Effect of Anomalous Photoabsorption on Parametric X-ray Radiation from Relativistic Electrons

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Abstract—Parametric x-ray radiation from relativistic electrons moving in a crystal is theoretically investigated in Bragg geometry. It is shown that the effect of anomalous photoabsorption can manifest itself within this geometry of the scattering of the pseudophoton field of a fast particle. In this case, the angular distribution of the radiation changes significantly, while the total radiation yield can increase by a factor of 3.

1. INTRODUCTION

The effect of anomalously low absorption (Borrmann effect [1]) can occur in the coherent scattering of x rays in a crystal. Theoretical investigations of a similar phenomenon—parametric x-ray radiation, which consists in the coherent scattering of the pseudophoton field of a fast particle moving in a crystal—revealed that there is no Borrmann effect in parametric x-ray radiation [2, 3]. It should be noted that the results presented in [2, 3] were obtained for parametric x-ray radiation in Laue geometry without taking into account the contribution to the formation of the radiation yield from transition radiation emitted by a fast particle at the entrance surface of a crystal. A more detailed analysis showed that anomalous photoabsorption can yet occur in parametric x-ray radiation owing to the Bragg diffraction of the aforementioned transition radiation from a fast electron at the entrance surface of the crystal [4].

Of considerably greater interest is nonetheless the possible manifestation of the Borrmann effect in the scattering of the pseudophoton field of a fast particle, since the yield of this process is proportional to the target thickness, in contrast to the yield of the diffracted transition radiation.

The objective of this study is to analyze in detail parametric x-ray radiation generated by relativistic electrons in a semi-infinite absorbing crystal. In contrast to what was done in [2, 3], we consider here Bragg geometry. Our basic result is the prediction of the Borrmann effect in parametric x-ray radiation under the conditions of the present analysis.

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2. GENERAL RELATIONS

Let us investigate the structure of the electromagnetic field excited by a relativistic electron moving from a vacuum and entering a crystal that is characterized by the periodically varying dielectric permittivity $\varepsilon(\omega, \mathbf{r}) = 1 + \chi_0(\omega) + \sum_{\mathbf{g}}' \chi_{\mathbf{g}}(\omega) e^{i\mathbf{g}\cdot\mathbf{r}}$, where $\mathbf{g}$ is a set of reciprocal-lattice vectors of the crystal. We assume that the crystal thickness exceeds the photoabsorption length; this makes it possible to consider the crystal as a semi-infinite one.

In order to find the Fourier transform of the excited electric field, $\mathbf{E}_{\omega\mathbf{k}} = (2\pi)^{-4} \int dt d^3r \times \exp(-i\mathbf{k}\cdot\mathbf{r} + i\omega t)\, \mathbf{E}(\mathbf{r}, t)$, we will make use of the conventional Maxwell equations

$$\left(k^2 - \omega^2\right) \mathbf{E}_{\omega\mathbf{k}} - \mathbf{k}\left(\mathbf{k}\cdot\mathbf{E}_{\omega\mathbf{k}}\right) - \omega^2 \chi^0 \mathbf{E}_{\omega\mathbf{k}} \quad (1)$$
$$-\omega^2 \sum_g{}' \chi_{-\mathbf{g}} \mathbf{E}_{\omega\mathbf{k}+\mathbf{g}} = \frac{i\omega e}{2\pi^2} \mathbf{v}\delta\left(\omega - \mathbf{k}\cdot\mathbf{v}\right),$$

where $\mathbf{v}$ is the velocity of the radiating electron.

Since the susceptibilities satisfy the condition χ_0, $\chi_{\mathbf{g}} \ll 1$ in the x-ray region, Eq. (1) can be solved within the well-known two-wave approximation of the dynamical theory of diffraction [5]. By considering that the components $\mathbf{E}_{\omega\mathbf{k}}$ and $\mathbf{E}_{\omega\mathbf{k}+\mathbf{g}}$ are virtually transverse in the x-ray range of frequencies for relativistic particles [6], we can straightforwardly reduce Eq. (1) to the simple set of equations

$$\left(k^2 - \omega^2 - \omega^2\chi_0\right) E_{\lambda 0} - \omega^2 \chi_{-\mathbf{g}} \alpha_\lambda E_{\lambda\mathbf{g}} \quad (2)$$
$$= \frac{i\omega e}{2\pi^2} \mathbf{e}_{\lambda 0}\cdot\mathbf{v}\delta\left(\omega - \mathbf{k}\cdot\mathbf{v}\right),$$
$$\left((\mathbf{k}+\mathbf{g})^2 - \omega^2 - \omega^2\chi_0\right) E_{\lambda\mathbf{g}} = \omega^2 \chi_{\mathbf{g}} \alpha_\lambda E_{\lambda 0},$$

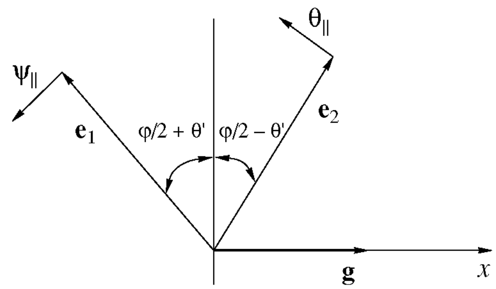

Fig. 1. Geometry of scattering.

where the as-yet-undefined quantities are given by

$$\mathbf{E}_{\omega\mathbf{k}} = \sum_{\lambda=1}^{2} \mathbf{e}_{\lambda 0} E_{\lambda 0}, \quad \mathbf{E}_{\omega\mathbf{k}+\mathbf{g}} = \sum_{\lambda=1}^{2} \mathbf{e}_{\lambda\mathbf{g}} E_{\lambda\mathbf{g}},$$

$$\mathbf{e}_{10} = \mathbf{e}_{1\mathbf{g}} = \frac{[\mathbf{k}_\parallel \times \mathbf{e}_x]}{k_\parallel}, \quad \mathbf{e}_{20} = \frac{[\mathbf{k} \times \mathbf{e}_{10}]}{k},$$

$$\mathbf{e}_{2\mathbf{g}} = \frac{[\mathbf{k}+\mathbf{g} \times \mathbf{e}_{10}]}{|\mathbf{k}+\mathbf{g}|}, \tag{3}$$

$$\alpha_1 = 1, \quad \alpha_2 = \mathbf{k}\cdot(\mathbf{k}+\mathbf{g})/k\,|\mathbf{k}+\mathbf{g}|,$$

$$\mathbf{k} = \mathbf{k}_\parallel + \mathbf{e}_x k_x, \quad \mathbf{e}_x \cdot \mathbf{k}_\parallel = 0.$$

Equations (2) describe the field in the target. The corresponding equations for this field in a vacuum (beyond the target),

$$\left(k^2-\omega^2\right) E^v_{\lambda 0} = \frac{i\omega e}{2\pi^2}\mathbf{e}_{\lambda 0}\cdot\mathbf{v}\,\delta(\omega - \mathbf{k}\cdot\mathbf{v}), \tag{4}$$

$$\left(k^2_{\mathbf{g}}-\omega^2\right) E^v_{\lambda g} = 0,$$

follow from (2) in the limit $\chi_0 = \chi_{\mathbf{g}} = 0$. Here, we have $\mathbf{k_g} = \mathbf{k} + \mathbf{g}$. Below, we assume that the reflecting crystallographic plane is parallel to the crystal surface. The reciprocal-lattice vector $\mathbf{g}$ is then parallel to the normal $\mathbf{e}_x$ to the crystal surface (see Fig. 1).

The radiation field $E^v_{\lambda\mathbf{g}}$ is determined as a solution to the corresponding equation in (4). The result is

$$E^v_{\lambda\mathbf{g}} = a_{\lambda\mathbf{k}_\parallel}\delta\left(k_{\mathbf{g}x} - p\right), \quad p = \sqrt{\omega^2 - k^2_\parallel}. \tag{5}$$

In order to determine an unknown coefficient $a_{\lambda\mathbf{k}_\parallel}$, it is necessary to find solutions to the remaining equations in (2) and (4) and to employ conventional boundary conditions for the electromagnetic field at the target surface. Considering that, in the x-ray frequency range, the photon wave vector in a vacuum differs insignificantly from that in a crystal, we introduce the variable ξ through the relation

$$k_{\mathbf{g}x} = p + \xi, \quad \xi \ll g. \tag{6}$$

By using relation (6), we can represent solutions to Eqs. (2) and (4) in the form

$$E_{\lambda\mathbf{g}} = b_{\lambda\mathbf{k}_\parallel}\delta(\xi - \xi_*) \tag{7}$$

$$-\frac{i\omega e}{2\pi^2}\frac{\omega^2\chi_{\mathbf{g}}\alpha_\lambda}{4p^2|v_x|}\frac{\mathbf{e}_{\lambda 0}\cdot\mathbf{v}}{(\xi-\xi_1)(\xi-\xi_2)}\delta(\xi-\xi_0),$$

$$E_{\lambda 0} = \frac{2p}{\omega^2\chi_{\mathbf{g}}\alpha_\lambda}\left(\xi - \frac{\omega^2}{2p}\chi_0\right)E_{\lambda\mathbf{g}},$$

$$E^v_{\lambda 0} = \frac{i\omega e}{2\pi^2}\frac{1}{2p|v_x|}\frac{\mathbf{e}_{\lambda 0}\cdot\mathbf{v}}{\Delta - \xi}\delta(\xi-\xi_0),$$

where

$$\xi_{1,2} = \frac{1}{2}\left(\Delta \pm \sqrt{\left(\Delta - \frac{\omega^2}{p}\chi_0\right)^2 - \beta^2_\lambda}\right), \tag{8}$$

$$\xi_0 = \frac{1}{v_x}\left(\omega - \mathbf{k}_\parallel\cdot\mathbf{v}_\parallel + pv_x\right) + \Delta,$$

$$\beta^2_\lambda = \frac{\omega^4}{p^2}\chi_{\mathbf{g}}\chi_{-\mathbf{g}}\alpha^2_\lambda, \quad \Delta = g\left(\frac{g}{2p} - 1\right) \ll g.$$

In the case of the Bragg geometry of scattering (and we consider precisely this case), the quantity ξ_* is determined by the relations

$$\xi_* = \begin{cases} \xi_2 \text{ for } \left(\Delta - \frac{\omega^2}{p}\chi_0\right)^2 < \beta^2_\lambda \\ \xi_1 \text{ for } \left(\Delta - \frac{\omega^2}{p}\chi_0\right)^2 > \beta^2_\lambda, \Delta - \frac{\omega^2}{p}\chi_0 > 0 \\ \xi_2 \text{ for } \left(\Delta - \frac{\omega^2}{p}\chi_0\right)^2 > \beta^2_\lambda,\ \Delta - \frac{\omega^2}{p}\chi_0 < 0. \end{cases} \tag{9}$$

By using Eqs. (5) and (7) and the boundary conditions

$$\int d\xi\left(E_{\lambda 0} - E^v_{\lambda 0}\right) = \int d\xi\left(E_{\lambda\mathbf{g}} - E^v_{\lambda\mathbf{g}}\right) = 0, \tag{10}$$

we find that the coefficient $a_{\lambda\mathbf{k}_\parallel}$ is given by the expression

$$a_{\lambda\mathbf{k}_\parallel} = \frac{i\omega e}{2\pi^2}\frac{\omega^2\chi_{\mathbf{g}}\alpha_\lambda}{4p^2|v_x|}\frac{\mathbf{e}_{\lambda 0}\cdot\mathbf{v}}{\xi_* - \frac{\omega^2}{2p}\chi_0} \tag{11}$$

$$\times\left(\frac{\xi_0 - \xi_*}{(\xi_0-\xi_1)(\xi_0-\xi_2)} - \frac{1}{\xi_0 - \Delta}\right),$$

which completely describes the properties of the radiation field.

3. CONTRIBUTION OF PARAMETRIC X RAYS: EFFECT OF ANOMALOUS PHOTOABSORPTION

In order to find the spectral and angular distribution of the radiation in question, we make use of the general expression (11). In order to determine the radiation amplitude A_λ, we apply the stationary-phase method to compute the Fourier integral

$$E^{\text{rad}}_\lambda = \int d^3k_{\mathbf{g}}e^{i\mathbf{k_g}\cdot\mathbf{n}r}E^v_{\lambda\mathbf{g}} \to A_\lambda\frac{e^{i\omega r}}{r}, \tag{12}$$

$$A_\lambda = -2\pi i\omega n_x a_{\lambda\omega\mathbf{n}_\parallel},$$

where $\mathbf{n} = \mathbf{n}_{\|} + \mathbf{e}_x n_x$ is a unit vector in the direction of radiation, $\mathbf{e}_x \cdot \mathbf{n}_{\|} = 0$.

For the purposes of the ensuing analysis, it is convenient to define the angular variables $\boldsymbol{\theta}$ and $\boldsymbol{\psi}$ through the relations (see Fig. 1)

$$\mathbf{v} = e_1 \left(1 - \frac{1}{2}\gamma^{-2} - \frac{1}{2}\psi^2\right) + \boldsymbol{\psi}, \quad \mathbf{e}_1 \cdot \boldsymbol{\psi} = 0,$$

$$\mathbf{n} = \mathbf{e}_2 \left(1 - \frac{1}{2}\theta^2\right) + \boldsymbol{\theta}, \tag{13}$$

$$\mathbf{e}_2 \cdot \boldsymbol{\theta} = 0, \quad \mathbf{e}_1 \cdot \mathbf{e}_2 = \cos\varphi$$

and the dielectric susceptibilities χ_0 and $\chi_{\mathbf{g}}$ as

$$\chi_0 = -\frac{\omega_0^2}{\omega^2} + i\chi_0'', \tag{14}$$

$$\chi_{\mathbf{g}} = \chi_{-\mathbf{g}} = -\frac{\omega_{\mathbf{g}}^2}{\omega^2} + i\chi_{\mathbf{g}}'',$$

where ω_0 is the plasmon frequency and $\omega_{\mathbf{g}}^2 = \omega_0^2 e^{-g^2u^2} \left(F(\mathbf{g})/Z\right)$, u, $F(\mathbf{g})$, and Z being, respectively, the root-mean-square amplitude of thermal vibrations of the atoms, the atomic form factor, and the number of electrons in an atom; we consider here a reflection for which the lattice structure factor is equal to unity.

From Eqs. (11)–(14), it follows that the spectral and angular distribution of the radiation being studied can be represented as

$$\omega\frac{dN_\lambda}{d\omega d^2\theta} = \frac{e^2}{\pi^2}\frac{\Omega_\lambda^2}{|\tau_\lambda \pm f_\lambda - i\delta_\lambda|^2}\left|A_\lambda^{\mathrm{PXR}} + A_\lambda^{\mathrm{DTR}}\right|^2,$$

$$A_\lambda^{\mathrm{PXR}} = \frac{\omega_{\mathbf{g}}^2}{\omega^2}|\alpha_\lambda|\frac{\tau_\lambda \pm f_\lambda}{\left(\gamma^{-2} + \frac{\omega_0^2}{\omega^2} + \Omega^2\right)\left(\gamma^{-2} + \frac{\omega_0^2}{\omega^2} + \Omega^2 - \frac{\omega_{\mathbf{g}}^2}{\omega^2}|\alpha_\lambda|(\tau_\lambda \pm f_\lambda)\right)}, \tag{15}$$

$$A_\lambda^{\mathrm{DTR}} = \frac{1}{\gamma^{-2} + \frac{\omega_0^2}{\omega^2} + \Omega^2} - \frac{1}{\gamma^{-2} + \Omega^2},$$

where

$$\tau_\lambda = \frac{g^2}{2\omega_{\mathbf{g}}^2|\alpha_\lambda|}\left(\frac{g}{2\omega n_x} - 1 + 2\frac{\omega_0^2}{g^2}\right),$$

$$f_\lambda = \sqrt{\tau_\lambda^2 - 1 - 2i\delta_\lambda(\tau_\lambda - \kappa_\lambda)},$$

$$\delta_\lambda = \frac{\omega^2}{2\omega_{\mathbf{g}}^2|\alpha_\lambda|}\chi_0'', \quad \kappa_\lambda = \frac{\chi_{\mathbf{g}}''}{\chi_0''}|\alpha_\lambda|, \tag{16}$$

$$\Omega_1 = \theta_\perp - \psi_\perp, \quad \Omega_2 = 2\theta' + \theta_{\|} + \psi_{\|},$$

$$\Omega^2 = \Omega_1^2 + \Omega_2^2, \quad \alpha_1 = 1, \quad \alpha_2 = \cos\varphi.$$

In Eq. (15), the quantities A_λ^{PXR} and A_λ^{DTR} represent the contributions of, respectively, parametric x rays and diffracted transition radiation (DTR) [7]. The plus (minus) sign corresponds to the case of $\xi_* = \xi_1$ ($\xi_* = \xi_2$) [see Eq. (9)].

Let us first consider the contribution of parametric x rays to the total radiation yield. According to (15), this contribution is given by

$$\omega\frac{dN_\lambda^{\mathrm{PXR}}}{d\omega d^2\theta} = \frac{e^2}{\pi^2}\frac{\omega_{\mathbf{g}}^4}{\omega^4}\frac{\Omega_\lambda^2\alpha_\lambda^2}{\left(\gamma^{-2} + \frac{\omega_0^2}{\omega^2} + \Omega^2\right)} \tag{17}$$

$$\times\left|\gamma^{-2} + \frac{\omega_0^2}{\omega^2} + \Omega^2 - \frac{\omega_{\mathbf{g}}^2}{\omega^2}|\alpha_\lambda|(\tau_\lambda \pm f_\lambda)\right|^{-2}.$$

It can easily be seen that the dependence of the distribution in (17) on the emitted-photon energy ω is associated primarily with the fast variable $\tau_\lambda(\omega)$ from (16), which shows that, at fixed values of the orientation angle θ' and of the observation angle $\theta_{\|}$, the spectrum of parametric x rays is concentrated in the vicinity of the frequency $\omega_{\mathrm{B}}' = \omega_{\mathrm{B}}\left(1 + \left(\theta' + \theta_{\|}\right)\cot(\varphi/2)\right)$, where $\omega_{\mathrm{B}} = g/2 \times \sin(\varphi/2)$ is the Bragg frequency, the relative frequency of the spectrum being very small: $\Delta\omega/\omega \sim 2\omega_{\mathbf{g}}^2/g^2 \sim 10^{-4}$.

For the ensuing analysis, it is convenient to separate the real and the imaginary part of the function f_λ:

$$f_\lambda = \begin{cases} f_\lambda' - i\,\mathrm{sgn}(\tau_\lambda - \kappa_\lambda)\,f_\lambda'', & \text{for } \tau_\lambda^2 > 1 \\ -\mathrm{sgn}(\tau_\lambda - \kappa_\lambda)\,f_\lambda' + if_\lambda'' & \text{for } \tau_\lambda^2 < 1, \end{cases}$$

$$f_\lambda' = \tfrac{1}{\sqrt{2}}\sqrt{\sqrt{\left(\tau_\lambda^2 - 1\right)^2 + 4\delta_\lambda^2(\tau_\lambda - \kappa_\lambda)^2} + \tau_\lambda^2 - 1},$$

$$f_\lambda'' = \tfrac{1}{\sqrt{2}}\sqrt{\sqrt{\left(\tau_\lambda^2 - 1\right)^2 + 4\delta_\lambda^2(\tau_\lambda - \kappa_\lambda)^2} - \tau_\lambda^2 + 1}. \tag{18}$$

We note that, because of the smallness of the absorption factor ($\delta_\lambda \ll 1$), the function f_λ'' is small in the region of normal dispersion ($\tau_\lambda^2 > 1$) and that

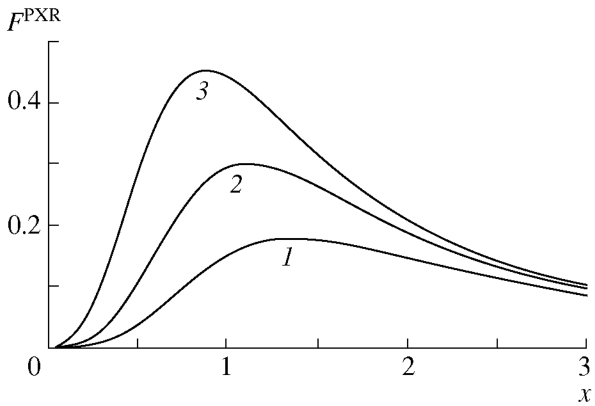

Fig. 2. Normalized angular distribution of parametric x rays with allowance for anomalous photoabsorption at $q_\lambda = 0.8$ and $x = \gamma_* \Omega$ for $\kappa_\lambda =$ (*1*) 0, (*2*) 0.7, and (*3*) 0.95.

the function f'_λ is small in the region of anomalous dispersion ($\tau_\lambda^2 < 1$). With allowance for Eqs. (9) and (18), expression (17) can be recast into the simple form

$$\omega \frac{dN_\lambda^{PXR}}{d\omega d^2\theta} = \frac{e^2}{\pi^2} \frac{\Omega_\lambda^2}{\left(\gamma^{-2} + \gamma_*^{-2} + \Omega^2\right)^2} R_\lambda^{PXR}(\tau_\lambda),$$

$$R_\lambda^{PXR} = \left[\left(p_\lambda - \tau_\lambda - \operatorname{sgn}(\tau_\lambda - \kappa_\lambda) f'_\lambda\right)^2 + \left(f''_\lambda\right)^2\right]^{-1}, \tag{19}$$

where

$$\gamma_* = \omega_B/\omega_0, \quad p_\lambda = \frac{\gamma^{-2} + \gamma_*^{-2} + \Omega^2}{\omega_{\mathbf{g}}^2 |\alpha_\lambda| / \omega_B^2}. \tag{20}$$

The above result demonstrates the physical essence of parametric x-ray radiation as the coherent Bragg scattering of the screened Coulomb field of a fast electron by a set of atomic crystal planes. Indeed, the factor appearing in front of the function R_λ^{PXR} in (19) describes the spectral and angular distribution of the Coulomb field of a particle in a medium whose mean dielectric permittivity is $\varepsilon(\omega) = 1 - \omega_0^2/\omega^2$ (in the vicinity of the Bragg frequency, $\omega \approx \omega_B$), while the quantity R_λ^{PXR} can be treated as the coefficient of reflection of this field by a crystal.

It can easily be shown that the denominator of the function $R_\lambda^{PXR}(\tau_\lambda(\omega))$ has a resonance character in the region of frequencies ω that correspond to the condition $\tau_\lambda(\omega) > 1$; that is, parametric x rays are formed in the region of normal dispersion. The position and the height of the peak of parametric x-ray radiation are determined by the single generalized parameter p_λ (20), which is greatly dependent on the radiating-particle energy, the photon-observation angle, and the orientation angle. Since the spectral width of the peak of parametric x-ray radiation is $\Delta\omega \sim 1$ eV at a fixed value of the parameter p_λ, the approximation

$$R_\lambda^{PXR} \to \frac{\pi}{f''_\lambda} \delta\left(p_\lambda - \tau_\lambda - f'_\lambda\right) \tag{21}$$

is sufficient for describing experiments that employ conventional x-ray detectors with an energy resolution of about 100 eV.

Considering that the absorption factor δ_λ is small, we find from (19) and (21) that

$$\omega \frac{dN_\lambda^{PXR}}{d\omega d^2\theta} \tag{22}$$

$$= \frac{e^2 \omega_{\mathbf{g}}^4}{\pi g^2} \frac{1}{\omega \chi''_0} \frac{\left(1 - 1/p_\lambda^2\right)^2}{\left(1 - 1/p_\lambda^2\right)^2 + 2(1 - \kappa_\lambda)/p_\lambda} \times \frac{\Omega_\lambda^2 - \alpha_\lambda^2}{\left(\gamma^{-2} + \gamma_*^{-2} + \Omega^2\right)^2} \delta(\omega - \omega_B).$$

This expression differs from the traditional formula in the kinematical theory of parametric x rays [8, 9] only by a factor that involves the parameters p_λ and κ_λ.

It can easily be seen that only in the region of sufficiently high radiating-particle energies can dynamical-scattering effects manifest themselves. To demonstrate this, we note that, in the region of low energies,

$$\gamma \ll \gamma_*, \tag{23}$$

it follows from (20) that $p_\lambda \gg 1$, irrespective of the observation-angle value. Expression (22) then coincides with that in kinematical theory.

In the energy region where the condition opposite to (23) is satisfied, the parameter p_λ is on the order of unity for observation angles in the region $\Omega_\lambda \leqslant \gamma_*^{-1}$, where the bulk of the radiation is concentrated. In this case, the distribution in (22) is extremely sensitive to variations in the parameter κ_λ. In order to demonstrate this, we consider, for $\gamma \gg \gamma_*$, the angular distribution of parametric x rays that follows from (22). We have

$$\omega \frac{dN_\lambda^{PXR}}{dx x} = \frac{e^2 \omega_{\mathbf{g}}^4 \alpha_\lambda^2 \sin^2(\varphi/2)}{g^4 \chi''_0} F^{PXR}(x, \kappa_\lambda, q_\lambda), \tag{24}$$

$$F^{PXR} = \frac{x^2}{(1 - q_\lambda + x^2)^2 + 2q_\lambda(1 - \kappa_\lambda)(1 + x^2)} \times \left(1 - \frac{q_\lambda^2}{(1 + x^2)^2}\right)^2,$$

where $x = \gamma_* \Omega$ and $q_\lambda = e^{-g^2 u^2/2} (F(\mathbf{g})/Z) |\alpha_\lambda|$. The function F^{PXR} calculated for a strong reflection ($q_\lambda \approx 1$) is illustrated by the curves in Fig. 2, which are plotted for various values of the parameter κ_λ. We can see that the polarization-bremsstrahlung yield grows significantly as κ_λ tends to unity, and this is the main result of the present study.

The effect being discussed is similar to the anomalous photoabsorption of x rays in a crystal [1]. The latter is manifested in the Bragg diffraction of x rays under the condition $\kappa_\lambda \approx 1$. In the diffraction of the pseudophoton field of a fast particle—it is precisely the case considered here—the effective absorption factor f''_λ also decreases for $\kappa_\lambda \to 1$. It is important to note that, according to formula (18), which determines the dependence $f''_\lambda(\tau_\lambda)$ (for example, we have $f''_\lambda \simeq \delta_\lambda\sqrt{(\tau_\lambda-1)/(\tau_\lambda+1)}$ in the limiting case of $\kappa_\lambda = 1$), the above suppression of photoabsorption is realized only near the region of anomalous dispersion, $\tau_\lambda(\omega) \approx 1$. In accordance with (21), the position of the maximum in the spectral distribution of parametric x-ray radiation is determined by the parameter p_λ [the corresponding value is $\tau_\lambda = \tau_\lambda^* \approx \left(p_\lambda^2+1\right)/2p_\lambda$]. At low radiating-particle energies that satisfy the condition in (23), the parameter p_λ is large. In this case, $f''_\lambda \simeq \delta_\lambda$; that is, the effect of anomalous photoabsorption does not manifest itself in kinematical parametric x-ray radiation. At high energies, $\gamma \gg \gamma_*$, we have $p_\lambda \approx \left(1+x^2\right)q_\lambda \sim 1$ for strong reflections ($q_\lambda \approx 1$) in the region $x \leqslant 1$, which is of particular interest [see formula (24)]. In this case, the effect of anomalously low photoabsorption, $f''_\lambda(\tau_\lambda^*) \ll \delta_\lambda$, can show up if the coefficient κ_λ is sufficiently close to unity.

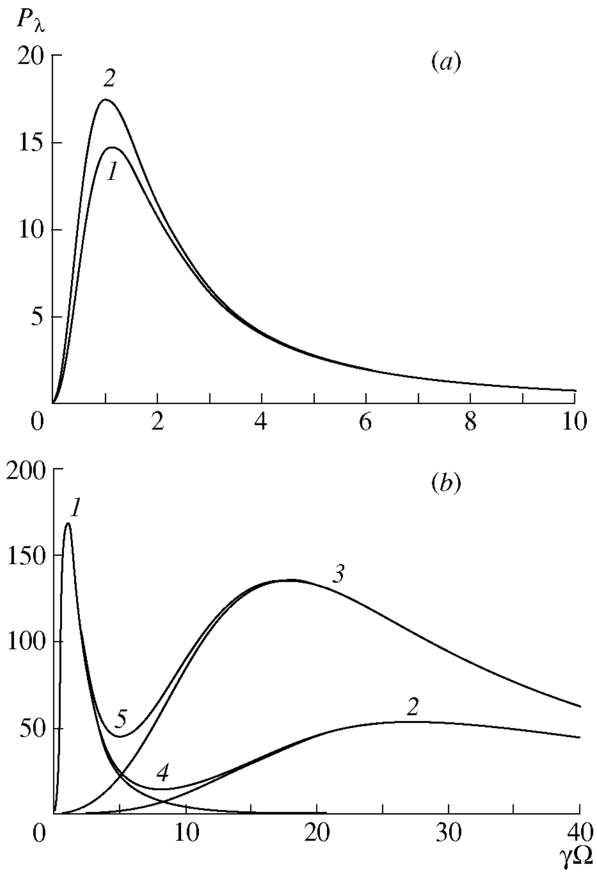

Fig. 3. Normalized angular distributions of parametric x-ray radiation, diffracted transition radiation, and total radiation with and without allowance for anomalous photoabsorption (presented in the figure are the quantities $P_\lambda^{\rm DTR}$, $P_\lambda^{\rm PXR}$, and $P_\lambda = P_\lambda^{\rm DTR} + P_\lambda^{\rm PXR}$ defined by the relations $dN_\lambda^{\rm DTR}/d^2\theta = A_\lambda P_\lambda^{\rm DTR}$ and $dN_\lambda^{\rm PXR}/d^2\theta = A_\lambda P_\lambda^{\rm PXR}$, where $A_\lambda = e^2 q_\lambda/4\pi\sin(\varphi/2)^2$: (*a*) curves *1* and *2* representing the contribution of parametric x rays at $\kappa_\lambda = 0$ and 0.95, respectively, correspond to $q_\lambda = 0.8$, $\gamma_*\chi''_0 = 1/300$, and $\gamma/\gamma_* = 0.5$ (there is virtually no contribution from diffracted transition radiation); (*b*) curves *1*, *2*, *3*, *4*, and *5* representing the contributions of, respectively, diffracted transition radiation, parametric x rays at $\kappa_\lambda = 0$, parametric x rays at $\kappa_\lambda = 0.95$, total radiation at $\kappa_\lambda = 0$, and total radiation at $\kappa_\lambda = 0.95$ correspond to $\gamma/\gamma_* = 20$, all other parameters being identical to those in Fig. 3*a*.

4. CONTRIBUTION OF DIFFRACTED TRANSITION RADIATION: INTERFERENCE OF DIFFRACTED TRANSITION RADIATION AND PARAMETRIC X-RAY RADIATION

Returning to the general formula (15), we consider the contribution of diffracted transition radiation, whose spectral and angular distribution can be represented in the form

$$\omega\frac{dN_\lambda^{\rm DTR}}{d\omega d^2\theta} = \frac{e^2}{\pi^2}\Omega_\lambda^2 \tag{25}$$

$$\times\left(\frac{1}{\gamma^{-2}+\Omega^2} - \frac{1}{\gamma^{-2}+\gamma_*^{-2}+\Omega^2}\right)^2 \times R_\lambda^{\rm DTR}(\tau_\lambda),$$

$$R_\lambda^{\rm DTR} = \left[\left(\tau_\lambda + \mathrm{sgn}(\tau_\lambda-\kappa_\lambda)f'_\lambda\right)^2 + \left(\delta_\lambda + f''_\lambda\right)^2\right]^{-1},$$

which is similar to that in (19). As follows from formula (25), diffracted transition radiation is generated when transition radiation from a relativistic electron that traverses the entrance surface of the crystal target undergoes Bragg reflection from the crystal [7].

It can easily be shown that the coefficient of reflection $R_\lambda^{\rm DTR}$ attains a maximum value of about unity in the anomalous-dispersion region $\tau_\lambda^2(\omega) < 1$.

Absorption reduces the yield of diffracted transition radiation; however, it does not play a crucial role, as it does in parametric x-ray radiation from a relativistic particle moving in a semi-infinite medium. The formula for the angular distribution of diffracted transition radiation—it follows from (25) without taking into account absorption—has the form

$$\frac{dN_\lambda^{\mathrm{DTR}}}{d^2\theta} = \frac{16e^2\omega_{\mathbf{g}}^2\,|\alpha_\lambda|}{3\pi^2 g^2}\Omega_\lambda^2 \times \left(\frac{1}{\gamma^{-2}+\Omega^2} - \frac{1}{\gamma^{-2}+\gamma_*^{-2}+\Omega^2}\right)^2. \tag{26}$$

This formula and the corresponding formula following from (22) for the angular distribution of parametric x rays make it possible to compare the contributions of the radiation mechanisms under investigation. First of all, we note that our analysis of the interference term in the general formula (15) has revealed that the interference between parametric x-ray radiation and diffracted transition radiation is insignificant.

The expression that follows from (22) and which describes the angular distribution of parametric x rays has the form

$$\frac{dN_\lambda^{\mathrm{PXR}}}{d^2\theta} = \frac{e^2\omega_{\mathbf{g}}^2\,|\alpha_\lambda|}{\pi g^2}\frac{q_\lambda}{\gamma_*^2\chi_0''}\frac{\Omega_\lambda^2}{\left(\gamma^{-2}+\gamma_*^{-2}+\Omega^2\right)^2} \times \frac{\left(1-1/p_\lambda^2\right)^2}{\left(1-1/p_\lambda\right)^2+2\left(1-\kappa_\lambda\right)/p_\lambda}, \tag{27}$$

where $p_\lambda = \left(\gamma^{-2}+\gamma_*^{-2}+\Omega^2\right)/q_\lambda\gamma_*^{-2}$.

In order to compare the dependences in (26) and (27), it is convenient to choose the quantity $\gamma\Omega$ for an angular variable. In this case, the ratio γ/γ_* becomes the main parameter in both distributions. The distributions calculated on the basis of (26) and (27) are displayed in Fig. 3, along with the computed distribution of the total radiation. The curves presented in this figure demonstrate that parametric x-ray radiation is dominant at low radiating-particle energies $\gamma < \gamma_*$ such that the effect of anomalous photoabsorption is not manifested here. At the same time, we can see that, at energies so high that $\gamma \gg \gamma_*$, the contribution of diffracted transition radiation dominates the observation-angle region $\Omega \leqslant \gamma_*^{-1}$, while the peak of parametric x-ray radiation is formed near the observation-angle value of $\Omega \approx \gamma_*^{-1}$. From these curves, we also see that the yield of parametric x-ray radiation increases considerably under the conditions where the effect of anomalous photoabsorption manifests itself and that the peak in the angular distribution of parametric x-ray radiation is shifted to the region of small observation angles.

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