and $f(\mathbf{r}')$ is the particle distribution function in the beam with respect to the center of the bunch.

Expressions (13)–(16) are derived here from the qualitative description of the phenomenon in terms of equivalent photons and from fundamental conservation laws. Furthermore, these results coincide with the results of the direct calculation based on Maxwell's equations in the vacuum ultraviolet and X-ray ranges for the calculation of radiation from a single charged particle (theory of diffraction radiation in [12] and the theory of Smith–Purcell radiation [9]) and for the calculation of radiation from bunches of particles with both a uniform distribution of particles [13–15] and a Gaussian distribution [16]. The dependence of the radiation field of type (11) also occurs for ideally conducting targets, i.e., in the long-wavelength part of the spectrum (see [17, 18]). Indeed, within a quite general approach of the matching method, the dependence of the properties of the radiation field on the current density appears through boundary conditions. Since the dependence of type (11) directly is contained in the Fourier transform of the current density, it is also present in the radiation field.

In contrast to the phase in Eq. (7), the phase in Eq. (11) is complex because the polarization mechanism of radiation generation is fundamentally different from bremsstrahlung mechanism when radiation is directly caused by a change in the magnitude or direction of the velocity of the charged particle. This is physically due to the following feature: the source of polarization radiation is matter excited by the field of charged particles, and the characteristics of radiation directly depend on the configuration of this polarizing field. However, for the complex value of the phase in Eq. (11) to be manifested, the target edge in the direction perpendicular to the trajectory of the particle beam should be at a finite distance.

Indeed, when the beam moves near the target edge, particles in the beam are located at different distances from the edge because of finite transverse sizes of the beam. Since the Fourier component of the Coulomb field decreases exponentially at a characteristic scale of $\gamma\beta\lambda/2\pi$ (λ is the radiation wavelength), charged particles located at different distances from the target edge make different contributions to the polarization of the target material. Mathematically, this means that the phase in Eq. (11) is complex for any processes involving polarization radiation whose characteristics depend on the existence of the target edge; the imaginary part of this phase corresponds to the exponential decrease in fields polarizing matter.

This is valid not only for diffraction radiation and Smith–Purcell radiation but also for Cherenkov, transition, and parametric X-ray radiation near the target edge. In the overwhelming majority of current works even concerning diffraction radiation and Smith–Purcell radiation, Eq. (7) is used in calculations instead of Eqs. (13)–(16). However, even this invalid approach can give adequate results when the transverse size of the beam is negligibly small.

3. ESTIMATE OF THE CONTRIBUTION OF THE INCOHERENT FORM FACTOR

To demonstrate differences in the form factors of radiation in the calculations by Eqs. (3) and (14), we discuss diffraction radiation from (a) a cylindrical bunch with a uniform distribution of electrons and (b) a Gaussian electron bunch.

(a) Let the cylindrical bunch with the length l and radius r_0 contain N uniformly distributed particles. Here and below, the coordinate system is chosen such that the bunch moves along the x axis and the z axis is perpendicular to the target surface. It is convenient to specify the unit vector of radiation observation as

$$\mathbf{n} = (\cos\theta, \sin\theta\cos\varphi, \sin\theta\sin\varphi). \tag{17}$$

Integration over the volume of such a bunch in Eq. (4) gives [1]

$$G_{\rm coh} = 4\left(\frac{\sin(\omega l/2v)}{\omega l/2v}\frac{J_1(r_0\sqrt{k_y^2+k_z^2})}{r_0\sqrt{k_y^2+k_z^2}}\right)^2, \tag{18}$$

where $J_1(r_0\sqrt{k_y^2+k_z^2})$ is the Bessel function of the first kind and $\mathbf{k} = \mathbf{n}\omega/c$.

At the same time, integration over the volume of the bunch in Eqs. (15) and (16) yields the correct formulas

$$F_{\rm inc} = 2\frac{I_1(2\rho r_0)}{2\rho r_0}, \tag{19}$$

$$F_{\rm coh} = 4\left(\frac{\sin(\omega l/2v)}{\omega l/2v}\frac{I_1(r_0\omega/c\beta\gamma)}{r_0\omega/c\beta\gamma}\right)^2, \tag{20}$$

where $\rho = \frac{\omega}{\gamma\beta c}\sqrt{1+\gamma^2\beta^2 n_y^2}$ and $I_1(x)$ is the modified Bessel function of the first kind. In contrast to Eq. (18), Eq. (20) is independent of the angles of radiation that appear in Eq. (18) through the k_y and k_z components of the wave vector $\mathbf{k}$.

The derived expressions are valid for particles of any energy if the loss on radiation is much smaller than the energy of the particle.

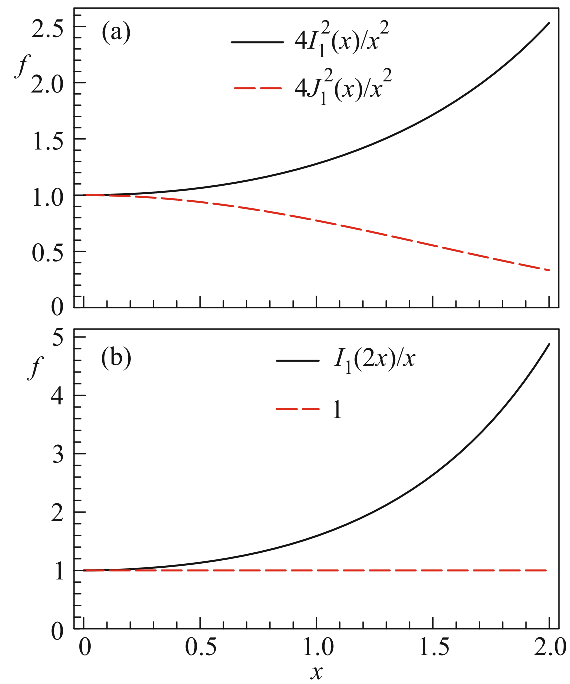

Fig. 1. (Color online) (a) Functions $4I_1^2(x)/x^2$ and $4J_1^2(x)/x^2$ determining the correct and incorrect behaviors of the transverse part of the coherent form factor, respectively. (b) Function $I_1(2x)/x$ determining the incoherent form factor.

For particles with a high energy, i.e., with $\gamma \gg 1$ and $\beta \approx 1$ ($v \approx c$), diffraction radiation is concentrated near the $n_y = 0$ plane containing the trajectory of the particle and the normal vector to the target surface. The characteristic angle of diffraction radiation is $\theta \approx \gamma^{-1}$, and the components of the unit observation vector are $n_x \approx 1$ and $n_z \approx \gamma^{-1}$. Then, the comparison of the coherent form factors (18) and (20) indicates that they differ only in the type of the Bessel function. These two Bessel functions behave similarly only at a small argument, as seen in Fig. 1a: at $r_0\omega/c\beta\gamma > 1/2$ or, in other words, at

$$r_0 > \frac{1}{2}\frac{\gamma\beta\lambda}{2\pi}, \tag{21}$$

the coherent form factors given by Eqs. (18) and (20) are noticeably different.

In turn, the difference of the incoherent form factor given by Eq. (19) from unity is obvious under the condition (see Fig. 1b)

$$r_0 > \frac{1}{2}\frac{\gamma\beta\lambda}{4\pi}. \tag{22}$$

It is noteworthy that an increase in the functions in Fig. 1 according to Eqs. (19) and (20) is generally limited by a decreasing exponential, which always appears in the spectral–angular distribution of radiation from a single particle in the presence of the target edge:

$$\frac{d^2W_1(\mathbf{n},\omega)}{d\omega d\Omega} \propto \exp[-2\rho h], \tag{23}$$

where h is the impact parameter, i.e., the shortest distance between the center of the bunch and the target surface. In this case, a geometric constraint occurs: it is clear that the transverse size of the bunch should be smaller than the distance between its center and the target surface, i.e.,

$$h \geq r_0. \tag{24}$$

We note that condition (24) in the practical case of particles with ultrarelativistic energies is taken with the more stringent form $h \gg r_0$, because beams are usually separated by distances noticeably exceeding the transverse size of the beam in order to prevent the induction of radioactivity in targets by the halo of the beam. However, beams with moderate energies, in particular, relativistic ones, are in practice guided maximally close to the target (diffraction grating in the case of generation of Smith–Purcell radiation), so that a significant heating of the target occurs [19, 20].

The radiation intensity decreases exponentially with an increase in the parameter ρh (see Eq. (23)). The maximum value of this parameter is

$$\rho_{\max} h = 2\pi h/\gamma\beta\lambda, \tag{25}$$

but the parameter h can be increased in the relativistic case by a factor of $b > 1$ until the exponential suppression is not too large and the sensitivity of the detector is enough to detect radiation. In this case, there is the range of the parameters,

$$\gamma\lambda\frac{b}{2\pi} > h > \gamma\lambda\frac{1}{8\pi}, \tag{26}$$

where radiation is still quite intense and can be detected experimentally, the effect of the incoherent form factor cannot be neglected, and the correct inclusion of the (transverse component) coherent form factor is necessary.

Figure 2a shows the wavelength dependence of the spectral–angular distribution of diffraction radiation from a cylindrical bunch calculated by (black solid line) Eq. (13) and (red dashed line) Eq. (1). The spectral–angular distribution of radiation from a single electron was taken from [14] for the X-ray frequency range. The length of the electron bunch was chosen such that radiation is incoherent. Figure 2b shows the spectral–angular distributions calculated by Eqs. (1) and (13) for coherent radiation.

When the particle moves parallel to the grating, Smith–Purcell radiation is concentrated in the $n_y = 0$ plane containing the trajectory of the particle and the normal vector to the target surface. The characteristic angles of Smith–Purcell radiation are determined by the dispersion relation [21]

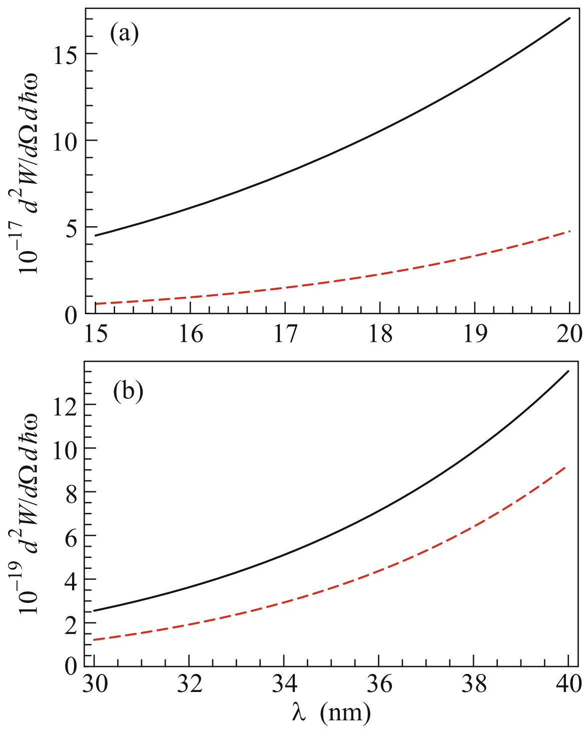

Fig. 2. (Color online) Wavelength dependence of the spectral–angular distribution of diffraction radiation of a cylindrical bunch for (a) incoherent radiation from the bunch with the radius r_0 = 40 μm and length l = 0.16 mm and (b) coherent radiation from the bunch with the radius r_0 = 50 μm and length l = 50 μm. The dark solid and red dashed lines correspond to Eqs. (13) and (1), respectively. All lines are obtained with the parameters $N = 10^8$, $\gamma = 10^4$, $\varphi = 0$, $\theta = \gamma^{-1}$, $\hbar\omega_p = 26.1$ eV (beryllium), impact parameter $h = 50$ μm, dielectric function of the target material $\varepsilon(\omega) = 1 - \omega_p^2/\omega^2$, and plate width $a = 165$ μm.

$$\lambda = \frac{d}{s}(\beta^{-1} - n_x), \quad s = 1, 2, \ldots, \tag{27}$$

where d is the period of the grating and s is a positive integer. Then, after the substitution of $n_x = \beta^{-1} - s\lambda/d$ and $n_z = \sqrt{1 - (\beta^{-1} - s\lambda/d)^2}$ into Eq. (18), it is seen that the difference between the transverse components of coherent form factors—incorrect (18) and correct (20)—both for Smith–Purcell radiation and for diffraction radiation discussed above is determined by mathematically different functions at any size of the bunch. A similar conclusion can be made on the difference of the incoherent form factor from unity.

(b) Let the particles in the bunch have Gaussian distributions in all three directions. Integration in Eq. (4) for such a bunch gives

$$G_{coh} = \exp\left[-\frac{k_x^2\sigma_x^2}{2} - \frac{k_y^2\sigma_y^2}{2} - \frac{k_z^2\sigma_z^2}{2}\right], \tag{28}$$

where σ_x, σ_y, and σ_z are the characteristic sizes of the bunch in the respective directions.

It is substantial that integration in Eqs. (15) and (16) is performed over the entire space, and limits can correspond only to the region of nonzero values of the distribution function of particles $f(\mathbf{r}')$. The function $f(\mathbf{r}')$ for the Gaussian beam is mathematically defined in the entire space. Consequently, strictly speaking, Eqs. (15) and (16) describe the form factor of the Gaussian beam, where some particles fly above the target surface and the other particles below it. In this case, radiation can be more intense if the center of the bunch moves very close to the surface. Such radiation can be described more strictly as done in [16]. However, for the qualitative consideration of the effect, it is sufficient to limit the distribution function:

$$f(\mathbf{r}') = \frac{2}{\sigma_x\sigma_y\sigma_z\pi\sqrt{\pi}}\frac{1}{1+\Phi(h/\sigma_z)} \times \exp\left[-\frac{x'^2}{\sigma_x^2} - \frac{y'^2}{\sigma_y^2} - \frac{z'^2}{\sigma_z^2}\right], \quad z' > -h. \tag{29}$$

The distribution function given by (29) is normalized to unity, so that the probability of intersection of the target by particles of the bunch is zero and integration in Eqs. (15) and (16) over dz' is performed from the target edge $-h$ to infinity. A simple integration gives

$$F_{inc} = \frac{1-\Phi(\sigma_z\rho - h/\sigma_z)}{1+\Phi(h/\sigma_z)}e^{\sigma_z^2\rho^2}, \tag{30}$$

$$F_{coh} = \frac{\left(1-\Phi\left(\frac{\sigma_z\rho}{2} - h/\sigma_z\right)\right)^2}{(1+\Phi(h/\sigma_z))^2}e^{\frac{-\omega^2\sigma_x^2}{2c^2\beta^2}}e^{-\frac{k_y^2\sigma_y^2}{2}}e^{\frac{\sigma_z^2\rho^2}{2}}, \tag{31}$$

where

$$\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \tag{32}$$

is the error function. Since ρ is proportional to ω, it seems that Eqs. (30) and (31) tend to infinity at an unlimited increase in the frequency. However, this does not occur. Indeed, using the known asymptotic formula

$$\Phi(x \to +\infty) \approx 1 - \frac{e^{-x^2}}{x\sqrt{\pi}}, \tag{33}$$

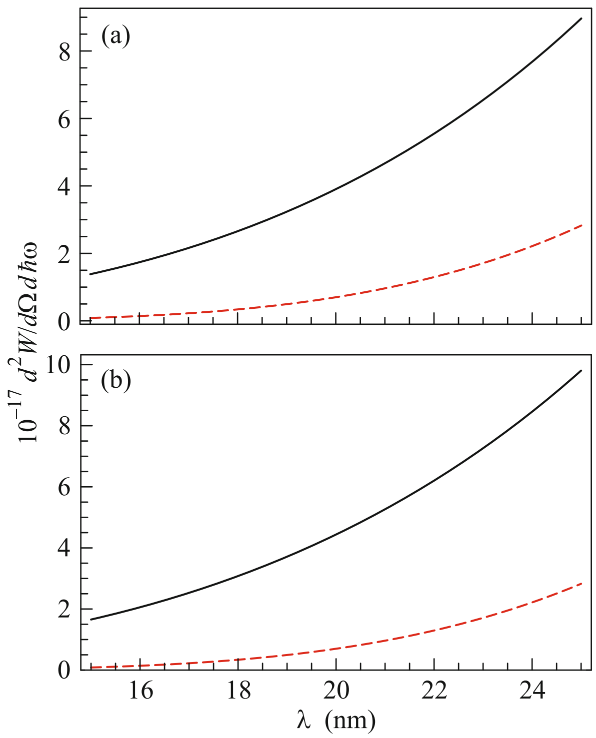

Fig. 3. (Color online) Wavelength dependence of the spectral–angular distribution of diffraction radiation from a Gaussian bunch for (a) incoherent radiation from the bunch with the characteristic transverse sizes $\sigma_y = \sigma_z = 40$ μm and length $\sigma_x = 0.16$ mm and (b) coherent radiation from the bunch with the characteristic transverse sizes $\sigma_y = \sigma_z = 50$ μm and length $\sigma_x = 50$ μm. The dark solid and red dashed lines correspond to Eqs. (13) and (1), respectively. The plate width is $a = 65$ μm and the other parameters are the same as in Fig. 2.

one can reduce Eq. (30) to the form

$$F_{inc} \overset{\omega\to+\infty}{=} \frac{1}{\sqrt{\pi}}\frac{e^{2\rho h}e^{-h^2/\sigma_z^2}}{(1+\Phi(h/\sigma_z))(\sigma_z\rho - h/\sigma_z)} \times e^{-\frac{\omega^2\sigma_x^2}{2c^2\beta^2}}e^{\frac{-k_y^2\sigma_y^2}{2}}. \tag{34}$$

When deriving Eq. (34), it is necessary to take into account that the geometry of the problem imposes the constraint $h/\sigma_z > 1$ and that $h/\sigma_z = \text{const}$ in the limit $\omega \to +\infty$. The growing exponential $e^{2\rho h}$ is compensated by the factor $e^{-2\rho h}$, which is always present in the spectral–angular distribution of radiation from a single particle with the impact parameter h. A similar consideration is also applicable to Eq. (31).

An expression for the Gaussian bunch similar to Eq. (15) was presented in [22] but without any justification, the distribution function was normalized to unity in the entire space, and integration was performed over a half-space.

Figure 3 shows dependences similar to those in Fig. 2 for diffraction radiation from the Gaussian bunch. It is seen that there is a wavelength range where the correct calculation gives a significantly different spectral–angular distribution of radiation.

We note that the above consideration is also applicable to a modulated beam [23, 24] or the measurement of polarization radiation [25].

4. CONCLUSIONS

To summarize, the incoherent form factor, along with the transverse part of the coherent form factor, appears when beams with finite transverse sizes move near the target edge. These physical effects occur because the Coulomb field of moving charged particles is limited in the transverse direction by a scale of $\gamma\beta\lambda/2\pi$; therefore, particles of the bunch located at different distances from the target edge make different contributions to the polarization of the target material. The effect has been revealed without complex calculations used in the theory of diffraction or Smith–Purcell radiation [3, 8, 17, 18, 22, 26, 27]. This is natural because this effect is due not to complicated methods of solution of boundary-value problems of electrodynamics but to a simple and clear physical circumstance: the Coulomb field of moving charged particles is limited in the transverse direction in the $(\mathbf{r}, \omega)$ variables. Similarly, the effect itself later named after S.J. Smith and E.M. Purcell [21], who experimentally discovered it, was predicted by I.M. Frank from simple qualitative arguments [28].

We note that when analyzing radiation from the bunch at the wavelength λ exceeding the size of the bunch σ_x,

$$\sigma_x < \lambda \tag{35}$$

(short bunch), both incoherent and coherent form factors depend on the same parameters, and it is possible that incoherent radiation will always be negligibly small compared to coherent radiation where the incoherent form factor is noticeably different from unity. For this reason, the analysis of the incoherent form factor in the case of short bunches is meaningful only in comparison with the coherent form factor.

In the case of long bunches,

$$\sigma_x > \lambda, \tag{36}$$

the coherent form factor (as well as coherent radiation) is suppressed, so that the incoherent form factor can and should be analyzed separately.

To conclude, we attract attention to an important item. In this work, the calculation has been performed for a limited class of targets. As we mentioned below Eq. (11), the description of coherent effects in terms of the form factor can be inapplicable for an arbitrary target, e.g., for a point one. A correct calculation method for the most general case is the calculation of radiation fields with the subsequent averaging of the square of their magnitude over all particles in the bunch. The parameters of a particular target determine whether the form factor will be separated in the result.

The existence of the incoherent form factor makes it possible in principle to perform noninvasive diagnostics of the transverse sizes of bunches in terms of incoherent radiation, which was thought to be fundamentally impossible because it was always accepted by default that the incoherent form factor is unity. Furthermore, even in the diagnostics of bunches of charged particles in terms of coherent radiation such as diffraction, Smith–Purcell, Cherenkov, and transition radiation, the inclusion of the transverse component of the form factor can be important in the measurement of not only the transverse sizes of the beam but also the longitudinal sizes because both the longitudinal and transverse parts of the form factor contribute to the experimentally measured quantities.

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