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Mathematical Model for the Formation of University Contingents on the Basis of Population Dynamics Equations

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Abstract

The mathematical model of two competitive universities for limited contingent of applicants, offered by L.A. Serkov, has been simplified up to level , allowing to investigate it by methods of the qualitative theory of dynamic systems. 8 critical points of the simplified dynamic system are defined and the analysis of their stability are made. It has allowed receiving all regimes of education system's behavior. Numerical experiments with the model confirmed the results qualitative analysis. The model spreads on n+1-dimensional case (n universities competing for the limited contingent of applicants).

Keywords: mathematical model, population dynamics equations, university competition,qualitative theory of dynamic systems, stability of critical points, university contingents.

Introduction

To describe a wide class of social systems V. Weidlich in 1988 in the " Stability and cyclicity in social systems " proposed nonlinear dynamic system of second order based on the logistic equation [1]:

$$\begin{cases} \frac{dx}{dt} = x[a(y)s - x] \\ \frac{dy}{dt} = y[b(x)s - y], \end{cases} \quad (1)$$

where piecewise variable influence functions *a(y)* and *b(x)* describe the cooperative or antagonistic interaction of variables. For example, if the impact of y on x is cooperative , then

$$\begin{cases} a(y) = a_- < 0, \ if \ \ 0 < y \le y_s \\ a(y) = a_+ > 0, \ if \ \ y_s < y \le \infty, \end{cases}$$

where y_s - is the switching point of the influence function .

Here are four possible options of interaction of macro variables *x* and *y*. This model was used in the work [2] to analyze the development of the education system in a competitive environment. In the work [3], the model (1) has been extended to three dimensions:

$$\begin{cases} \frac{dx}{dt} = x[a(y,z)s - x] \\ \frac{dy}{dt} = y[b(x,z)s - y] \\ \frac{dz}{dt} = z[c(x,y)s - z], \end{cases} \tag{3}$$

when specifying the following influence functions :

$$\begin{aligned} a(y,z) &= -A_{yx} th[k(y - y_{sx})] + A_{zx} th[k(z - z_{sx})] \\ b(x,z) &= -A_{xy} th[k(x - x_{sy})] + A_{zy} th[k(z - z_{sy})] \\ c(x,y) &= -A_{xz} th[k(x - x_{sz})] + A_{yz} th[k(y - y_{sz})], \end{aligned} \tag{4}$$

where x_{sy}, x_{sz}, y_{sx}, y_{sz}, z_{sx}, z_{sy} - are the switching points of the influence functions. Their crossing changes the macro variables interaction nature (from suppression to strengthening or vice versa).

Numerical experiments with this model are done with the basic parameters $s = 5$, $k = 1$, $A_{ij} = 1$, $x_{sy} = x_{sz} = x_s = 4{,}5$, $y_{sx} = y_{sz} = y_s = 4{,}25$, $z_{sx} = z_{sy} = z_s = 4{,}2$, and the model is interpreted as a competition between two one-profile universities (*x, y* - the number of students in them) for some limited resource of applicants (*z* - number of applicants) .

As a result of these experiments only bistable mode of behavior of the educational system were obtained in the work [3]: $(x_s^*, 0, z_s^*)$; $(0, y_s^*, z_s^*)$; and complexity of the influence functions setting (4) did not allow to do qualitative analysis.

Mathematical model and its qualitative analysis

If the influence functions are set to a first approximation as linear functions, the dynamical system (3) would be able to be explored with the simplest methods of the qualitative theory of dynamical systems . This is conveniently done in terms of the models of the "predator - prey" type with the presentation of their coefficients as in competitive models of pairwise competitive-cooperative interactions [4, 5] In this case, the system of equations (3) can be written as :

$$\begin{cases} \dfrac{dx}{dt} = \alpha_1 x - \beta_1 x^2 - \gamma_1 xy + \varepsilon_1 xz \\ \dfrac{dy}{dt} = \alpha_2 y - \beta_2 y^2 - \gamma_2 xy + \varepsilon_2 yz, \\ \dfrac{dz}{dt} = \alpha_3 z - \beta_3 z^2 - \varepsilon_1 xz - \varepsilon_2 yz. \end{cases} \tag{5}$$

where x - number of students in the first university , y - the number of students in the second university, z - number of applicants wishing to enroll in these two higher educational institutions, $\alpha_i > 0$ - growth factors, $\beta_i > 0$ -intra-contingent (intra university),competition factors, $\gamma_i > 0$ - inter- contingent (inter-university)competition factors , $\varepsilon_i > 0$ - coefficients of students-applicants cooperation.

In model (5) in addition to the three logistics members and inter-university competition (members: $-\gamma_i xy$)) the balance of students- applicants interactions (for example , an influx of applicants the first university ($\varepsilon_1 xz$) is equal to the outflow of applicants from their total number($-\varepsilon_1 xz$)) is taken into account. If the right-hand sides of equations (5) we will take out of brackets *x*, *y* and *z*, then we will see in the brackets the above mentioned linear functions of influence in the model of L.A.Serkov .

We will find singular points of the dynamical system (5) in the solutions of the following system of algebraic equations :

$$\begin{cases} x\left(\alpha_1 - \beta_1 x - \gamma_1 y + \varepsilon_1 z\right) = 0 \\ y\left(\alpha_2 - \beta_2 y - \gamma_2 x + \varepsilon_2 z\right) = 0 \\ z\left(\alpha_3 - \beta_3 z - \varepsilon_1 x - \varepsilon_2 y\right) = 0. \end{cases} \tag{6}$$

This system of equations has eight solutions which determine the coordinates of the singular points . We will write them in order:

1. $x_* = 0$, $y_* = 0$, $z_* = 0$;

2. $x_* = \dfrac{\alpha_1}{\beta_1}$, $y_* = 0$, $z_* = 0$;

3. $x_* = 0$, $y_* = \dfrac{\alpha_2}{\beta_2}$, $z_* = 0$;

4. $x_* = 0$, $y_* = 0$, $z_* = \dfrac{\alpha_3}{\beta_3}$;

5. $x_* = 0$, $y_* = \dfrac{\alpha_2\beta_3 + \alpha_3\varepsilon_2}{\beta_2\beta_3 + \varepsilon_2^2}$, $z_* = \dfrac{\alpha_3\beta_2 - \alpha_2\varepsilon_2}{\beta_2\beta_3 + \varepsilon_2^2}$;

6. $x_* = \dfrac{\alpha_1\beta_3 + \alpha_3\varepsilon_1}{\beta_1\beta_3 + \varepsilon_1^2}$, $y_* = 0$, $z_* = \dfrac{\alpha_3\beta_1 - \alpha_1\varepsilon_1}{\beta_1\beta_3 + \varepsilon_1^2}$;

7. $x_* = \dfrac{\alpha_1\beta_2 - \alpha_2\gamma_1}{\beta_1\beta_2 - \gamma_1\gamma_2}$, $y_* = \dfrac{\alpha_2\beta_1 - \alpha_1\gamma_2}{\beta_1\beta_2 - \gamma_1\gamma_2}$, $z_* = 0$;

8. $$x_* = \begin{vmatrix} \alpha_1 & \gamma_1 & -\varepsilon_1 \\ \alpha_2 & \beta_2 & -\varepsilon_2 \\ \alpha_3 & \varepsilon_2 & \beta_3 \end{vmatrix} \Bigg/ \begin{vmatrix} \beta_1 & \gamma_1 & -\varepsilon_1 \\ \gamma_2 & \beta_2 & -\varepsilon_2 \\ \varepsilon_1 & \varepsilon_2 & \beta_3 \end{vmatrix},$$

$$y_* = \begin{vmatrix} \beta_1 & \alpha_1 & -\varepsilon_1 \\ \gamma_2 & \alpha_2 & -\varepsilon_2 \\ \varepsilon_1 & \alpha_3 & \beta_3 \end{vmatrix} \Bigg/ \begin{vmatrix} \beta_1 & \gamma_1 & -\varepsilon_1 \\ \gamma_2 & \beta_2 & -\varepsilon_2 \\ \varepsilon_1 & \varepsilon_2 & \beta_3 \end{vmatrix},$$

$$z_* = \begin{vmatrix} \beta_1 & \gamma_1 & \alpha_1 \\ \gamma_2 & \beta_2 & \alpha_2 \\ \varepsilon_1 & \varepsilon_2 & \alpha_3 \end{vmatrix} \Bigg/ \begin{vmatrix} \beta_1 & \gamma_1 & -\varepsilon_1 \\ \gamma_2 & \beta_2 & -\varepsilon_2 \\ \varepsilon_1 & \varepsilon_2 & \beta_3 \end{vmatrix}$$

Jacobi matrix of the linearized dynamic system (5) has the form :

$$A = \begin{pmatrix} \alpha_1 - 2\beta_1 x_* - \gamma_1 y_* + \varepsilon_1 z_* & -\gamma_1 x_* & \varepsilon_1 x_* \\ -\gamma_2 y_* & \alpha_2 - 2\beta_2 y_* - \gamma_2 x_* + \varepsilon_2 z_* & \varepsilon_2 y_* \\ -\varepsilon_1 z_* & -\varepsilon_2 z_* & \alpha_3 - 2\beta_3 z_* - \varepsilon_1 x_* - \varepsilon_2 y_* \end{pmatrix}. \quad (7)$$

The characteristic equation for this matrix has the form :

$$|A - \lambda I| = \begin{vmatrix} \alpha_1 - 2\beta_1 x_* - \gamma_1 y_* + \varepsilon_1 z_* - \lambda & -\gamma_1 x_* & \varepsilon_1 x_* \\ -\gamma_2 y_* & \alpha_2 - 2\beta_2 y_* - \gamma_2 x_* + \varepsilon_2 z_* - \lambda & \varepsilon_2 y_* \\ -\varepsilon_1 z_* & -\varepsilon_2 z_* & \alpha_3 - 2\beta_3 z_* - \varepsilon_1 x_* - \varepsilon_2 y_* - \lambda \end{vmatrix} = 0. \quad (8)$$

For the first trivial singular point the characteristic equation (8) takes the form :

$$(\alpha_1-\lambda)(\alpha_2-\lambda)(\alpha_3-\lambda)=0,$$

and therefore , the point is unstable node.

For the second singular point the equation (8) takes the form :

$$|A-\lambda I|=\begin{vmatrix} -\alpha_1-\lambda & -\gamma_1\dfrac{\alpha_1}{\beta_1} & \varepsilon_1\dfrac{\alpha_1}{\beta_1} \\ 0 & \alpha_2-\gamma_2\dfrac{\alpha_1}{\beta_1}-\lambda & 0 \\ 0 & 0 & \alpha_3-\varepsilon_1\dfrac{\alpha_1}{\beta_1}-\lambda \end{vmatrix}=$$

$$(-\alpha_1-\lambda)\left(\alpha_2-\frac{\gamma_2\alpha_1}{\beta_1}-\lambda\right)\left(\alpha_3-\frac{\varepsilon_1\alpha_1}{\beta_1}-\lambda\right)=0.$$

A stable node occurs here, if

$$\alpha_2\beta_1-\gamma_2\alpha_1<0,\alpha_3\beta_1-\varepsilon_1\alpha_1<0,$$

and in other cases we come to an unstable saddle point. A similar situation occurs for the third singular point, which is a stable node at $\alpha_1\beta_2-\gamma_1\alpha_2<0$, $\alpha_3\beta_2-\varepsilon_2\alpha_2<0$.

For the fourth singular point the equation (8) reduces to:

$$\left(\alpha_1+\frac{\varepsilon_1\alpha_3}{\beta_3}-\lambda\right)\left(\alpha_2+\frac{\varepsilon_2\alpha_3}{\beta_3}-\lambda\right)(-\alpha_3-\lambda)=0$$

and therefore , this point is a saddle .

For the fifth singular point the equation (8) reduces to:

$$(\alpha_1-\gamma_1y_*+\varepsilon_1z_*-\lambda)\left[\lambda^2+\lambda(-\alpha_2-\alpha_3+2\beta_3z_*+2\beta_2y_*+\varepsilon_2y_*-\varepsilon_2z_*)+\alpha_2\alpha_3-2\beta_2\alpha_3y_*+\right.$$
$$\left.+\alpha_3\varepsilon_2z_*-2\beta_3\alpha_2z_*+4\beta_2\beta_3y_*z_*-2\beta_3\varepsilon_2z_*^2-\varepsilon_2\alpha_2y_*+2\beta_2\varepsilon_2y_*^2\right]=0. \quad (9)$$

Consider the simple case when $\alpha_i=\alpha$, $\beta_i=\beta$, $\gamma_i=\gamma$, $\varepsilon_i=\varepsilon$, then equation (9) reduces to:

$$\left[\frac{\alpha(\beta^2-\gamma\beta-\gamma\varepsilon+\varepsilon\beta)}{\beta^2+\varepsilon^2}-\lambda\right]\left[\lambda^2+\frac{2\alpha\beta^2}{\beta^2+\varepsilon^2}\lambda+\frac{\alpha^2(\beta^4+\varepsilon^4-2\beta\varepsilon^3)}{(\beta^2+\varepsilon^2)^2}\right]=0, \quad (10)$$

wherein $y_*=\dfrac{\alpha(\beta+\varepsilon)}{\beta^2+\varepsilon^2}$, $z_*=\dfrac{\alpha(\beta-\varepsilon)}{\beta^2+\varepsilon^2}>0$ when $\beta>\varepsilon$.

Of the cubic equation (10) we define the eigenvalues:

$$\lambda_1 = \frac{\alpha\left(\beta^2 - \gamma\beta - \gamma\varepsilon + \varepsilon\beta\right)}{\beta^2 + \varepsilon^2}, \ \lambda_{2,3} = -\frac{\alpha}{\beta^2 + \varepsilon^2}\left[\beta \pm \varepsilon\sqrt{\varepsilon(2\beta - \varepsilon)}\right]. \quad (11)$$

One can show that the first eigenvalue is negative while satisfying the following inequality:

$$\lambda_1 < 0 \Leftrightarrow 0 < \beta < -\frac{(\varepsilon - \gamma)}{2} + \sqrt{\frac{(\varepsilon - \gamma)^2}{4} + \varepsilon\gamma}$$

Next λ_2 (plus sign in the expression (11)) is real and negative, if $2\beta \geq \varepsilon$, λ_3 is real and negative when $\beta > \varepsilon$.

Thus, the conditions of positivity of the singular points coordinates and negativity of the eigenvalues requires the carrying out of the inequalities

$$0 < \beta < -\frac{(\varepsilon - \gamma)}{2} + \sqrt{\frac{(\varepsilon - \gamma)^2}{4} + \varepsilon\gamma}, \ \beta > \varepsilon. \quad (12)$$

In this case, the singular point $(0, y_*, z_*)$ is a stable node.

For the sixth singular point equation (8) reduces to:

$$\left(\alpha_2 - \gamma_2 x_* + \varepsilon_2 z_* - \lambda\right)\left[\lambda^2 + \lambda\left(-\alpha_1 - \alpha_3 + 2\beta_1 x_* + 2\beta_3 z_* + \varepsilon_1 x_* - \varepsilon_2 z_*\right) + \alpha_1\alpha_3 - 2\alpha_1\beta_3 z_* - \right.$$
$$\left. -\alpha_1\varepsilon_1 x_* - 2\beta_1\alpha_3 x_* + 4\beta_1\beta_3 x_* z_* + 2\beta_1\varepsilon_1 x_*^2 + \alpha_3\varepsilon_1 z_* - 2\beta_3\varepsilon_1 z_*^2 - \varepsilon_1^2 z_* x_* + \varepsilon_1^2 z_*^2\right] = 0. \quad (13)$$

Simplified case $\alpha_i = \alpha$, $\beta_i = \beta$, $\gamma_i = \gamma$, $\varepsilon_i = \varepsilon$, $x_* = \dfrac{\alpha(\beta + \varepsilon)}{\beta^2 + \varepsilon^2}$, $z_* = \dfrac{\alpha(\beta - \varepsilon)}{\beta^2 + \varepsilon^2}$ leads to the same results , which were obtained for the fifth singular point.

For the seventh singular point equation (8) reduces to:

$$\left(\alpha_3 - \varepsilon_1 x_* - \varepsilon_2 y_* - \lambda\right)\left[\lambda^2 + \lambda\left(-\alpha_1 - \alpha_2 + 2\beta_1 x_* + 2\beta_2 y_* + \gamma_1 y_* + \gamma_2 x_*\right) + \alpha_1\alpha_2 - 2\alpha_1\beta_2 y_* - \right.$$
$$\left. -\alpha_1\gamma_2 x_* - 2\beta_1\alpha_2 x_* + 4\beta_1\beta_2 x_* y_* + 2\beta_1\gamma_2 x_*^2 - \gamma_1\alpha_2 y_* + 2\beta_2\gamma_1 y_*^2\right] = 0. \quad (14)$$

Simplified case $\alpha_i = \alpha$, $\beta_i = \beta$, $\gamma_i = \gamma$, $\varepsilon_i = \varepsilon$, $x_* = y_* = \dfrac{\alpha}{\beta + \gamma}$, where $\beta \neq \gamma$, leads to the following cubic equation:

$$\left[\alpha\left(\frac{\beta + \gamma - 2\varepsilon}{\beta + \gamma}\right) - \lambda\right]\left[\lambda^2 + \left(\frac{2\alpha\beta}{\beta + \gamma}\right)\lambda + \frac{\alpha^2(\beta - \gamma)}{(\beta + \gamma)}\right] = 0. \quad (15)$$

The solution of this equation is:

$$\lambda_1 = \alpha\left(\frac{\beta + \gamma - 2\varepsilon}{\beta + \gamma}\right), \ \lambda_2 = \frac{\alpha(\gamma - \beta)}{\beta + \gamma}, \ \lambda_3 = -\alpha. \quad (16)$$

Negative eigenvalues λ_i and, consequently, a stable node for the seventh singular point will take place when the following restrictions on the parameters of the model:

$$\frac{2\varepsilon}{\beta+\gamma}>1,\ \gamma<\beta. \tag{17}$$

In case $\beta=\gamma$ we have $x_*+y_*=\frac{\alpha}{\beta}$, $\lambda_1=\alpha-\varepsilon(x_*+y_*)=\alpha\left(1-\frac{\varepsilon}{\beta}\right)$, $\lambda_2=0$, $\lambda_3=-\frac{2\alpha\beta}{\beta+\gamma}<0$.

Thus, in this case when $\varepsilon\geq\beta$ we have a stable node.

For the eighth nontrivial singular point equation (8) we are to consider for the simplified case $\alpha_i=\alpha$, $\beta_i=\beta$, $\gamma_i=\gamma$, $\varepsilon_i=\varepsilon$, when

$$x_*=y_*=\frac{\alpha\left(\beta^2-\gamma\beta-\varepsilon\gamma+\beta\varepsilon\right)}{\beta^3+2\beta\varepsilon^2-2\gamma\varepsilon^2-\gamma^2\beta},\ z_*=\frac{\alpha\left(\beta^2-2\beta\varepsilon+2\varepsilon\gamma-\gamma^2\right)}{\beta^3+2\beta\varepsilon^2-2\gamma\varepsilon^2-\gamma^2\beta}. \tag{18}$$

After a fairly cumbersome transformations it has been reduced to a cubic equation

$$\lambda^3-(2A+B)\lambda^2+\left[A^2\left(1-\frac{\gamma^2}{\beta^2}\right)+2AB\left(1+\frac{\varepsilon^2}{\beta^2}\right)\right]\lambda+A^2B\left(\frac{2\gamma\varepsilon^2}{\beta^3}+\frac{\gamma^2}{\beta^2}-\frac{\varepsilon^2}{\beta^2}-1\right)-AB^2\frac{\varepsilon^2}{\beta^2}=0, \tag{19}$$

where

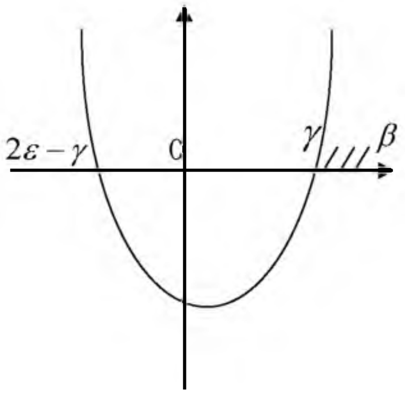
$$A=-\beta x_*=\frac{\alpha\beta\left(-\beta^2+\gamma\beta+\varepsilon\gamma-\varepsilon\beta\right)}{\beta^3+2\beta\varepsilon^2-2\gamma\varepsilon^2-\gamma^2\beta},\ B=-\beta z_*=\frac{\alpha\beta\left(-\beta^2+\gamma^2+2\beta\varepsilon-2\varepsilon\gamma\right)}{\beta^3+2\beta\varepsilon^2-2\gamma\varepsilon^2-\gamma^2\beta}.$$

We obtain conditions of singular points coordinates positivity. If $\beta>\gamma$ it follows automatically that $x_*>0$, $y_*>0$. To fulfill the conditions $z_*>0$ it is necessary to satisfy the system of inequalities

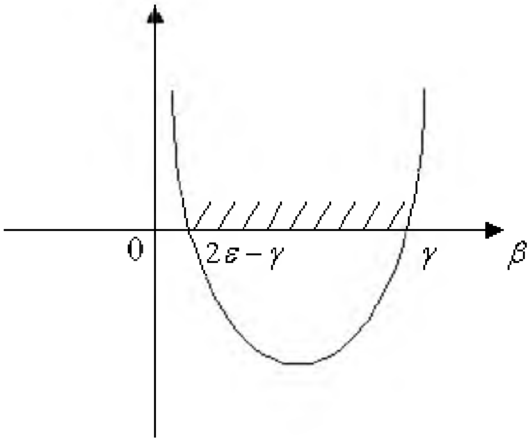
$$\begin{cases}\beta^2-2\beta\varepsilon+2\varepsilon\gamma-\gamma^2>0\\ \beta>\gamma\end{cases}. \tag{20}$$

The roots of a quadratic equation $\beta^2-2\beta\varepsilon+2\varepsilon\gamma-\gamma^2=0$ have the form $\beta_1=2\varepsilon-\gamma$, $\beta_2=\gamma$ when $\varepsilon<\gamma$ and $\beta_1=\gamma$, $\beta_2=2\varepsilon-\gamma$ when $\varepsilon>\gamma$. To solve this system of inequality it is necessary to consider three cases:

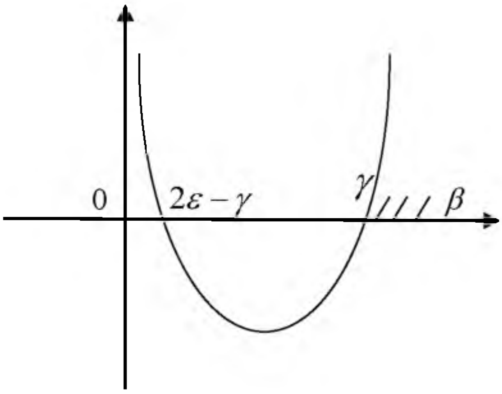
1. $2\varepsilon-\gamma<0$



2. $0<\gamma<2\varepsilon-\gamma \Leftrightarrow 0<\gamma<\varepsilon$



3. $0<2\varepsilon-\gamma<\gamma \Leftrightarrow 0<\varepsilon<\gamma$



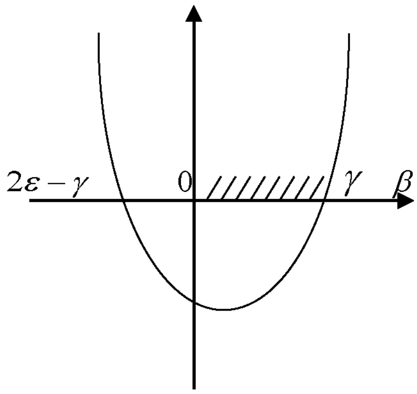
Thus, the first and third case system of inequality (20) has the solution $\beta>\gamma$, in the second - $\beta>2\varepsilon-\gamma$.

If $0<\beta<\gamma$ it follows automatically that $x_*>0, y_*>0$. To fulfill the conditions $z_*>0$ it is necessary to satisfy the system of inequalities

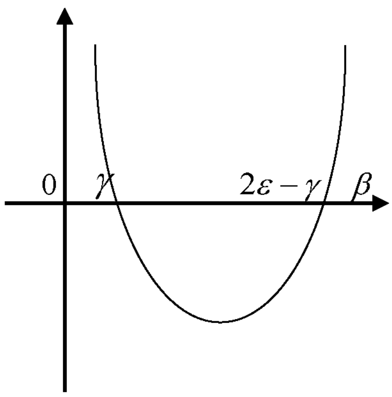
$$\begin{cases} \beta^2-2\beta\varepsilon+2\varepsilon\gamma-\gamma^2<0 \\ 0<\beta<\gamma \end{cases} \quad (21)$$

As before, to solve this system of inequality it is necessary to consider three cases:

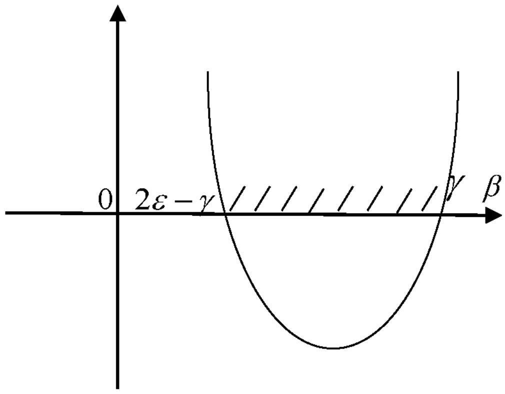
1. $2\varepsilon - \gamma < 0$



2. $0 < \gamma < 2\varepsilon - \gamma \Leftrightarrow 0 < \gamma < \varepsilon$



3. $0 < 2\varepsilon - \gamma < \gamma \Leftrightarrow 0 < \varepsilon < \gamma$



That way, in the first instance system of inequality (21) possesses a solution $0 < \beta < \gamma$, in the second case there in no solution, in the third case this inequality possesses a solution $2\varepsilon - \gamma < \beta < \gamma$.

The stability of eight singular point of simplified dynamic system will be revealed, when real root parts of cubic equation (19) are negative. According to the Routh - Hurwitz conditions, in order that roots of arbitrary cubic equation

$$a_0\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 \tag{22}$$

with a real coefficients have negative real parts, it's essential and sufficient that all main diagonal minor determinants of Hurwitz matrix for equation (22)

$$G = \begin{pmatrix} a_1 & a_3 & 0 \\ a_0 & a_2 & 0 \\ 0 & a_1 & a_3 \end{pmatrix} \tag{23}$$

are positive.

$$\begin{vmatrix} a_2 & 0 \\ a_1 & a_3 \end{vmatrix} = a_2 a_3 > 0, \begin{vmatrix} a_1 & 0 \\ 0 & a_3 \end{vmatrix} = a_1 a_3 > 0, \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix} = a_1 a_2 - a_0 a_3 > 0. \tag{24}$$

For our actual equation (19) the conditions (24) will be:

$$\begin{cases} \left[A^2\left(1-\frac{\gamma^2}{\beta^2}\right)+2AB\left(1+\frac{\varepsilon^2}{\beta^2}\right)\right]\left[A^2B\left(\frac{2\gamma\varepsilon^2}{\beta^3}+\frac{\gamma^2}{\beta^2}-\frac{\varepsilon^2}{\beta^2}-1\right)-AB^2\frac{\varepsilon^2}{\beta^2}\right]>0 \\ -(2A+B)\left[A^2B\left(\frac{2\gamma\varepsilon^2}{\beta^3}+\frac{\gamma^2}{\beta^2}-\frac{\varepsilon^2}{\beta^2}-1\right)-AB^2\frac{\varepsilon^2}{\beta^2}\right]>0 \\ -(2A+B)\left[A^2\left(1-\frac{\gamma^2}{\beta^2}\right)+2AB\left(1+\frac{\varepsilon^2}{\beta^2}\right)\right]-A^2B\left(\frac{2\gamma\varepsilon^2}{\beta^3}+\frac{\gamma^2}{\beta^2}-\frac{\varepsilon^2}{\beta^2}-1\right)+AB^2\frac{\varepsilon^2}{\beta^2}>0. \end{cases} \tag{25}$$

Let's analyze the case $\beta > \gamma$, if the first factor of first system inequality (25) is positive, as $A < 0, B < 0$. And thus, the second factor of this inequality has to be positive. It follows that second inequality is fulfilled by itself. The third inequality transforms to the form:

$$2A^3\left(\frac{\gamma^2}{\beta^2}-1\right)-4A^2B-3A^2B\frac{\varepsilon^2}{\beta^2}-2AB^2-\frac{AB^2\varepsilon^2}{\beta^2}-\frac{2A^2B\gamma\varepsilon^2}{\beta^3}>0$$

it follows that, for the same conditions $(\beta > \gamma,\ A < 0,\ B < 0)$ is fulfilled by itself.

By this means, all roots of cubic equation (19) will have negative and real components as:

$$A^2B\left(\frac{2\gamma\varepsilon^2}{\beta^3}+\frac{\gamma^2}{\beta^2}-\frac{\varepsilon^2}{\beta^2}-1\right)-AB^2\frac{\varepsilon^2}{\beta^2}>0 \Longleftrightarrow \frac{2\gamma}{\beta}-1+\left(\frac{\gamma^2-\beta^2}{\varepsilon^2}\right)>\frac{B}{A}=\frac{z_*}{x_*}. \tag{26}$$

For the case $\beta > 2\gamma$ this inequality admittedly is not fulfilled as its left part is becoming negative. Therefore the stable node or stable focus for the eighth singular point will be in the next changing parameters fields of shortcut dynamic system (5):

$$\begin{cases} 2\gamma / \beta + (\gamma^2 - \beta^2)/\varepsilon^2 - 1 > B / A \\ \gamma < \beta < 2\gamma \\ 2\varepsilon - \gamma < 0 \end{cases} \tag{27}$$

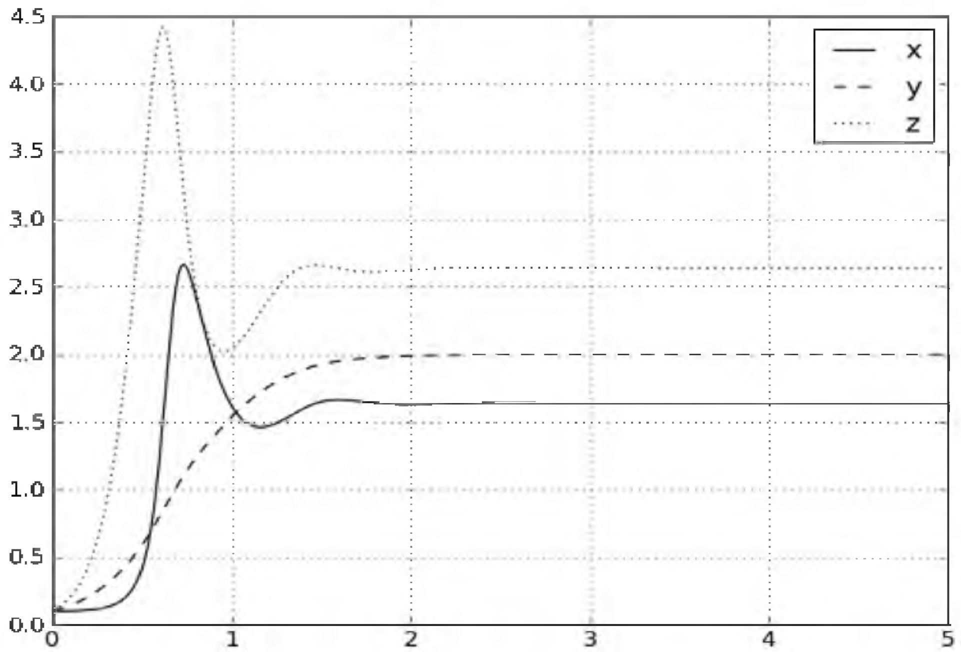
The analytic study of other cases is quite difficult and will not be considered. Singular case for $\beta = \gamma$ the leads to the nonexistence of solution of algebraic system (6) for the determination of singular point coordinates.

Therefore qualitative assay of dynamic system (5) shows that there are two regimes of its behavior with full suppression of one university by other with full exhausting of stationary applicants reserve. (singular points 2 and 3), two similar regimes if there is stationary applicants reserve (singular points 5 and 6), two regimes of mutual coexistence of universities with presence (singular points 8) or exhaustion (singular points 7) of stationary applicants reserve, two unstable regimes of educational system's behavior (singular points 1 and 4).It should be noted that for the singular points 5, 6 and 8 there are situations when $z_* = 0$ (exhaustion of stationary applicants reserve).

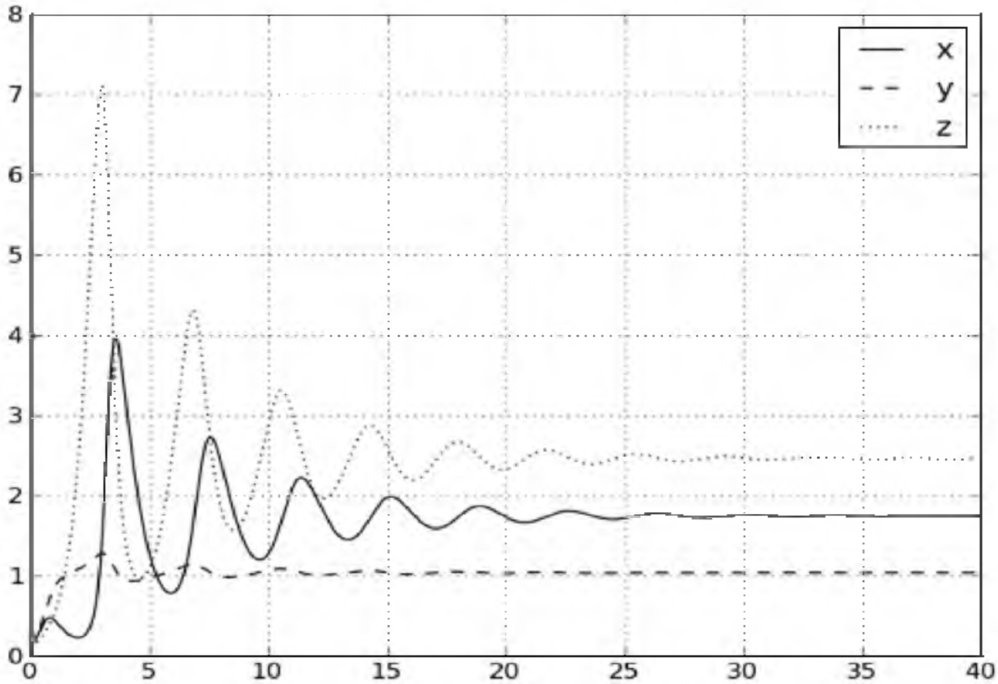
Other, more complicated regimes of dynamic system's behavior (5) (for example self-oscillations), might be detected in the result of numerical experiments.

Numerical experiments

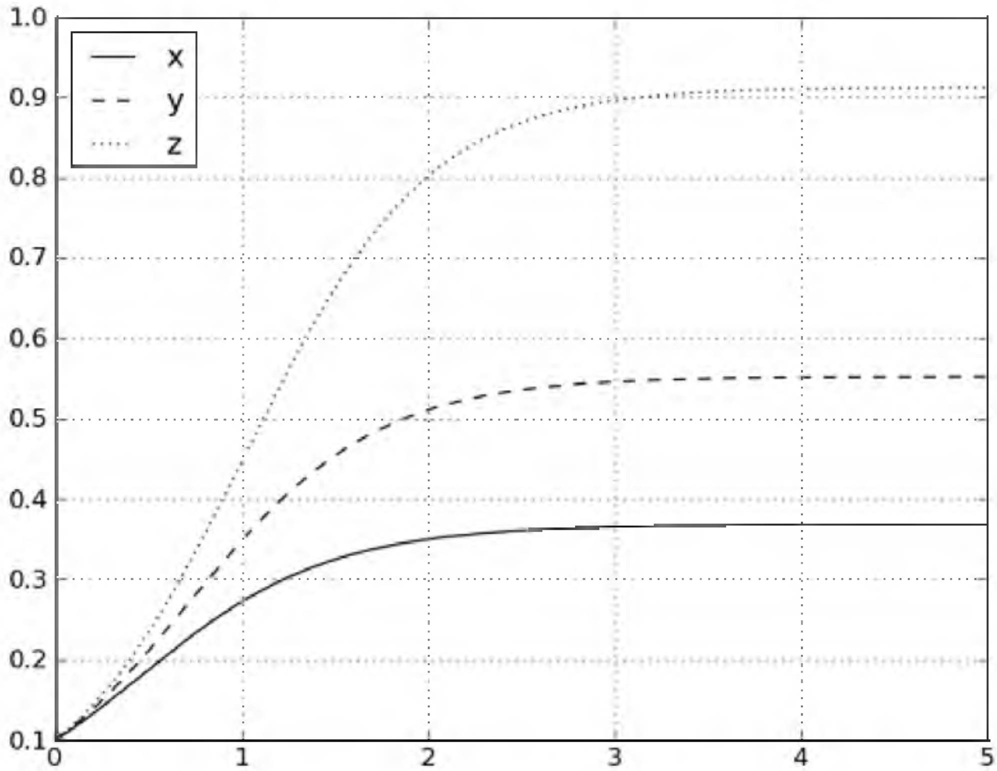
The examples of numerical experiments with model (5) are shown in figure 1



Variant 1. Model parameters and calculated coordinates of nontrivial singular point: $\alpha_1 = 0.2, \alpha_2 = 4.0$, $\alpha_3 = 8.0$, $\beta_1 = 4.0$, $\beta_2 = 2.1$, $\beta_3 = 0.4$,$\gamma_1 = 2.1$, $\gamma_1 = 0.2$,$\varepsilon_1 = 4.0, \varepsilon_2 = 0.2$, $x_* = 1.64$, $y_* = 2.0$, $z_* = 2.64$



Variant 2. Model parameters and calculated coordinates of nontrivial singular point: $\alpha_1 = 4.0$, $\alpha_2 = 4.0$, $\alpha_3 = 2.1$, $\beta_1 = 0.2$, $\beta_2 = 4.0$, $\beta_3 = 0.06$,$\gamma_1 = 5.9$, $\gamma_2 = 0.2$,$\varepsilon_1 = 1.0, \varepsilon_2 = 0.2$, $x_* = 1.74$, $y_* = 1.04$, $z_* = 2.46$



Variant 3. Model parameters and calculated coordinates of nontrivial singular point: $\alpha_1 = 2.1$, $\alpha_2 = 2.1$,$\alpha_3 = 2.1$, $\beta_1 = 5.9$, $\beta_2 = 4.0$, $\beta_3 = 2.1$,$\gamma_1 = 0.2$, $\gamma_2 = 0.2$,$\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.2$, $x*$ =0.37, $y*$= 0.56, $z*$ =0.91

Figure 1. The result of numerical experiments according to the model (5)

The numerical experiments show that system's behavior seems the type of stable focus in the first two variations, and the third variation – by type of stable node.

Conclusion

Dynamic system (5) can be easily spreaded to $n+1$ - dimensional case (n of universities, competing for the limited applicants' contingent). In this case it is as follows:

$$\begin{cases} \dfrac{dx_1}{dt} = \alpha_1 x_1 - \beta_1 x_1^2 - \gamma_{12} x_1 x_2 - \ldots - \gamma_{1i} x_1 x_i - \ldots - \gamma_{1n} x_1 x_n + \varepsilon_1 x_1 z \\ \dfrac{dx_2}{dt} = \alpha_2 x_2 - \beta_2 x_2^2 - \gamma_{21} x_2 x_1 - \ldots - \gamma_{2i} x_2 x_i - \ldots - \gamma_{2n} x_2 x_n + \varepsilon_2 x_2 z \\ \vdots \\ \dfrac{dx_i}{dt} = \alpha_i x_i - \beta_i x_i^2 - \gamma_{i1} x_i x_1 - \ldots - \gamma_{ij} x_i x_j - \ldots - \gamma_{in} x_i x_n + \varepsilon_i x_i z \\ \vdots \\ \dfrac{dx_n}{dt} = \alpha_n x_n - \beta_n x_n^2 - \gamma_{n1} x_n x_1 - \ldots - \gamma_{ni} x_n x_i - \ldots - \gamma_{n,n-1} x_n x_{n-1} + \varepsilon_n x_n z \\ \dfrac{dz}{dt} = \alpha_{n+1} z - \beta_{n+1} z^2 - \varepsilon_1 x_1 z - \ldots - \varepsilon_i x_i z - \ldots - \varepsilon_n x_n z \end{cases} \quad (28)$$

Singular points of dynamic system (28) are defined from the solution of algebraic equation system

$$\begin{cases} x_1 \left(\alpha_1 - \beta_1 x_1 - \gamma_{12} x_2 - \ldots - \gamma_{1i} x_i - \ldots - \gamma_{1n} x_n + \varepsilon_1 z \right) = 0 \\ x_2 \left(\alpha_2 - \beta_2 x_2 - \gamma_{21} x_1 - \ldots - \gamma_{2i} x_i - \ldots - \gamma_{2n} x_n + \varepsilon_2 z \right) = 0 \\ \vdots \\ x_i \left(\alpha_i - \beta_i x_i - \gamma_{i1} x_1 - \ldots - \gamma_{ij} x_j - \ldots - \gamma_{in} x_n + \varepsilon_i z \right) = 0 \\ \vdots \\ x_n \left(\alpha_n - \beta_n x_n - \gamma_{n1} x_1 - \ldots - \gamma_{ni} x_i - \ldots - \gamma_{n,n-1} x_{n-1} + \varepsilon_n z \right) = 0 \\ z \left(\alpha_{n+1} - \beta_{n+1} z - \varepsilon_1 x_1 - \ldots - \varepsilon_i x_i - \ldots - \varepsilon_n x_n \right) = 0 \end{cases} \quad (29)$$

The general amount of singular points with different combination of zero and nonzero elements is equal to 2^{n+1} [6]. By analogy with analysis of 3 - dimensional dynamic system, there are different suppression regimes of ones universities by others. Different university coalitions might appear which will suppress others with time. Let us assume that, five one-profile universities compete with each other in educational market, than, for example, coalition of three universities, there is a possibility to form (three combinations of five): $C_5^3 = 10$.

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