Statistics of Mesoplasma Channel Formation upon Thermal Breakdown Stabilization in Thin Semiconductor Films

N. V. Andreeva and Yu. P. Virchenko*

Belgorod State University, Belgorod, Russia

** e-mail: virch@bsu.edu.ru*

Abstract—A phenomenological model of the development of thermal breakdown in semiconductor films is considered, and it is shown that the average number of mesoplasma channels experimentally observed in the regime of thermal breakdown stabilization can substantially vary. The probability distribution function for the number of such luminous channels arising as a result of thermal fluctuations in a thin semiconductor film is calculated within the framework of a simple approximation.

This paper briefly describes a new theoretical approach to the description of statistics of mesoplasma channels, which are formed in thin semiconductor films in the regime of thermal breakdown stabilization. The description is based on the model developed in [1, 2], which is analogous to a model corresponding to the Fock theory [3] of the thermal breakdown in dielectrics. In contrast to the Fock theory, our model takes into account that the voltage drop in the film exhibits a self-consistent variation rather than remaining constant. This change is caused by an increase in the conductivity and the redistribution of voltage between elements of a related electric circuit. Previously, we have demonstrated [4] that this variation makes possible the stabilization of thermal breakdown at a sufficiently large active resistance of the external part of this circuit. As a result of this stabilization, the film features the formation of small regions (on the order of so-called fundamental length [1]) in which the temperature is significantly higher than that of the thermal background.

The overheated regions can be experimentally observed [5] as manifested by luminous points (sparkles) in the film, which are called mesoplasma channels. Depending on the ambient conditions, the average number of such channels formed in the film can substantially vary [6]. In particular, a single channel was usually observed in [5] after a certain transient process. The dependence of the average number of mesoplasma channels on the external conditions has to be explained. Since these channels appear as a result of random thermal fluctuations of sufficiently large amplitude, their number is naturally random and, hence, the mean value should be evaluated from the probability distribution function (PDF) of this random quantity. The PDF must be dependent on the external conditions, which can be varied in experiment. Thus, in order to explain the experimentally observed results, it is necessary to find the PDF of the number of channels (random variable) and study its dependence on the experimental parameters.

This Letter offers an approach to the solution of this problem and presents the results of calculations for the simplest model.

Let us consider thermal fluctuations leading to the development of thermal breakdown in a thin semiconductor film, which appear as small regions with randomly temperature above the background temperature T_0 in the film. The magnitudes of fluctuations are assumed to be statistically independent at the onset of breakdown, after which the temperature exhibits a rapid increase. This assumption is related to a small density of fluctuations in the film. Upon stabilization of the breakdown regime, some fluctuations of sufficiently high magnitude transform into mesoplasma channels with growing temperature and, hence, increasing electric conductivity [1]. The temperature dependence of the conductivity gives rise to a competition between channels in this respect, so that not all fluctuations transform into mesoplasma channels and reach a temperature T_* at which they can generate a visible emission. Thus, it is necessary to find the PDF for the number n of luminous channels under the conditions when the initial number N of fluctuations is sufficiently large: $N \propto 10^4$ (we relate this number to the density of dislocations).

The temporal evolution of the temperature distribution $T(\mathbf{x}, t)$ in the film is described by the equation of heat conduction with a Joule heat source [1, 2]:

$$\rho c\dot{T} = \nabla(\kappa(T)\nabla T) + \sigma(T)E^2(t), \quad (1)$$

where ρ is the material density, c is the specific heat, $\kappa(T)$ is the thermal conductivity, $\sigma(T)$ is the electric conductivity,

$$E(t) = E\left(1 + (\bar{\sigma}|\Sigma|)^{-1}\int_{\Sigma}\sigma(T(\mathbf{y}, t))d\mathbf{y}\right)^{-1}$$

is the voltage drop in the film, E is the electromotive force, $\bar{\sigma}$ is the average electric conductivity of the electric circuit, and the integration is performed over the film area Σ.

We will consider the simplest model with the temperature dependences of $\kappa(T)$ and $\sigma(T)$ leading to the development of instability [2], whereby a fluctuation transforms into a thermal breakdown: $\kappa(T) = \kappa$; $\sigma(T) = \sigma_0 + \sigma'(T - T_m)$, $\sigma' > 0$. If the fluctuations are absent, then the above equation yields $T(\mathbf{x}, t) = T_0(t)$, where $\dot{T}_0 = \sigma(T_0)E^2(t)$. Solutions to this equation have the form of slowly varying functions of time (as compared to the characteristic breakdown time). Therefore, we can ignore the time dependence of the thermal background temperature $T_0(t)$ and consider the deviation $\Theta(\mathbf{x}, t) = T(\mathbf{x}, t) - T_0(t)$ obeying the equation $\rho c\dot{\Theta} = \kappa\Delta\Theta + \sigma' E^2(t)\Theta(t)$. It will be also assumed that $\Theta(\mathbf{x}, t)$ is a sum of independent thermal fluctuations, $\Theta(\mathbf{x}, t) = \sum_i \Theta_i(\mathbf{x}, t)$, and that the fluctuations are not interacting in the course of their evolution.

Since the temperatures of fluctuations having transformed into mesoplasma channels are much higher than the background temperature, it will be natural to describe their subsequent evolution on a rough scale by introducing the effective fluctuation temperatures $\Theta_j(t)$, which are determined by averaging $\Theta_j(\mathbf{x}, t)$ over the corresponding small spatial regions. As a result, Eq. (1) yields a system of equations for $\Theta_i(t)$ functions ($j = 1, 2, \ldots, N$):

$$\rho c\dot{\Theta}_i = -\frac{2\kappa_0}{r_0^2}\Theta_i + \sigma' E^2(t)\Theta_i,$$

where N is the number of fluctuations and r_0 is the radius of the typical thermal channel. Upon averaging, the voltage drop $E(t)$ is given by the following formula:

$$E(t) = E\left(1 + \frac{\sigma(T_0)}{\bar{\sigma}}\right)^{-1}\left(1 + \eta\sum_{i=1}^{N}\Theta_i(t)\right)^{-1},$$

$$\eta = \frac{\sigma'\pi r_0^2}{(\sigma(T_0) + \bar{\sigma})|\Sigma|}.$$

Introducing the positive parameters

$$a = \frac{2\kappa_0}{c\rho r_0^2}, \quad b = \frac{\sigma' E^2}{c\rho((1 + \sigma(T_0))/\bar{\sigma})^2},$$

we obtain the following dynamical system for the ensemble of fluctuations:

$$\dot{\Theta}_i = -a\Theta_i + b\Theta_i\left(1 + \eta\sum_{j=1}^{N}\Theta_j\right)^{-2}, \quad i = 1, \ldots, N. \quad (2)$$

This system has a trivial immobile point $\Theta_j = 0$ ($j = 1, 2, \ldots, N$) and a continuous set of immobile points representing a part of the hyperplane $\sum_{j=1}^{N}\Theta_j = \Theta_\infty$, $\Theta_\infty = \eta^{-1}[(b/a)^{1/2} - 1]$, which is cut by the sector $\Theta_j \geq 0$ ($j = 1, 2, \ldots, N$). The first (unstable) point corresponds to a thermal equilibrium in the film. As a fluctuation $\Theta_j > 0$ arises, the system trajectory on the hyperplane tends to one of the stable points. If the melting temperature T_{melt} is such that $T_{\text{melt}} - T_0 < \Theta_\infty$, thermal breakdown is possible, whereby at least one fluctuation will transform into a melt-through channel in the film. Should this difference exceed Θ_∞, no breakdown takes place, and the system exhibits stabilization. As the thermal fluctuation reaches a level of T^*, the corresponding mesoplasma channel will generate visible emission.

Let us consider the case when $\Theta_* = T_* - T_0 < \Theta_\infty$. If $\Theta_* \ll \Theta_\infty$, then all channels will become luminous because all $\Theta_j(t)$ cross the Θ_* level when the trajectory of system (1) tends to any of the immobile points on the hyperplane. In the opposite situation, we have $\Theta_* \approx \Theta_\infty$. Let all the initial fluctuations $\Theta_j(0)$ ($j = 1, 2, \ldots, N$) be independent and distributed, in the simplest case, with a density of $f(\theta) = \theta_0^{-1}\exp(-\theta/\theta_0)$, where θ_0 is the average fluctuation ($\theta_0 \ll \Theta_*$). Then, the probability $P_N(n)$ of the event, whereby the limiting point $\Theta_j(\infty)$ ($j = 1, 2, \ldots, N$) of the trajectory corresponds to the formation of n luminous channels from N initial fluctuations, is given by the integral over θ_k ($k = 1, 2, \ldots, N$) in the phase space of $\Theta_k(0)$ values:

$$P_N(n) = \binom{N}{n}\int_0^{\Theta_*}\left(\prod_{k=1}^{N}f(\theta_k)\right)\left(\prod_{k=1}^{n}\chi(\Theta_k(\infty) - \Theta_*)\right) \times\left(\prod_{k=n+1}^{N}\chi(\Theta_* - \Theta_k(\infty))\right)d\theta_1\ldots d\theta_N. \quad (3)$$

Here, $\chi(\cdot)$ is the Heaviside function and $\Theta_j(\infty)$ ($j = 1, 2, \ldots, N$) are functions of the initial values of θ_k ($k = 1, 2, \ldots, N$).

System of equations (2) admits exact integration, and we obtain the following explicit expressions for the limiting functions:

$$\Theta_j(\infty) = \Theta_\infty \theta_j \left(\sum_{k=1}^{\infty} \theta_k \right)^{-1}. \quad (4)$$

Substituting expressions (4) into relation (3) and replacing the Heaviside functions by their Fourier representations, we obtain a multiple integral of the type of a statistical sum. Calculating this integral by the method of steepest descent for $N \longrightarrow \infty$ and $\theta_0/\Theta_* \longrightarrow 0$ and using transformations analogous to those involved in the Poisson limit theorem, we obtain in the first approximation a Poisson distribution $P_N(n) \propto (\nu^n/n!)\exp(-\nu)$ with the index $\nu = N\exp(-\Theta_*/\theta_0)$. Substantial results are that (i) the average number of mesoplasma channels ν depends only on the number of fluctuations N and (ii) the ratio of the temperature of the channel formation (which can vary within broad limits, beginning with $\approx 3 \times 10^2$ K) to the thermal fluctuation amplitude (~10–30 K) is independent of the characteristics of the film material (these characteristics enter into the parameters of dynamical system (2) and must only provide for the possibility of the regime of breakdown stabilization [4]).

By controlling the density of thermal fluctuations through a change in the material purity and the temperature of the mesoplasma channel formation, it is possible to observe significantly different average numbers of channels, from one (for $\nu \ll 1$) [5] to a large number on the order of ν (for $\nu \gg 1$) [6].

REFERENCES

1. Yu. P. Virchenko and A. A. Vodyanitskii, Funct. Mater. **8**, 428 (2001).
2. Yu. P. Virchenko and A. A. Vodyanitskii, Funct. Mater. **9**, 601 (2002).
3. V. A. Fock, Arch. Elektrotech. (Berlin) **19**, 71 (1927).
4. N. V. Andreyeva and Yu. P. Virchenko, Probl. At. Sci. Technol. NASU, No. 5, 126 (2004).
5. T. Puritis, D. Volks, and J. Kaupuzs, Solid-State Electron. **38**, 258 (1995).
6. I. V. Grekhov and Yu. N. Serezhkin, *Avalanche Breakdown of p–n Junctions in Semiconductors* (Énergiya, Leningrad, 1980) [in Russian].

Translated by P. Pozdeev