Mathematical Model of Message Delivery in a Mobile Ad Hoc Network

Igor Konstantinov, Kostiantyn Polshchykov, Sergej Lazarev, Olha Polshchykova
Belgorod State University
Belgorod, Russia
polshchikov@bsu.edu.ru

***Abstract*— Analytic expressions for calculating the average time for delivery of a message, taking into account the effect of subscriber mobility and destructive factors are obtained. The effect of premature discontinuities of virtual connections on the efficiency of data delivery is studied.**

***Index Terms*— mobile ad hoc networks, message delivery, virtual connections, probability-time graph, generating function.**

I. Introduction

A Mobile Ad Hoc Networks (MANET) function on the basis of decentralized management of network components, absence of base stations (fixed nodes), ability of each node to perform router functions [1]. Merits of MANET usage are rapid deployment, high subsistence, ability to provide communication in case of dynamically changing network topology. The data traffic in MANET is formed by the messages containing control signals, text, graphic and table information [2–7]. Upon delivery of these messages virtual connections mode is used in order to support the reliability, integrity of transmitted data and avoiding network overloads by the transport layer means. At the same time message delivery process includes three main stages:

- at the first stage the virtual connection between a source-node and an addressee-node addressee source is set;
- at the second stage transmission of the following flows is carried out: from a source-node to an addressee-node data packets are transmitted, and then in the opposite direction – confirmations (the service packets confirming successful receiving of data packets by addressee-nodes) are transferred;
- at the third stage the virtual connection between a source-node and an addressee-node is closed.

Transmission of information flows to MANET can be interrupted due to premature ruptures of the physical and virtual connections caused by rapid changing of network topology. The major factors causing premature ruptures of the virtual connections in MANET include [8–15]:

- mobility of network nodes, since relocation of nodes can lead to their exit from certain coverage zones of a network;
- destructive influences which lead to nodes destruction or losses of their working capacity;
- other factors, for instance, influence of network overloads and unfavorable jamming situation.

In such conditions the average time of message delivery can significantly exceed the average time required for transmission of the appropriate data stream excluding procedures of establishment, closing and possible premature ruptures of the virtual connections. To estimate the average message delivery time in MANET, it is required to receive analytical relation of this value to the average time of transmission of the relevant traffic. At the same time there shall be values of virtual connections ruptures probability preset, which could be caused by mobility of subscribers, influence of the destructive influences and other factors specific to MANET.

II. Task Set

Let the following values be set:

- t_T — the average time of traffic transmission during the message delivery;
- t_C — average virtual connection establishing time;
- t_R — the average time required for repeated establishment of the virtual connection;
- t_{CC} — average virtual connection closing time;
- v_D — intensity of virtual connections ruptures caused by mobility of subscribers;
- v_M — intensity of virtual connections ruptures happening due to influence of destructive factors;
- v_O — intensity of virtual connections ruptures due to influence of other factors;
- n — the permitted number of repeatedly set virtual connections during the process of message delivery.

It is required to receive an expression in an analytical type for value t_D calculation – the average time of message delivery in MANET.

Dependence of virtual connection premature rupture probability value from time value is subordinate to the exponential law. Then the following expression could be used for probability calculation of that during an interval of time τ there will be a virtual connection rupture:

$$p=1-\exp\{-(v_M+v_D+v_O)\tau\}. \quad (1)$$

Sum of v_M, v_D and v_O used in formula (1) could be used for average virtual connection existence time evaluation:

$$t_V=1/(v_M+v_D+v_O). \quad (2)$$

While modelling time interval value τ could be represented in the following way:

$$\tau=t_T/m. \quad (3)$$

Where value m corresponds to number of identical time intervals, into which the message delivery time process is being split. The bigger the value m, the largest number of conditions of the studied process is considered during modeling and, therefore, its characteristics are defined more precisely.

III. Model development

Process of message delivery in MANET at m=2 and n=2 is also possible to present the probability-time graph represented in the Fig. 1. The node "S" corresponds to the beginning of message delivery from source-node to addressee-node. During average time t_C virtual connection establishment is carried out. The state in which virtual connection is established for the first time corresponds to the node "C". Transition from node "S" to node "C" is being modeled by function of the corresponding edge:

$$f_C(z)=z^{tc}. \quad (4)$$

z – function parameter.

Transition from node «C» to node «B1» models an event, which is the premature rupture of virtual connection during the time interval τ. The described transition relates to the following function:

$$f_B(z)=pz^{\tau}. \quad (5)$$

Transition from node «C» to node «T1» relates to data transfer premature rupture of virtual connection during the time interval τ and relates to the following function:

$$f_T(z)=(1-p)z^{\tau}. \quad (6)$$

After the first premature rupture of virtual connection it is established again during the average time t_R, which relates node «B1»-to-node «R1» transition and is being modeled by the following function:

$$f_R(z)=z^{t_R}. \quad (7)$$

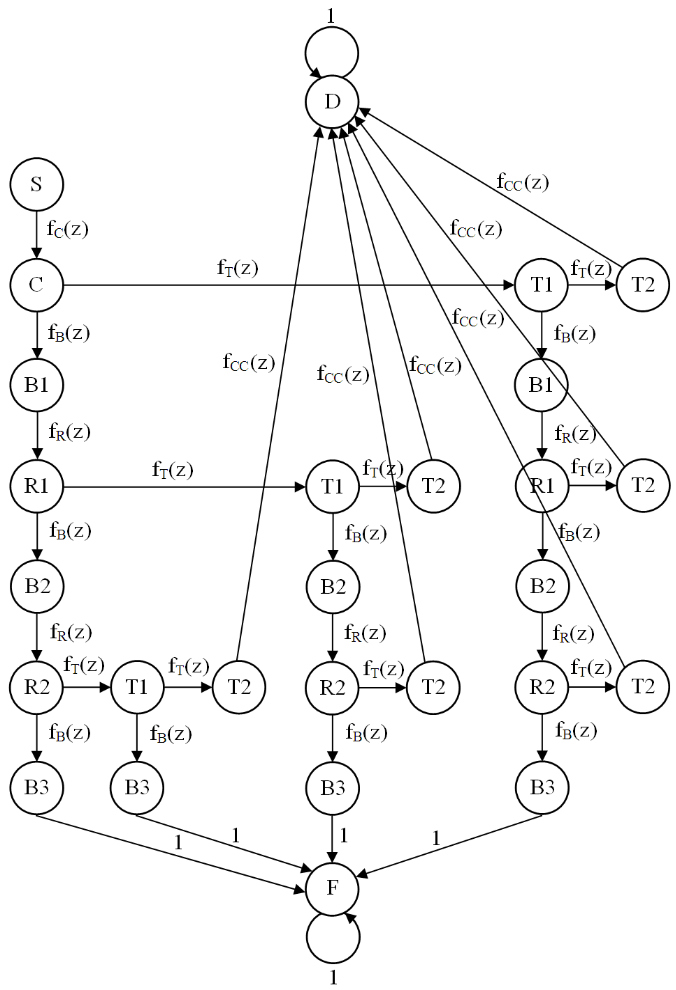

Fig. 1. Probability-time graph, modeling data transfer to MANET at m=2 and n=2.

Other graph nodes are corresponding to the following conditions of analyzed process:

- node «B2» – virtual connection premature rupture happened for the second time;
- node «B3» – virtual connection premature rupture happened for the third time;
- node «R2» – virtual connection is being established repetitively for the second time;
- node «T2» – during the second time interval τ data has been transferred without virtual connection premature rupture;
- node «F» – repetitive establishment of virtual connection is unacceptable, message delivery stopped;
- node «D» – virtual connection closed, message delivery successfully completed.

All transitions to nodes «B1», «B2» and «B3» are being modeled by function $f_B(z)$, to nodes «R1» and «R2» – by function $f_R(z)$, to nodes «T1» and «T2» – by function $f_T(z)$. Each transition from node «T2» to node «D» corresponds to the function:

$$f_{CC}(z)=z^{t_{CC}}. \quad (8)$$

For T_D evaluation it is required to take all transition functions of node «S» to node «D» transitions:

$$f_{D1}(z) = f_C(z)f_T^2(z)f_{CC}(z); \quad (9)$$

$$f_{D2}(z) = f_{D4}(z) = f_C(z)f_T^2(z)f_B(z)f_R(z)f_{CC}(z); \quad (10)$$

$$f_{D3}(z)=f_{D4}(z)=f_{D5}(z)=f_{D6}(z)=f_C(z)f_T^2(z)f_B^2(z)f_R^2(z)f_{CC}(z). \quad (11)$$

Equivalence conversions allow presenting the graph in a simplified way (Fig. 2).

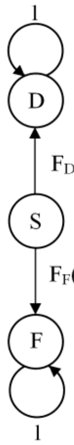

Fig. 2. Graph after equivalence conversions

Generating function modeling the node «S» to node «D» transition is represented below:

$$F_D(z) = f_C(z)\, f_{CC}(z)f_T^2(z)(1+2f_B(z)f_R(z)+3f_B^2(z)f_R^2(z); \quad (12)$$

Average message delivery time could be calculated by formula:

$$T_D = (dF_D(z) / dz) \mid z=1; \quad (13)$$

Mathematical models of message delivery process in MANET were developed for other m and n values as well. As a result of trends analyze, discovered in such models, the general formula has been obtained for calculation of value $F_D(z)$ at m>=2 and n>=2:

$$F_D(z) = f_C(z)\, f_{CC}(z)f_T^m(z)(a_{m1}+a_{m2}f_B(z)f_R(z)+ a_{m3}(f_B(z)f_R(z))^2+$$

$$+ a_{mk}(f_B(z)f_R(z))^{(k-1)}+ \ldots + a_{m(n+1)}(f_B(z)f_R(z))^n). \quad (14)$$

In formula (14) values $a_{m1}, a_{m2}, \ldots, a_{mk}, \ldots a_{m(n+1)}$, are defined with the following relations:

$$a_{m1}=1, \quad a_{2k}=k, \quad a_{mk}= a_{(m-1)k} + a_{m(k-1)}, \quad (15)$$

$k=1, 2, \ldots, n + 1$.

IV. Calculation experiments

The basic data used for performance of calculation experiments on modeling of various scenarios of MANET application are represented in TABLE I.

TABLE I. BASIC DATA FOR T_D VALUE CALCULATION

Parameter	Value
m	10
n	6
t_C	2 s
t_R	4 s
t_{CC}	6 s
v_M	0,005 s^{-1}
v_D	0,001 s^{-1}; 0,011 s^{-1}; 0,031 s^{-1}
v_O	0,004 s^{-1}

The calculation results are represented in the Fig. 3.

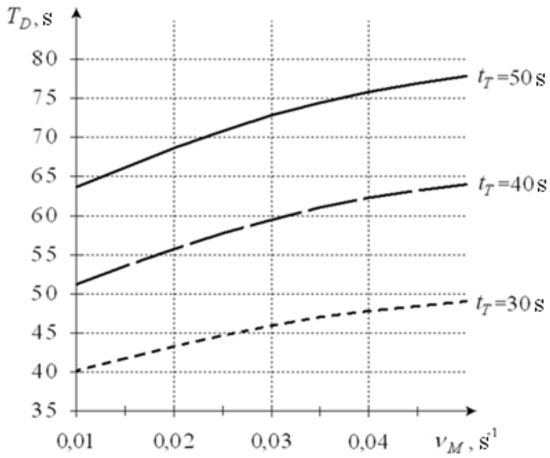

Fig. 3. The calculation results

The analysis of the specified curves shows that growth of intensity of virtual connections ruptures caused by influence of destructive influences and mobility of MANET subscribers leads to increase in average time of message delivery.

Results of simulation showed that in the process of MANET functioning the average time of message delivery significantly exceeds the average time of appropriate data transmission. Value of the relative exceeding of message delivery average time can be calculated with the following formula:

$$\delta = (T_D - t_T) / t_T; \quad (16)$$

In TABLE II calculation experiments results are represented, reflecting the δ growth, defined by the growth of t_v values.

The analysis of TABLE II data shows that the reduction of virtual connection existence average time observed in operating conditions by MANET leads to the essential growth of message delivery average time.

TABLE II. CALCULATION EXPERIMENTS RESULTS

Parameter	Value			
t_T	40 s			
t_v	100 s	75 s	50 s	25 s
T_D	48.9 s	50.3 s	52.1 s	58.2 s
δ	22.3 %	25.8 %	30.3 %	45.5 %

V. CONCLUSIONS

For an assessment of message delivery average time taking into account MANET specifics the analytical models developed on the basis of mathematical apparatus application of probability-time graphs and generating functions are offered. The expressions allow to calculate value of message delivery average time on the basis of the available information regarding the message transmission average time, duration of establishment and closing of the virtual connection, distribution of probabilities of the virtual connection rupture.

Results of analytical simulation showed that the reduction of virtual connection existence average time caused by influence of subscribers mobility and destructive factors leads to the fact that the message delivery average time in MANET can increase by 45,5% in relation to the average time of transmission of the appropriate data traffic.

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