Features of Bremsstrahlung from Relativistic Electrons in Solid Targets

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It has been shown that the classical Bethe–Heitler result can be inapplicable for describing the bremsstrahlung spectrum of relativistic electrons in real solid targets.

1. The Bethe–Heitler formula for the bremsstrahlung spectrum of a relativistic electron on an atom [1] describes well the experimental results obtained with amorphous targets under conditions when the Landau–Pomeranchuk–Migdal effect (suppression of radiation owing to the multiple scattering of emitting electrons [2, 3]) and Ter-Mikaelyan effect (suppression of radiation owing to the polarization of the electrons of the target [4]) can be disregarded. It is necessary to take into account that bremsstrahlung is experimentally investigated primarily with solid targets whose atomic structure is usually crystalline or polycrystalline. It is well known that the ordered arrangement of atoms in the single-crystal lattice is responsible for fundamental change in the properties of bremsstrahlung (see, e.g., [5–7]). At the same time, the agreement of the measurement data with the predictions of the Bethe–Heitler theory indicates that the effect of partial ordering of the atomic structure of polycrystals on bremsstrahlung is insignificant. Nevertheless, the commonly accepted opinion that amorphous and polycrystalline radiators are identical seems to be inadequate. The aim of this study is to show that bremsstrahlung in a real polycrystalline target can be significantly different in its characteristics from bremsstrahlung in an amorphous medium owing to the effect of texture that is often manifested in polycrystalline materials.

The predicted effect seems to be very important, because the Bethe–Heitler plateau is a reference for interpretation of the measured characteristics of bremsstrahlung (special caution is required for the description of the properties of bremsstrahlung from thin films, where the probability of appearing texture is particularly high).

In this study, the relativistic system of units is used with $\hbar = c = 1$.

2. Let us consider radiation from relativistic electrons passing through a substance layer with a thickness of L along normal unit vector $\mathbf{e}_z$. In the frequency range of interest, $\gamma\omega_0 \ll \omega \ll m\gamma$ (γ is the Lorentz factor of the electron, ω_0 is the plasma frequency of the target substance, and m is the electron mass), the radiation amplitude is determined primarily by the bremsstrahlung mechanism:

$$\mathbf{A}_\mathbf{n} = \frac{e}{\pi}\int_0^L dt \exp\left[\frac{i\omega}{2}\int_0^t d\tau(\gamma^{-2} + (\boldsymbol{\Psi}_\tau - \boldsymbol{\Theta})^2)\right] \times \frac{d}{dt}\frac{\boldsymbol{\Psi}_t - \boldsymbol{\Theta}}{\gamma^{-2} + (\boldsymbol{\Psi}_t - \boldsymbol{\Theta})^2}, \quad (1)$$

where the scattering angle Ψ_t and observation angle Θ determine the emitting electron velocity $\mathbf{V}(t) = \mathbf{e}_z\left(1 - \frac{1}{2}\gamma^{-2} - \frac{1}{2}\Psi_t^2\right) + \Psi_t$, $\mathbf{e}_z\Psi_t = 0$, and the unit vector in the emission direction $\mathbf{n} = \mathbf{e}_z\left(1 - \frac{1}{2}\Theta^2\right) + \Theta$, respectively.

Under the condition of the dipole bremsstrahlung process ($\gamma^2\Psi_L^2 \ll 1$), we determine the scattering angle Ψ_t from the relativistic equation of motion $\frac{d}{dt}\mathbf{P} = -e\nabla\varphi$ (φ is the total potential of the atoms of the target) in the rectilinear-trajectory approximation and perform the corresponding expansion in Eq. (1). Simple calculations yield the following formula for the spectral angular distribution of bremsstrahlung:

$$\omega\frac{dN}{d\omega d^2\Theta} = \langle|\mathbf{A}_\mathbf{n}|^2\rangle,$$

$$\mathbf{A}_{\mathbf{n}} = \frac{ie^2}{\pi m\gamma}\frac{1}{\gamma^{-2}+\Theta^2}\int d^3k\left(\mathbf{k}_\perp - 2\frac{\boldsymbol{\Theta}(\boldsymbol{\Theta}\mathbf{k}_\perp)}{\gamma^{-2}+\Theta^2}\right)\varphi_{a\mathbf{k}} \quad (2)$$

$$\times\sum_n e^{i\mathbf{k}\mathbf{r}_n}\frac{\exp\left[i\left(\frac{\omega}{2}(\gamma^{-2}+\Theta^2)-k_z\right)L\right]-1}{i\left(\frac{\omega}{2}(\gamma^{-2}+\Theta^2)-k_z\right)},$$

where $\varphi_{a\mathbf{k}}$ is the Fourier transform of the atomic potential, $\mathbf{r}_n$ is the radius vector of the nth atom, $\mathbf{k}_\perp = \mathbf{k} - \mathbf{e}_z k_z$, and the angular brackets mean the averaging over the coordinates $\mathbf{r}_\mathbf{n}$.

Performing averaging in Eq. (2), one should take into account that the polycrystalline target under consideration consists of microcrystallites with an ideal lattice that are randomly oriented in space and make an independent contribution to the scattering and emission of a fast electron. First, we average over the positions of atoms in a fixed microcrystallite (for simplicity, we suggest that a unit cell contains one atom so that $\mathbf{r}_n = \mathbf{R}_n + \mathbf{u}_n$, where $\mathbf{R}_n$ and $\mathbf{u}_n$ are the equilibrium position and thermal displacement of the nth atom, respectively). Using the known averaging result

$$\left\langle \sum_n\sum_l e^{i\mathbf{k}\mathbf{r}_n - i\mathbf{k}'\mathbf{r}_l}\right\rangle = (2\pi)^3 n_a \quad (3)$$

$$\times\left[1 - e^{-k^2u_T^2} + (2\pi)^3 n_a\sum_{\mathbf{g}} e^{-g^2u_T^2}\delta(\mathbf{k}-\mathbf{g})\right]\delta(\mathbf{k}'-\mathbf{k}),$$

where n_a is the density of the atoms, u_T is the rms amplitude of the thermal vibrations of the atoms, and $\mathbf{g}$ is the reciprocal lattice vector, we perform averaging over the orientations of $\mathbf{g}$. Let us consider the potential of a microcrystallite averaged over the thermal vibrations of the atoms as the total potential of atomic chains with the common axis $\mathbf{e}$ coinciding with one of the principal crystallographic directions. In this case, $\mathbf{g} = \mathbf{e}g_\parallel + \mathbf{g}_\perp$, $\mathbf{e}\mathbf{g}_\perp = 0$. The coherent scattering of the emitting electron on the averaged potential of the atomic chain corresponds to the condition $g_\parallel = 0$. Assuming that the probability of the capture of the fast electron into planar channels formed by the atomic chains is small, we pass to the model of a "gas of chains" by changing summation over $\mathbf{g}_\perp$ in Eq. (3) to integration [$\sum_{\mathbf{g}_\perp} \longrightarrow (2\pi/a_\perp)^{-2}\int d^2 g_\perp$, where $a_\perp$ is the distance between atomic chains].

Let us consider the most interesting case, where the normal unit vector $\mathbf{e}_z$ to the target surface coincides with the direction of the predominant orientation of microcrystallites and perform averaging over the directions of the chain axis $\mathbf{e}$ in Eq. (2) in the framework of the simplest model of the uniform distribution of $\mathbf{e}$ over the solid angle in the limits $(\Theta, 2\pi(1-\cos\xi))$. It is easy to see that the model in the limit $\xi \longrightarrow \pi$ describes bremsstrahlung in a polycrystal with completely disoriented microcrystallites. The limit $\xi \longrightarrow 0$ corresponds to the transition to a single crystal. Performing necessary integrations in Eq. (2) with the use of the model of the atom with exponential screening [$(\varphi_a = (Ze/r)\exp(-r/R))$, where Z is the atomic number and R is the radius of the electron screening], we arrive at the simple formula

$$\frac{dN}{d\omega} \simeq \frac{dN_{\text{B–H}}}{d\omega}\left[1 - \frac{\ln(R/u_T)}{\ln(mR)} + \frac{\pi R/a_\parallel}{\ln(mR)}\frac{\xi}{1-\cos\xi}\right], \quad (4)$$

$$\frac{dN_{\text{B–H}}}{d\omega} = \frac{16Z^2e^6n_a\ln(mR)L}{3m^2\omega},$$

where $a_\parallel$ is the distance between the atoms in the chain.

3. Expression (4) contains two corrections to the Bethe–Heitler formula. The first of them describing a certain decrease (~10%) in the yield of bremsstrahlung owing to the periodicity of the positions of the atoms in the lattice was indicated by Ter-Mikaelyan. It is necessary to take into account that this correction is calculated with logarithmic accuracy requiring the condition $mu_T \gg 1$.

The second correction is attributed to the contribution of coherent bremsstrahlung on atomic chains. According to Eq. (4), this correction is smaller than the first correction in the case of usual or weakly textured polycrystals ($\xi \sim 1$), because result (4) theoretically justifies the hypothesis that amorphous and polycrystalline radiators are identical.

At the same time, bremsstrahlung yields from strongly textured polycrystals and amorphous targets can be significantly different. Estimates show that the corresponding addition in Eq. (4) can reach 50% even at $\xi \sim 0.2$–0.3. Thus, polycrystalline targets used in the experimental investigations of bremsstrahlung should be tested to reveal the presence of texture.

Note that the formula for the correction under consideration at small ξ parameter values coincides with the corresponding result for coherent bremsstrahlung on the atomic chain [7], which is valid in the approximation of the rectangular motion of the fast electron through the chain at an angle of ξ (a factor of $R/a_\parallel\xi$ describes the number of chain atoms that are located along the electron trajectory and introduce the coherent contribution to the bremsstrahlung yield). According to the formulation of this problem, all the electrons pass through the chains at angles $\psi < \xi$; for this reason, the coherent gain of bremsstrahlung diverges in the limit $\xi \longrightarrow 0$. However, it is known that the electron trajectory at the orientation angles $\psi \approx \psi_c$ (ψ_c is the critical angle of the axial channeling) is curved by the averaged potential of the atomic chain and the coherent increase in bremsstrahlung is terminated. Therefore, Eq. (4) correctly describes the predicted effect under the condition $\xi \gg \psi_c$, which means that the rectilinear-motion

approximation is valid for the overwhelming majority of beam electrons.

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