

Conditions and Numerical Simulation of Dissipative Structure Formation in Imperfect Double-Component Reaction-Diffusion Systems

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ABSTRACT: Imperfection of a reaction-diffusion system is investigated. This imperfection causes modification of Fick's equation. We provide imperfection of diffusion fluxes on the basis of a chemical reaction model of unimolecular and bimolecular processes with two intermediate substances. We demonstrate that allowing for imperfection of diffusion processes leads to introducing the spatial derivatives of fourth order in the evolution equations for concentrations of intermediate substances. It is important that dissipative structures including oscillations can be formed in the system due to the availability of these derivatives in the evolution equations. It is proved that the necessary condition of forming the dissipative structures can appear due to imperfection of diffusion processes in the systems considered. It is shown by linear analysis that concentration of the intermediate substance near a stationary state can exhibit oscillatory behavior under certain conditions. Phase-plane portrait peculiarities of a nonlinear system are investigated at different values of the initial parameters. It is shown by numerical simulation that dependence of concentrations of the intermediate substances has a quasi-periodical character.

Key words: reaction-diffusion system; evolution equation; dissipative structure

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Introduction

The description of forming dissipative structures in chemical reaction-diffusion systems has long been of interest of nonlinear dynamics [1, 2]. It is connected with modeling chemical reactions on the basis of nonlinear differential equations, such as kinetical or reaction-diffusion equations. What character of nonlinearity must the kinetic equation have for the dissipative structures to be formed in the systems described by this equation? This question is of a great interest. Some general conclusions of the possibility of forming the dissipative structures and oscillator regimes in such systems are given in Refs. [1–3]. In particular, proof is given of the impossibility to form the dissipative structures, including a limited cycle in the system with two intermediate substances involved by unimolecular and bimolecular reaction mechanisms in the works [3, 4, 5]. In this work, we will show that the necessary condition of forming the dissipative structures can descend from the presence of inhomogeneous diffusion fluxes in such systems.

It is necessary to note that limited cycles often take place in systems with other reaction mechanisms. The best known of these are the brusselyator [1–3] and the experimentally observed celebrated Belousov–Zhabotinsky reaction [6, 7]. The brusselyator model is the one with a trimolecular reaction mechanism. Its equations hold cubic nonlinearity. As is well known, cubic nonlinearity leads to formation of the oscillator regime. In such systems with cubic nonlinearity, the availability of ideal and no ideal diffusion gives no idea in principle.

In this work, new features of a model involving only the unimolecular and bimolecular mechanisms of reactions are considered on the basis of equations holding quadratic nonlinearity with coefficients of definite signs. As noted in Ref. [3], the above necessary condition cannot be realized by ideal diffusion in such systems. We show that the situation changes radically if one considers nonideal diffusion processes.

As is well known, allowing for higher derivatives in the kinetical equation for reagent concentration [8, 9] leads to the nonideality and inhomogeneity of diffusion processes of chemical reactions. These terms can play a main role for appearing the time dissipative structures including oscillator regime.

As shown in Refs. [1, 3], there are many more possibilities of arising instabilities in a nonideal

system than in an ideal one. The influence of nonideality has been analyzed in this system in terms of the nonideality of a model having a regular solution; this has led to changes in chemical potential expression. In our work, another nonideality is considered, conveniently called “diffusion nonideality”; analogously we analyze how it is done in Ref. [3].

Modeling of Formation of Spatial Structures

We consider the open chemical system consisting of substances A , B , X_1 , and X_2 mixed. Substance A transmutes into substance B by means of substances X_1 and X_2 . The processes that take place in this system are isothermal and isobaric, with the system in the state of mechanical (but not thermodynamic) equilibrium, and not under external fields. Let concentrations of substances X_1 and X_2 depend on time and spatial coordinates r , and the concentrations of substances A and B are constant.

We modify Fick’s law, with free energy represented as the sum of two parts [3, 8, 9]. The first part $f_{0i}[X_i(r)]$ ($i = 1, 2$) corresponds to the free energy of a homogeneous liquid, and the second part takes account of phase inhomogeneity through the term proportional to the square of concentration gradient ∇X_i : $f_i[X_i(r)] = f_{0i}[X_i(r)] + k[\nabla X_i(r)]^2/2$, where coefficient $K > 0$ characterizes the inhomogeneity of diffusion fluxes. Further, K will be called an inhomogeneity parameter and will be constant. For simplicity, it is proposed to be almost the same for substances X_1 and X_2 . Taking this into account the densities of corresponding diffusion fluxes j_i are written by the modified Fick’s law in the one-dimensional case as:

$$j_i = -D_i \frac{\partial}{\partial x} \left(X_i - K \frac{\partial^2}{\partial x^2} X_i \right). \quad (1)$$

On the basis of this, nonideal systems are understood to be systems for which $K \neq 0$ (nonideality of diffusion processes).

As the balance equations for concentration in the one-dimensional case take the form

$$\left(\frac{\partial X_i}{\partial t} \right)_{\text{diff}} = - \frac{\partial j_i}{\partial x'}$$

it follows from Eq. (1):

$$\begin{cases} \frac{\partial X_1}{\partial t} = D_1 L_D [X_1] + F_1(X_1, X_2), \\ \frac{\partial X_2}{\partial t} = D_2 L_D [X_2] + F_2(X_1, X_2), \end{cases} \quad (2)$$

where X_1 and X_2 are concentrations of two components, respectively, $L_D = (\partial^2/\partial x^2) - K(\partial^4/\partial x^4)$. The general forms of the functions $F_{1,2}(X_1, X_2)$ are written by the following expressions [3]:

$$\begin{aligned} F_1(X_1, X_2) &= a_0 \pm a_1 X_1 \\ &\quad + a_2 X_2 - a_3 X_1^2 \pm a_4 X_1 X_2 + a_5 X_2^2, \\ F_2(X_1, X_2) &= b_0 \pm b_1 X_2 \\ &\quad + b_2 X_1 - b_3 X_2^2 \pm b_4 X_1 X_2 + b_5 X_1^2, \end{aligned} \quad (3)$$

where a_j and b_j ($j = 1, \dots, 5$) are positive. The system is supposed to be isothermal and isobar, and concentrations A and B are constant. It is interesting to note that the one-dimensional analogues of Eq. (2) with the operator like L_D have been applied to describe self-organization processes of dislocation ensembles in a solid state (see, e.g., Ref. [10]).

We suppose that stationary homogeneous states $X_1^{(s)}$ and $X_2^{(s)}$ have been found from the system of equations

$$F_1(X_1^{(s)}, X_2^{(s)}) = 0, F_2(X_1^{(s)}, X_2^{(s)}) = 0.$$

Now we linearize the equation system (2) near the stationary homogeneous states $X_1^{(s)}$, $X_2^{(s)}$ and put $\mathbf{Z} = (Z_1, Z_2)$, where $Z_j = X_j - X_j^{(s)}$ ($j = 1, 2$). As a result, we obtain system equations in the matrix form

$$\frac{\partial \mathbf{Z}}{\partial t} = \hat{\mathbf{L}}[\mathbf{Z}], \quad (4)$$

where the matrix

$$\hat{\mathbf{L}} = \begin{pmatrix} L_{11} + D_1 L_D & L_{12} \\ L_{21} & L_{22} + D_2 L_D \end{pmatrix}, \quad L_{ij} = \left. \frac{\partial F_i}{\partial X_j} \right|_{X_j = X_j^{(s)}}.$$

The solution of the equation system (4) has the form $\mathbf{Z} = \mathbf{h} \psi_{s,a}(x) \exp(\omega t)$, where $\psi_{s,a}(x)$ is the eigen-

function of the operator L_D , and \mathbf{h} is a constant eigenvector of the matrix

$$\hat{\mathbf{L}}(\lambda_{s,a}) = \begin{pmatrix} L_{11} + D_1 \lambda_{s,a} & L_{12} \\ L_{21} & L_{22} + D_2 \lambda_{s,a} \end{pmatrix}, \quad (5)$$

$\lambda_{s,a}$ is the eigenvalue of operator L_D , and ω is eigenvalue of the matrix (5).

The eigenfunctions and eigenvalues of operator L_D can be easily found for stationary boundary conditions corresponding to the equality to zero of function $\psi(x)$ and its derivative on the chemical system bounds at $x = \pm l$: $\psi(l) = \psi(-l) = \psi'(l) = \psi'(-l) = 0$. As the system have a symmetry with respect to the origin, then the problem eigenfunction can be separated into two parts: $\psi(x) = \psi_s(x) + \psi_a(x)$, where $\psi_s(x)$ is a symmetrical state and $\psi_a(x)$ is an antisymmetrical state [11, 12]. As a result, such eigenfunctions of the operator L_D describe the spatial distributions of two types

1. Quasi-periodic structures:

$$\begin{aligned} \psi_s(x) &= C_s \left(\cos \mu_s x - \frac{\cos \mu_s l}{ch \nu_s l} ch \nu_s x \right), \\ \psi_a(x) &= C_a \left(\sin \mu_a x - \frac{\sin \mu_a l}{sh \nu_a l} sh \nu_a x \right), \end{aligned} \quad (6)$$

where $C_{s,a}$ are normalization constants, $\mu_{s,a} = (1/K + \eta_{s,a}^2/l^2)^{1/2}$, $\nu_{s,a} = \eta_{s,a}/l$, quantities η_s are the real solutions of the equation

$$\eta_s th \eta_s = -(l^2/K + \eta_s^2)^{1/2} tg(l^2/K + \eta_s^2)^{1/2}, \quad (7)$$

and quantities η_a are the real solutions of the equation

$$\eta_a cth \eta_a = (l^2/K + \eta_a^2)^{1/2} ctg(l^2/K + \eta_a^2)^{1/2}. \quad (8)$$

The quasi-periodic structures (6) correspond to the negative eigenvalues

$$\lambda_{s,a}^{(q)} = -\frac{\eta_{s,a}^2}{l^2} \left(1 + K \frac{\eta_{s,a}^2}{l^2} \right), \quad (9)$$

where $\eta_{s,a}$ are the real solutions of Eqs. (7) and (8), respectively.

2. *Nonperiodic structures:*

$$\begin{aligned}\psi_s(x) &= C_s \left(chq_{1s}x - \frac{chq_{1s}l}{chq_{2s}l} chq_{2s}x \right), \\ \psi_a(x) &= C_a \left(shq_{1a}x - \frac{shq_{1a}l}{shq_{2a}l} shq_{2a}x \right),\end{aligned}\quad (10)$$

where $q_{2s,a} = \zeta_{s,a}/l$, $q_{1s,a} = (1/K - \zeta_{s,a}^2/l^2)^{1/2}$, quantities ζ_s are the real solutions of the equation

$$\zeta_s th \zeta_s = (l^2/K - \zeta_s^2)^{1/2} th(l^2/K - \zeta_s^2)^{1/2},\quad (11)$$

and ζ_a are the real solutions of the equation

$$\zeta_a cth \zeta_a = (l^2/K - \zeta_a^2)^{1/2} cth(l^2/K - \zeta_a^2)^{1/2}.\quad (12)$$

The real solutions of Eqs. (11) and (12) exist at carrying out the following condition: $l/\sqrt{K} < \zeta_{s,a} < l/\sqrt{K}$. Under this condition, the eigenvalues of the operator L_D for the nonperiodical structures (10) are positive:

$$\lambda_{s,a}^{(n)} = \frac{\zeta_{s,a}^2}{l^2} \left(1 - K \frac{\zeta_{s,a}^2}{l^2} \right),\quad (13)$$

where $\zeta_{s,a}$ are the real solutions of Eqs. (11) and (12). Note that eigenvalues (9) and (13) depend only on K and l .

It follows that the nonperiodical structures (10) cannot be formed, and the quasi-periodical structures are converted to the one-component periodical structures in the case of the ideal diffusive processes ($K = 0$), as shown in Refs. [11, 12] for the extensional Shogle model. This means that forming such inhomogeneous spatial structures is due exclusively to the inhomogeneity of diffusion fluxes in the reaction system.

Necessary Condition of Existing Dissipative Structure in Imperfect System

It is known (see Refs. [1, 3]) that diagonal elements of the matrix (36) must have the different signs ($\hat{L}_{11}\hat{L}_{22} < 0$) for the dissipative structures to be formed in the system considered. It is also known that $L_{11} < 0$ and $L_{22} < 0$ in the two-component

system with the uni- and bimolecular reaction mechanisms [3].

In the system with ideal diffusion ($K = 0$), the operator L_D becomes the ordinary Laplace operator having the negative eigenvalues $\lambda = -k_n^2$, where k_n has the discrete values determined by only l and the boundary conditions. It leads to $\hat{L}_{11} = -|L_{11}| - D_1k^2 < 0$ and $\hat{L}_{22} = -|L_{22}| - D_2k^2 < 0$. As a result, $\hat{L}_{11}\hat{L}_{22} > 0$. It means that the dissipative structures cannot appear in the system considered with ideal diffusion [1, 3].

The situation changes essentially if diffusive processes of the chemical system are considered non-ideal ($K \neq 0$). In this case, the nonperiodical structures (10) with the positive eigenvalues (13) are of the most interest. In a result, one can satisfy the considered necessary condition $\hat{L}_{11}\hat{L}_{22} < 0$, as now $\hat{L}_{11} = -|L_{11}| + D_1\lambda_{s,a}^{(n)}$ and $\hat{L}_{22} = -|L_{22}| + D_2\lambda_{s,a}^{(n)}$. The situation interesting us $\hat{L}_{11}\hat{L}_{22} < 0$ can be realized in the two following cases:

1. $\hat{L}_{11} > 0$ and $\hat{L}_{22} < 0$
as it follows from $\frac{|L_{11}|}{D_1} < \lambda_{s,a}^{(n)} < \frac{|L_{22}|}{D_2}$;
2. $\hat{L}_{11} < 0$ and $\hat{L}_{22} > 0$
as it follows from $\frac{|L_{22}|}{D_2} < \lambda_{s,a}^{(n)} < \frac{|L_{11}|}{D_1}$.

This proves that the nonideality of diffusive processes in the two-component system with the uni-molecular and bimolecular reaction mechanisms leads to the possibility of carrying out the condition of dissipative structures, including the limit cycle.

Oscillation Regime

Eigenvalues (5) of the considered state take the form:

$$\begin{aligned}\omega_{s,a}^{(n)} &= \frac{1}{2} \{ \lambda_{s,a}^{(n)}(D_1 + D_2) - |L_{11}| - |L_{22}| \pm [(\lambda_{s,a}^{(n)}(D_1 \\ &\quad - D_2) - |L_{11}| + |L_{22}|)^2 + 4L_{12}L_{21}]^{1/2} \}.\end{aligned}\quad (14)$$

A series of additional conditions imposed on the solution of Eqs. (11) and (12) exist for the stationary nonperiodical states to be appeared. These are determined by the maximum of the dependence (14) at the definite critical value $\zeta_{s,a}^{(c)}$ and the requirement $\omega_{s,a}^{(n)}(\zeta_{s,a}^{(c)}) \geq 0$. The availability of inhomogeneous diffusive processes in the system leads to appearing two couples of critical points in an every kind of state:

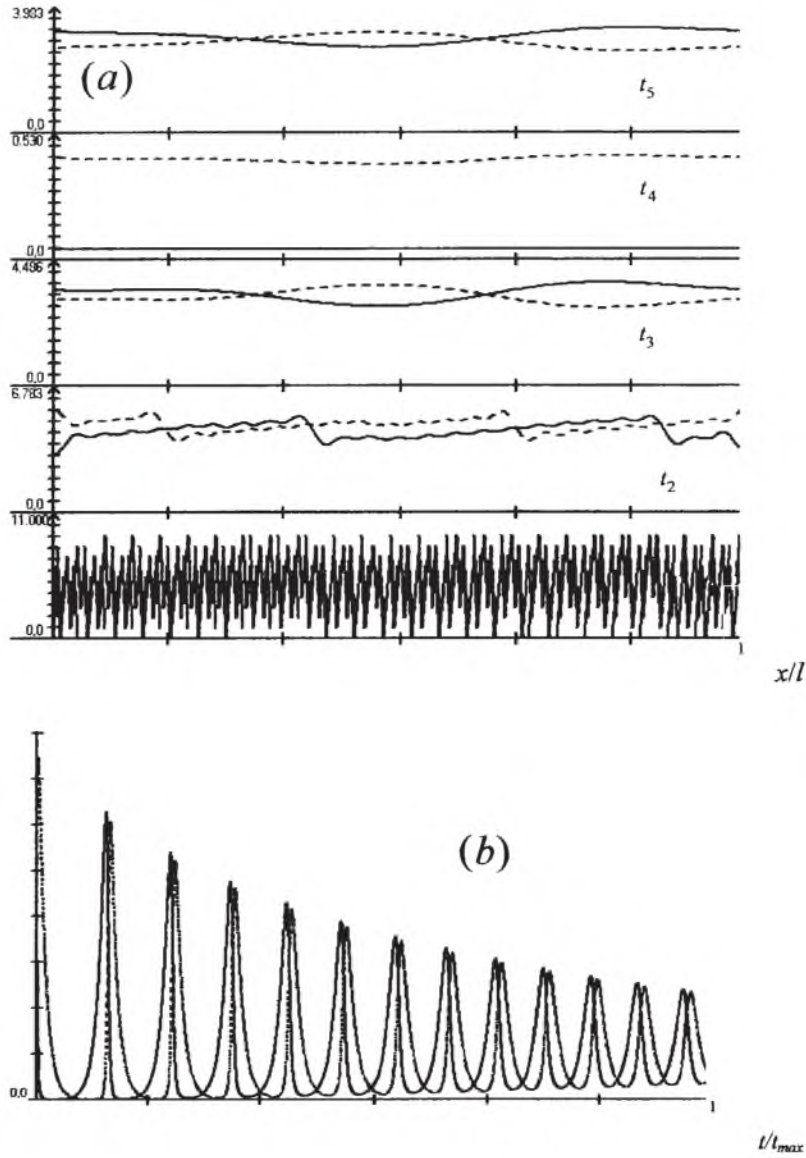


FIGURE 1. Numerical solution of system (2) at $a_0 = 0$; $a_1 = 1$; $a_2 = 0.001$; $a_3 = 0.01$; $a_4 = -1$; $a_5 = 0.0001$; $b_0 = 0$; $b_1 = -1$; $b_2 = 0.001$; $b_3 = 0.01$; $b_4 = 1$; $b_5 = 0.001$; $K = 1$; $D_1 = 1$; $D_2 = 1$; $l = 50$. Solid line, X_1 ; dashed line, X_2 . (a) The spatial distribution of concentrations at the moments $t_1 = 0$; $t_2 = 0.006$; $t_3 = 20.39$; $t_4 = 23.44$; $t_5 = 29.29$. (b) Changing concentrations during time in the point $x = 25$, $t_{\max} = 100$.

$$\zeta_{s,a}^{(c)2} = \frac{l^2}{2K} \pm \left\{ \frac{l^4}{4K^2} - \frac{1}{2K} \left(\frac{|L_{11}|}{D_1} + \frac{|L_{22}|}{D_2} \right) \right\}^{1/2}. \quad (15)$$

Such points can exist not for all nonideality parameter values, but at $K \leq K_c$, where $K_c = D_1 D_2 / 2(D_1 |L_{22}| + D_2 |L_{11}|)$. Only the one critical point will be at $K = K_c$ (as well as for ideal diffusion), and they disappear at $K \geq K_c$. The requirement $\omega_{s,a}^{(n)}(\zeta_{s,a}^{(c)}) \geq 0$ leads to restriction put on the values of the control parameters

$$|L_{11}| \frac{D_2}{D_1} - 2 \left(|L_{12} L_{21}| \frac{D_2}{D_1} \right)^{1/2} \leq |L_{22}| \leq |L_{11}| \frac{D_2}{D_1} + 2 \left(|L_{12} L_{21}| \frac{D_2}{D_1} \right)^{1/2},$$

where it takes account of the inequality $L_{12} L_{21} < 0$ that is held in the instable reaction system.

We now consider a question on stability of the stationary inhomogeneous states of the equation

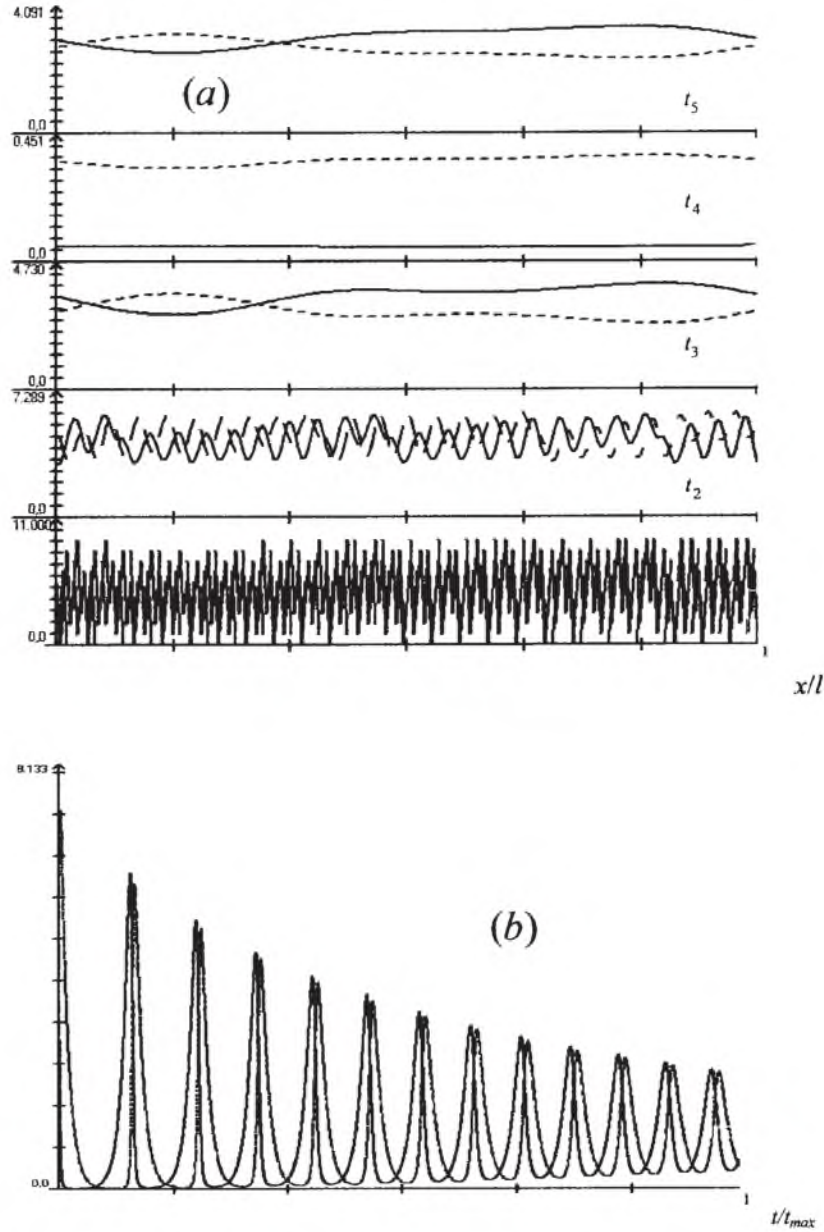


FIGURE 2. Numerical solution of the system (2) at $a_0 = 0$; $a_1 = 1$; $a_2 = 0.001$; $a_3 = 0.1$; $a_4 = -1$; $a_5 = 0.1$; $b_0 = 0$; $b_1 = -1$; $b_2 = 0.001$; $b_3 = 0.1$; $b_4 = 1$; $b_5 = 0.1$; $K = 1$; $D_1 = 1$; $D_2 = 1$; $l = 50$. Solid line, X_1 ; dashed line, X_2 . (a) The spatial distribution of concentrations at the moments $t_1 = 0$; $t_2 = 0.003$; $t_3 = 20.19$; $t_4 = 23.44$; $t_5 = 29.004$. (b) Changing concentrations during time in the point $x = 25$, $t_{\max} = 100$.

system (4), allowing for $L_{11}L_{22} < 0$ and $L_{12}L_{21} < 0$. It is convenient to represent the eigenvalues of the matrix (5) as $\omega = -\gamma \pm i\Omega$, where the quantity

$$\gamma = -\text{Re } \omega = -(\hat{L}_{11} + \hat{L}_{22})/2 \quad (16)$$

can be called damping and the imaginary part takes the form

$$\Omega = \text{Im } \omega = \{|L_{12}L_{21}| - (\hat{L}_{11} - \hat{L}_{22})^2/4\}^{1/2}. \quad (17)$$

It is true $\Omega > 0$ at $|L_{12}L_{21}| > (\hat{L}_{11} - \hat{L}_{22})^2/4$. This can be fulfilled in both the system without diffusion and the system with ideal diffusion, as the nondiagonal elements of the matrix (5) do not depend on the eigenvalue λ of the differential operator of the matrix (5) describing diffusion ($\hat{L}_{12} = L_{12}$ and $\hat{L}_{21} =$

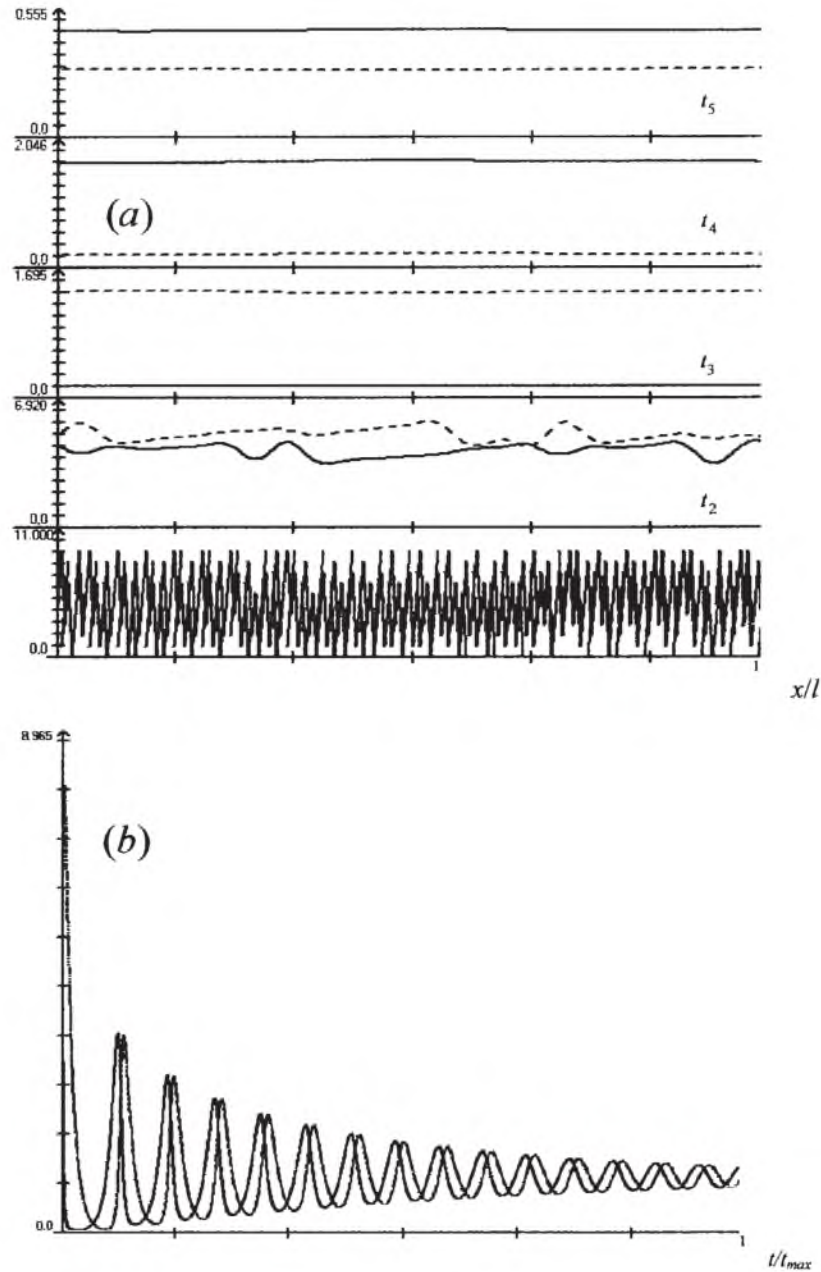


FIGURE 3. Numerical solution of the system (2) at $a_0 = 1$; $a_1 = 1$; $a_2 = 0.001$; $a_3 = 0.1$; $a_4 = -1$; $a_5 = 0.1$; $b_0 = 1$; $b_1 = -1$; $b_2 = 0.001$; $b_3 = 0.1$; $b_4 = 1$; $b_5 = 0.1$; $K = 1$; $D_1 = 1$; $D_2 = 1$; $l = 50$. Solid line, X_1 ; dashed line, X_2 . (a) The spatial distribution of concentrations at the moments $t_1 = 0$; $t_2 = 0.003$; $t_3 = 10.37$; $t_4 = 14.65$; $t_5 = 19.92$. (b) Changing concentrations during time in the point $x = 25$, $t_{\max} = 100$.

L_{21}). Nonideality of diffusion does not influence on the condition $\Omega > 0$.

However, nonideality of diffusion is important to analyze a sign of damping $\gamma = (|L_{11}| + |L_{22}|)/2 > 0$ in the system without diffusion that means asymptotic stability. The situation does not change in con-

sidering ideal diffusion, as the eigenvalues of the one-dimensional Laplace operator are always negative, and $\gamma = \{|L_{11}| + |L_{22}| + (D_1 + D_2)k_n^2\}/2 > 0$. When considering nonideal diffusion processes, the above eigenvalues (13) can be positive for the nonperiodical structures. In a result, it is possible $\gamma =$

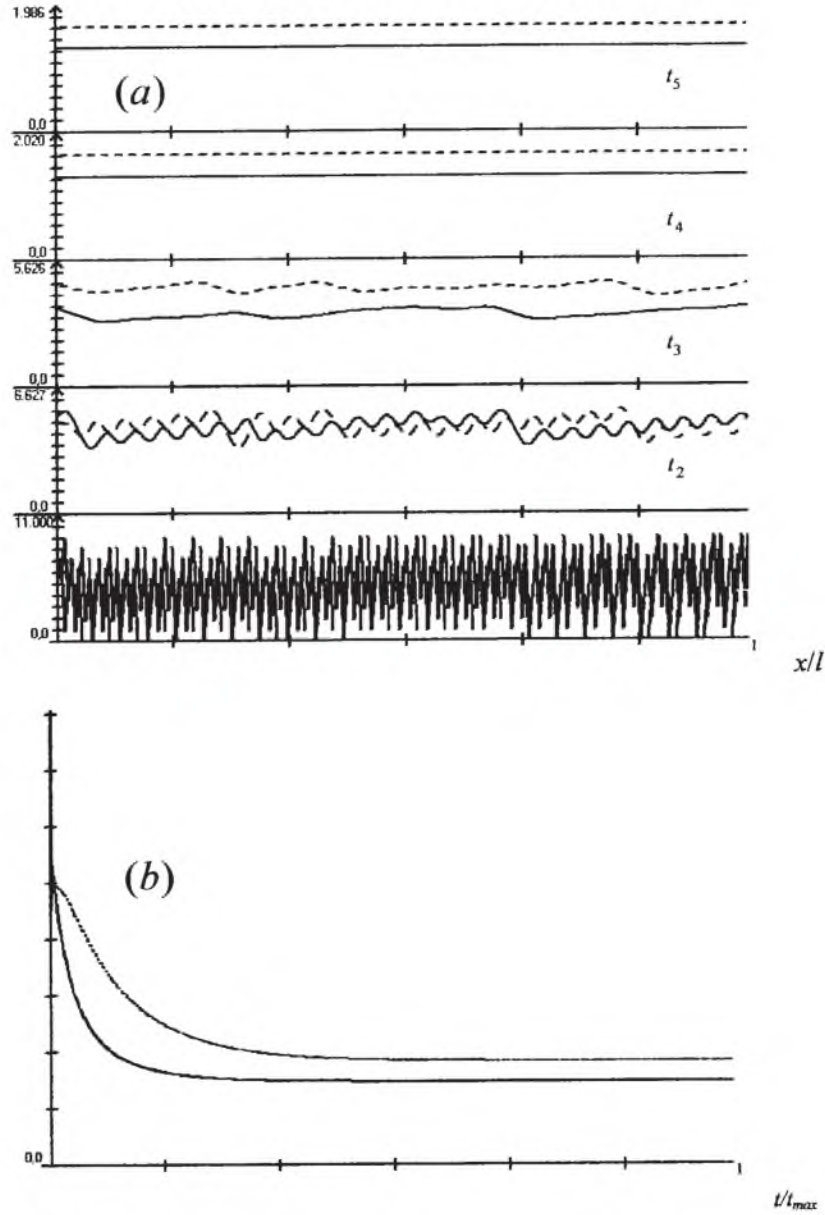


FIGURE 4. Numerical solution of the system (2) at $a_0 = 1$; $a_1 = 1$; $a_2 = 0.001$; $a_3 = 0.1$; $a_4 = -1$; $a_5 = 1.1$; $b_0 = 1$; $b_1 = -1$; $b_2 = 0.001$; $b_3 = 0.1$; $b_4 = 1$; $b_5 = 1.1$; $K = 1$; $D_1 = 1$; $D_2 = 1$; $l = 50$. Solid line, X_1 ; dashed line, X_2 . (a) The spatial distribution of concentrations at the moments $t_1 = 0$; $t_2 = 0.003$; $t_3 = 0.059$; $t_4 = 1.46$; $t_5 = 2.93$. (b) Changing concentrations during time in the point $x = 25$, $t_{\max} = 20$.

$\{|L_{11}| + |L_{22}| - (D_1 + D_2)\lambda_{s,a}^{(n)}\} / 2 \leq 0$ at $|L_{11}| + |L_{22}| \leq (D_1 + D_2)\lambda_{s,a}^{(n)}$, but it cannot be in the ideal system.

Case $\gamma = 0$ is of special interest when the eigenvalues of the matrix (5) become imaginary: $\omega = \pm i\Omega$. It is possible only in the system with nonideality at $|L_{11}| + |L_{22}| = (D_1 + D_2)\lambda_{s,a}^{(n)}$. Substituting this condition into Eq. (17) one obtains:

$$\Omega = \{|L_{12}L_{21}| - (D_1\lambda_{s,a}^{(n)} - |L_{11}|)^2\}^{1/2}. \quad (18)$$

Quantity Ω is real at the additional condition

$$D_1\lambda_{s,a}^{(n)} - |L_{12}L_{21}|^{1/2} < |L_{11}| < D_1\lambda_{s,a}^{(n)} + |L_{12}L_{21}|^{1/2}, \quad (19)$$

which can be ensured by the appropriate eigenvalues (13), as they do not depend on the elements L_{ij} . That is the solution of the equation system (4) os-

cillates in time under the inequality (19) carried out the solution of the equation system (4) oscillates in time with the frequency determined by (18):

$$Z(x, t) = h\psi_{s,a}(x)\exp(\pm i\Omega t),$$

where the spatial part $\psi_{s,a}(x)$ is given by Eq. (10).

Numerical Simulation

Temporal evolution and spatial distribution of concentrations X_1 and X_2 , which obey the equation system (2), are analyzed by numerical methods. It is assumed that initially, the concentrations of these mixture components are distributed uniformly over the reaction volume $0 < x < l$. Such initial conditions are simulated by random distribution that is uniform along interval $[0, l]$. There are also periodical boundary conditions that correspond to the equality of functions $X_1(0, t) = X_2(l, t)$ and their derivatives up to the third order including $X_{1x}(0, t) = X_{2x}(l, t)$, $X_{1xx}(0, t) = X_{2xx}(l, t)$, $X_{1xxx}(0, t) = X_{2xxx}(l, t)$ on the interval ends. The results of the numerical solution of the systems (2) are shown in Figures 1–4.

Numerical modeling has shown that concentrations X_1 and X_2 perform damping oscillations (Figs. 1–3). As shown in Figures 1 and 2, the spatially inhomogeneous distributions of concentrations are stable enough during time. The regime in which rapid transition goes to homogeneous states performing the damping oscillations, in turn, also quickly going to the stationary states is shown in Figure 3. The solutions corresponding to the nonperiodical regime are shown in Figure 4. Increasing coefficient K does not change the solution pattern essentially. Changing coefficients a_0 and b_0 essentially affect the oscillation amplitude of functions X_1 and X_2 over time. Functions X_1 and X_2 degenerate in the right lines when increasing coefficients a_2, a_3, b_2, b_3 . At that their oscillatory behavior along time axis does not stop long enough (Fig. 3). Increasing these coefficients leads to stabilization of the numerical results; that is, oscillations along the time axis stop (Fig. 4). Decreasing coefficients $a_2, a_3, a_5, b_2, b_3, b_5$ leads to slower damping oscillations and the system (2) goes into the system exactly corresponding to the Lotka–Volterra model at $K = a_0 = a_2 = a_3 = a_5 = b_0 = b_2 = b_3 = b_5 = 0$ [1–3]. As is known, undamped oscillations continue during time and stationary states correspond to peculiar

points of a center type at any initial condition of the Lotka–Volterra model.

Conclusions

It is demonstrated that the necessary condition of forming dissipative structures can be carried out in a two-component system with unimolecular and bimolecular reaction mechanisms, with the inhomogeneity of diffusive processes the main factor. Solving this problem is considered to be principal, as the necessary condition isn't carried out in the systems with the above reaction mechanisms and when there is only ideal diffusion.

Taking into account nonideal diffusion also leads to the formation of new types of spatial structures that are conditionally named by quasi-periodical and nonperiodical ones. Both phenomena considered in this work show the importance of analyzing nonideal systems with respect to diffusion in modeling chemical reactions.

It follows that we have only shown analytically that the necessary condition discussed can be fulfilled in a chemical nonideal system. Establishing the fact of forming a limit cycle requires solving a total nonlinear problem. These problems are of special interest and will be the subject of further investigation.

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