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Mathematical Modeling of Training and Dynamics of Scientific Personnel

Vladimir M. Moskovkin[1], Bilal N.E. Suleiman[1], Ruslan V. Lesovik[2], Evgeniy I. Evtushenko[2], Valeriy A. Kuznetsov [2]

[1]Belgorod State University, Russia, 308015, Belgorod, Pobedy str., 85
e-mail: moskovkin@bsu.edu.ru
[2] Belgorod State Technological University, Russia, 308011, Belgorod, Kostyukov str., 46

Abstract

Qualitative and numerical analysis of mathematical model of training and dynamics of scientific personnel, which is a dynamic system of third order, describing the interaction of scientific personnel without degrees, candidates and doctors of sciences were done.

In the absence of intra-group competition, and when the doctors of sciences prepared only with the consultants, who are doctors sciences, a stability analysis of non-trivial singular point of the dynamical system by using the Routh-Hurwitz criteria were carried out. Numerical experiments carried out with the model shown on the compliance obtained analytically Routh-Hurwitz criteria.

Key words: equation of population dynamics, dynamics of scientific personnel, dynamic third-order system, Routh-Hurwitz criteria, singular points, linear stability analysis

Introduction

Currently great attention is paid to mathematic modeling of the education and research processes. Among the most fundamental research we would like to mention the works [1-4]. In those works the above-mentioned processes are simulated by means of the difference and ordinary differential equations systems as well as using the dynamic optimization methods.

The use of the population dynamics equations [5-7] also seems to be promising in terms of the education and research processes and systems [8–10].

By construction of the mathematic model of preparation and dynamics of the scientific personnel it is conveniently to rely upon the accounting phenomenological equations and act like by construction of the population dynamics equations.

Model construction

In the study [8] the model of preparation and dynamics of the scientific personnel in terms of the population dynamics equations has been suggested

$$\begin{cases} \dfrac{dx}{dt} = \alpha_1 x - \beta_1 xy - \gamma_1 xz - \varepsilon_1 x^2 \\ \dfrac{dy}{dt} = -\alpha_2 y - \gamma_2 y + \beta_1 xy + \gamma_1 xz - \beta_2 yz - \varepsilon_2 y^2, \\ \dfrac{dz}{dt} = -\alpha_3 z + \beta_2 yz + \gamma_2 y - \varepsilon_3 z^2. \end{cases} \quad (1)$$

where x – number of the scientific workers without academic degree, y – number of candidates of sciences, z – number of doctors of sciences.

This model describes the processes of the scientific manpower reproduction, irreversible withdrawal as well as transitions from one category to another. Here $\alpha_1 x$ means reproduction of the scientific personnel without academic degree (the difference between their preparation and withdrawal is not related to transition to the category of candidates of sciences per unit time), $\beta_1 xy$ – rate of preparation of the candidates of sciences from amongst the non-degree scientific personnel (x) by the candidates of sciences (y), $\gamma_1 xz$ – rate of preparation of the candidates of sciences from amongst the non-degree scientific personnel (x) by the doctors of sciences (z), $\alpha_2 y$ – rate of withdrawal by the candidates of sciences from the scientific personnel not related to transition to the category of doctors of sciences (withdrawal as the result of death, intellectual migration, transfer to another area of activities), $\gamma_2 y$ – rate of self-preparation of the candidates of sciences up to the level of doctors of sciences, $\beta_2 yz$ – rate of preparation of the doctors of sciences from amongst the candidates of sciences (y) by the doctors of sciences (z), $\alpha_3 z$ – rate of withdrawal by the doctors of sciences from the scientific personnel; $\varepsilon_1 x^2$, $\varepsilon_2 y^2$, $\varepsilon_3 z^2$ – terms describing the intra-group competition within the relevant categories (standard terms of the population dynamics equations that are responsible for the growth self-restriction).

In the model (1) it was assumed that the candidates of sciences are trained only under the academic guidance on the part of candidates (term – $\beta_1 xy$) and doctors (term – $\gamma_1 xz$) of sciences, and the doctors of sciences are trained on their own (term – $\gamma_2 y$) or through an academic adviser – doctor of sciences (term – $\beta_2 yz$).

The specified model holds for the post-Soviet system of scientific personnel training though it can be easily adjusted to the three-level system – bachelor, master and PhD, which are characterized by a high degree of education.

In the study [8] for the sake of the analysis simplicity there was considered the case when $\gamma_2 = 0$ (corresponds to the present-day practice when the doctors of sciences are trained exclusively with the participation of the academic advisers being the doctors of sciences) and $\varepsilon_i = 0$ (absence of the intra-group competition). The singular dynamic system points were obtained within the frameworks of this study (1):

$$x^* = y^* = z^* = 0$$

$$x^* = \frac{\alpha_2}{\beta_1}, y^* = \frac{\alpha_1}{\beta_1}, z^* = 0$$

$$x^* = \frac{\alpha_3(\alpha_1\beta_2 - \alpha_3\beta_1 + \alpha_2\gamma_1)}{\alpha_1\beta_2\gamma_1}, y^* = \frac{\alpha_3}{\beta_2}, z^* = \frac{\alpha_1\beta_2 - \alpha_3\beta_1}{\beta_2\gamma_1}.$$

And the linear analysis of the stability of the first two singular points was conducted. It was shown that the first singular point had an unstable form (saddle) and the second at $\alpha_1\beta_2 - \alpha_3\beta_1 > 0$ – was an unstable focus point, at $\alpha_1\beta_2 - \alpha_3\beta_1 < 0$ – a stable focus point, at $\alpha_1\beta_2 - \alpha_3\beta_1 = 0$ – the center (here no bifurcation of cycle generation occurs since $det\,A = 0$, A – Jacobian matrix of the linear system (1)).

In the study [8] the matrix A and the characteristic equation $|A - \lambda I| = 0$ were written out for the third singular point:

$$A = \begin{pmatrix} 0 & \frac{M\beta_1}{\gamma_1} & M \\ \alpha_1 & N & \frac{\alpha_3(\alpha_2\gamma_1 - \alpha_3\beta_1)}{\alpha_1\beta_2} \\ 0 & \frac{\alpha_1\beta_2 - \alpha_3\beta_1}{\gamma_1} & 0 \end{pmatrix}, \quad (2)$$

$$|A - \lambda I| = \frac{(\alpha_3\beta_1 - \alpha_1\beta_2)}{\gamma_1}\left[\frac{(\alpha_3\beta_1 - \alpha_2\gamma_1)\alpha_3}{\alpha_1\beta_2}\lambda - \alpha_1 M\right] - \lambda\left[(\lambda - N)\lambda - \frac{\alpha_1\beta_1 M}{\gamma_1}\right] = 0 \quad (3)$$

where $M = \frac{-\alpha_3(\alpha_1\beta_2 - \alpha_3\beta_1 + \alpha_2\gamma_1)}{\alpha_1\beta_2}$,

$$N = \frac{\alpha_3\beta_1(2\alpha_1\beta_2 + \alpha_2\gamma_1) - \alpha_3^2\beta_1^2 - \alpha_1^2\beta_2^2}{\alpha_1\beta_2\gamma_1} - \alpha_2.$$

In the same study, since $\alpha_1\beta_2 - \alpha_3\beta_1 > 0 <=> Z^* > 0$ and $M < 0$, it was also shown that:

$$detA = \frac{(\alpha_1\beta_2 - \alpha_3\beta_1)\alpha_1 M}{\gamma_1} < 0.$$

Qualitative study of the nontrivial singular model point and numerical experiments

It can be shown that the third (nontrivial) singular point becomes nonphysical ($z^* < 0$) if the second singular point is the stable focus. As noted above, this happens at $\alpha_1\beta_2 - \alpha_3\beta_1 < 0$. We show next $\alpha_1\beta_2 - \alpha_3\beta_1 = 0$ that at the third (nontrivial) singular point degenerates into the second one. Indeed, according to this equation the coordinates of the third singular point go into the coordinates of the second singular point:

$$x^* = \frac{\alpha_3\alpha_2\gamma_1}{\alpha_1\beta_2\gamma_1} = \frac{\alpha_3\alpha_2}{\alpha_1\beta_2} = \left(\frac{\alpha_1\beta_2}{\beta_1}\right)\frac{\alpha_2}{\alpha_1\beta_2} = \frac{\alpha_2}{\beta_1};$$

$$y^* = \frac{\alpha_3}{\beta_2} = \left(\frac{\alpha_1\beta_2}{\beta_1}\right)\frac{1}{\beta_2} = \frac{\alpha_1}{\beta_1}; z^* = 0.$$

Let us put the above-mentioned criteria equation as follows: $\frac{\alpha_1}{\alpha_3} = \frac{\beta_1}{\beta_2}$.

It states that the ratio between the rate of reproduction of the scientific personnel without academic degree (α_1) and rate of withdrawal of the doctors of sciences from the scientific personnel (α_3) equals to the ratio between the rate of training of the candidates of sciences from amongst the non-degree scientific personnel by the candidates of sciences (β_1) and rate of training of the doctors of sciences from amongst the candidates of sciences by the doctors of sciences (β_2). Given that the number of the non-degree scientific personnel exceeds the number of the candidates of sciences the inequality $\alpha_2 > \alpha_1$ shall be assumed for the position of the second singular point.

Within our study we will conduct the analysis of stability of the third (nontrivial) singular point of the dynamic system under consideration with the use of the Routh-Hurwitz criteria. To do this we will reduce the characteristic equation (3) to the standard cubic equation and expand the expression into factors for N:

$$\lambda^3 - N\lambda^2 - \left[\frac{\alpha_1\beta_1 M}{\gamma_1} + \frac{(\alpha_3\beta_1 - \alpha_1\beta_2)(\alpha_3\beta_1 - \alpha_2\gamma_1)\alpha_3}{\alpha_1\beta_2\gamma_1}\right]\lambda + \frac{(\alpha_3\beta_1 - \alpha_1\beta_2)\alpha_1 M}{\gamma_1} = 0 \qquad (4)$$

$$N = \frac{(\alpha_3\beta_1 - \alpha_1\beta_2)(\alpha_2\gamma_1 - \alpha_3\beta_1 + \alpha_1\beta_2)}{\alpha_1\beta_2\gamma_1}. \qquad (5)$$

The cubic equation (4) coefficients are reduced to the notations used by writing-out the Routh-Hurwitz criteria:

$$\alpha_0 = 1, \alpha_1 = -N, \alpha_2 = -\frac{\alpha_1\beta_1 M}{\gamma_1} - \frac{(\alpha_3\beta_1 - \alpha_1\beta_2)(\alpha_3\beta_1 - \alpha_2\gamma_1)\alpha_3}{\alpha_1\beta_2\gamma_1},$$

$$\alpha_3 = \frac{(\alpha_3\beta_1 - \alpha_1\beta_2)\alpha_1 M}{\gamma_1}.$$

The criteria themselves appear as follows:

$$\begin{cases} \alpha_1\alpha_3 > 0 <=> \dfrac{(-N)(\alpha_3\beta_1 - \alpha_1\beta_2)\alpha_1 M}{\gamma_1} > 0, \\ \alpha_2\alpha_3 > 0 <=> \left[-\dfrac{\alpha_1\beta_1 M}{\gamma_1} - \dfrac{(\alpha_3\beta_1 - \alpha_1\beta_2)(\alpha_3\beta_1 - \alpha_2\gamma_1)\alpha_3}{\alpha_1\beta_2\gamma_1}\right] * \\ * \dfrac{(\alpha_3\beta_1 - \alpha_1\beta_2)\alpha_1 M}{\gamma_1} = \dfrac{\alpha_3}{\alpha_1\beta_2\gamma_1} * \\ * [\alpha_1\beta_1(\alpha_1\beta_2 - \alpha_3\beta_1) + \alpha_1\alpha_2\beta_1\gamma_1 + (\alpha_1\beta_2 - \alpha_3\beta_1)(\alpha_3\beta_1 - \alpha_2\gamma_1)] > 0, \\ \alpha_1\alpha_2 - \alpha_0\alpha_3 > 0 <=> \dfrac{M}{\gamma_1^2\beta_2}(\alpha_3\beta_1 - \alpha_1\beta_2) * \\ * [\gamma_1(\alpha_2\beta_1 - \alpha_1\beta_2) + \beta_1(\alpha_1\beta_2 - \alpha_3\beta_1)] > 0. \end{cases} \quad (6)$$

It follows from the expression for N (5) at the base condition $z^* > 0 <=> \alpha_1\beta_2 - \alpha_3\beta_1 > 0$ that $N < 0$. Thus, the first criterion is always satisfied.

It follows from the Routh-Hurwitz criteria that the stability of the nontrivial singular point within our model will be held in the following system of restrictions on its parameters:

$$\begin{cases} \alpha_1\beta_2 - \alpha_3\beta_1 > 0, \\ \alpha_1\beta_1(\alpha_1\beta_2 - \alpha_3\beta_1) + \alpha_1\alpha_2\beta_1\gamma_1 + (\alpha_1\beta_2 - \alpha_3\beta_1)(\alpha_3\beta_1 - \alpha_2\gamma_1) > 0, \\ \gamma_1(\alpha_2\beta_1 - \alpha_1\beta_2) + \beta_1(\alpha_1\beta_2 - \alpha_3\beta_1) > 0. \end{cases} \quad (7)$$

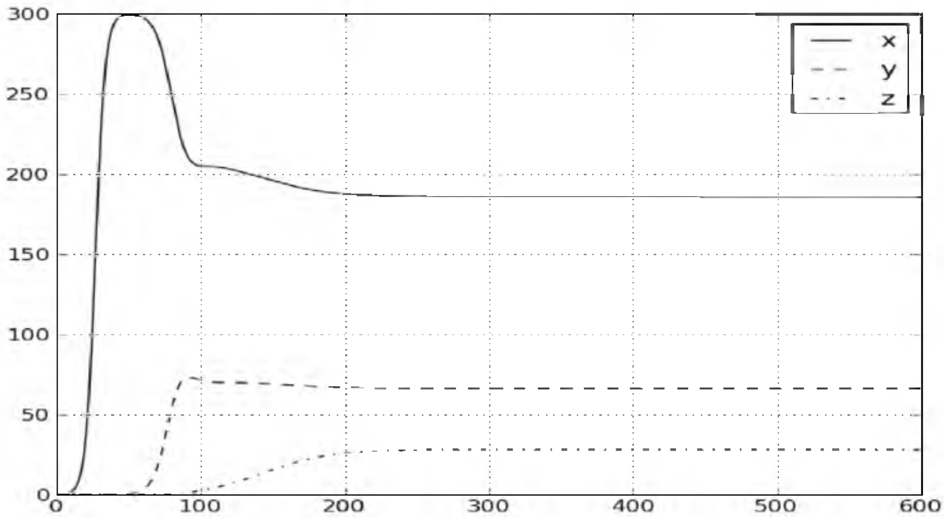
Given that $x^* > y^* > z^*$ (the number of the scientific personnel without academic degree exceeds the number of the candidates of sciences, the number of the candidates of sciences exceeds the number of the doctors of sciences) we will arrive at the following equation:

$$\frac{\alpha_1\beta_2 - \alpha_3\beta_1 + \alpha_2\gamma_1}{\alpha_1\gamma_1} > 1 > \frac{\alpha_1\beta_2 - \alpha_3\beta_1}{\alpha_3\gamma_1}. \quad (8)$$

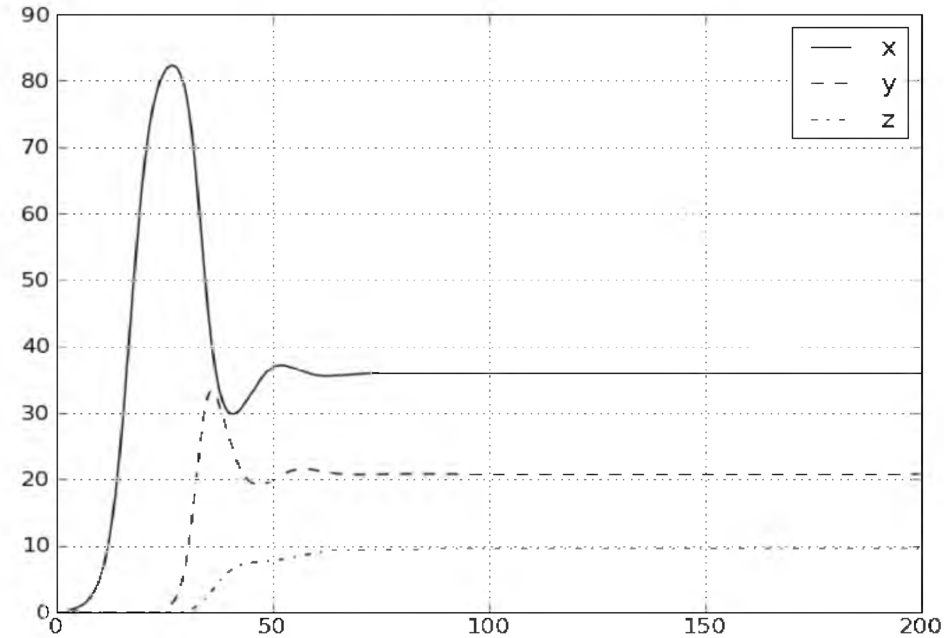
Having combined the inequalities (7) and (8), we will arrive at the following system of inequalities:

$$\begin{cases} \alpha_1\beta_2 - \alpha_3\beta_1 > 0 \\ (\alpha_1\beta_2 - \alpha_3\beta_1)(\alpha_1\beta_1 + \alpha_3\beta_1 - \alpha_2\gamma_1) + \alpha_1\alpha_2\beta_1\gamma_1 \\ \gamma_1(\alpha_2\beta_1 - \alpha_1\beta_2) + \beta_1(\alpha_1\beta_2 - \alpha_3\beta_1) > 0 \\ \alpha_1\beta_2 - \alpha_3\beta_1 + \alpha_2\gamma_1 - \alpha_1\gamma_1 > 0 \\ \alpha_3\gamma_1 - \alpha_1\beta_2 + \alpha_3\beta_1 > 0. \end{cases} \quad (9)$$

The numeric experiments conducted with the model (1) are represented in the Figure 1.



Model parameters and calculated coordinates of the nontrivial singular point: $\alpha_1 = 0.3$, $\alpha_2 = 0.2$, $\alpha_3 = 0.1$, $\beta_1 = 0.0013$, $\beta_2 = 0.0019$, $\gamma_1 = 0.001$, $\gamma_2 = 0.001$, $\varepsilon_1 = 0.001$, $\varepsilon_2 = 0.001$, $\varepsilon_3 = 0.001$, $x^* = 185.881$, $y^* = 66.176$, $z^* = 28.09$.



Model parameters and calculated coordinates of the nontrivial singular point: $\alpha_1 = 0.4$, $\alpha_2 = 0.24$, $\alpha_3 = 0.13$, $\beta_1 = 0.01$, $\beta_2 = 0.006$, $\gamma_1 = 0.0025$, $\gamma_2 = 0.02$, $\varepsilon_1 = 0.0047$, $\varepsilon_2 = 0.004$, $\varepsilon_3 = 0.004$, $x^* = 35.9$, $y^* = 20.75$, $z^* = 9.52$.

Figure 1. Results of the numeric experiments on the basis of the model (1)

The model parameters were matched in order to meet the conditions (9), despite that they were obtained by $\varepsilon_i = 0$, $\gamma_2 = 0$.

If within the simplified dynamic system (1) at $\varepsilon_i = 0, \gamma_2 = 0$ instead of generating the non-degree scientific personnel according to the Malthusian model there will be put the training of the personnel by the higher educational institutions at the constant rate a, as well as their withdrawal not related to transition to the category of the candidates of sciences with the coefficient α_1 then we will arrive at the following dynamic system:

$$\begin{cases} \dfrac{dx}{dt} = a - \alpha_1 x - \beta_1 xy - \gamma_1 xz \\ \dfrac{dy}{dt} = -\alpha_2 y - \beta_1 xy + \gamma_1 xz - \beta_2 yz \\ \dfrac{dz}{dt} = -\alpha_3 z + \beta_2 yz. \end{cases} \tag{10}$$

Here in order to ensure uniformity of representation of the withdrawal rate coefficients (α_i) for the non-degree (α_i) scientific personnel it was denoted by α_1 though in the previous dynamic system (1) α_1 expressed the reproduction rate (the aggregate growth ratio).

In contrast to the previous model the singular zero point is absent in the dynamic system (10). Here the two kinds of the singular points are present:

$$1.\, x^* = \frac{\alpha_2}{\beta_1}, y^* = \frac{a\beta_1 - \alpha_1\alpha_2}{\alpha_1\beta_1}, z^* = 0$$

$$2.\, x^*_{1,2} = \frac{1}{2}\left(\frac{a}{\alpha_1} + \frac{\alpha^2{}_3\beta_1}{\alpha_1\beta_2\gamma_1} + \frac{\alpha_3}{\gamma_1} - \frac{\alpha_2\alpha_3}{\alpha_1\beta_2}\right) \pm \tag{11}$$

$$\pm\sqrt{\frac{1}{4}\left(\frac{a}{\alpha_1} + \frac{\alpha^2{}_3\beta_1}{\alpha_1\beta_2\gamma_1} + \frac{\alpha_3}{\gamma_1} - \frac{\alpha_2\alpha_3}{\alpha_1\beta_2}\right)^2 - \frac{a\alpha_3}{\alpha_1\gamma_1}}, y^* = \frac{\alpha_3}{\beta_2}, z^* = \frac{a}{\alpha_3} - \frac{\alpha_2}{\beta_2} - \frac{\alpha_1}{\alpha_3}x^*.$$

The Jacobian matrix for the present linear system appears as a arbitrary of singular point as follows:

$$A = \begin{pmatrix} -\alpha_1 - \beta_1 y^* - \gamma_1 z^* & -\beta_1 x^* & -\gamma_1 x^* \\ \beta_1 y^* + \gamma_1 z^* & -\alpha_2 + \beta_1 x^* - \beta_2 z^* & \gamma_1 x^* - \beta_2 y^* \\ 0 & \beta_2 z^* & -\alpha_3 + \beta_2 y^* \end{pmatrix}. \tag{12}$$

Let's immediately write out the characteristic equation for the first singular point:

$$|A - \lambda I| = \left[\frac{\beta_2(a\beta_1 - \alpha_1\alpha_2) - \alpha_2\alpha_3\beta_1}{\alpha_2\beta_1} - \lambda\right][\lambda^2 + (\alpha_1 + \beta_1 y^*)\lambda + \beta_1^2 x^* y^*] = 0 \tag{13}$$

Here for the sake of convenience of the further analysis the singular point coordinate in the second square brackets was left in the general form. The eigenvalues were obtained as follows:

$$\lambda_1 = \frac{\beta_2(a\beta_1 - \alpha_1\alpha_2) - \alpha_2\alpha_3\beta_1}{\alpha_2\beta_1} \tag{14}$$

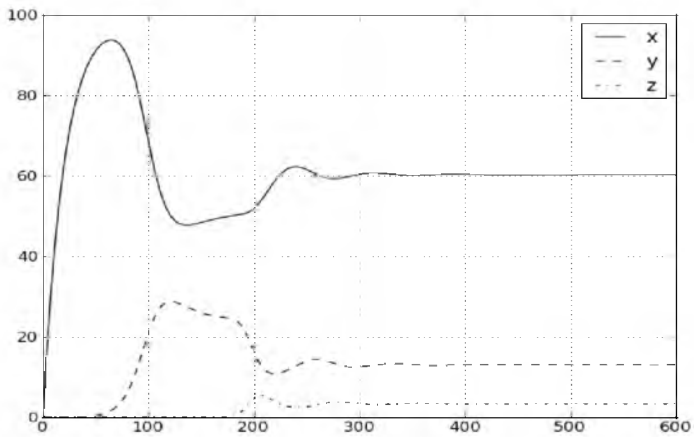
$$\lambda_{2,3} = \frac{-(\alpha_1 + \beta_1 y^*) \pm \sqrt{(\alpha_1 + \beta_1 y^*)^2 - 4\beta_1^2 x^* y^*}}{2} = \frac{1}{2}\left[-\frac{a\beta_1}{\alpha_2} \pm \sqrt{\frac{a^2\beta_1^2}{\alpha_2^2} + 4(\alpha_1\alpha_2 - a\beta_1)}\right]$$

It follows that at $\beta_2(a\beta_1 - \alpha_1\alpha_2) - \alpha_2\alpha_3\beta < 0$ in the case $\frac{a^2{\beta^2}_1}{{\alpha^2}_2} + 4\left(\alpha_1\alpha_2 - a\beta_1\right) \geq 0$, taking into consideration the expressions for $\lambda_{2,3}$ for the arbitrary singular point we will arrive at the stable node, and in the case $\frac{a^2{\beta^2}_1}{{\alpha^2}_2} + 4\left(\alpha_1\alpha_2 - a\beta_1\right) < 0$ – at the stable focus.

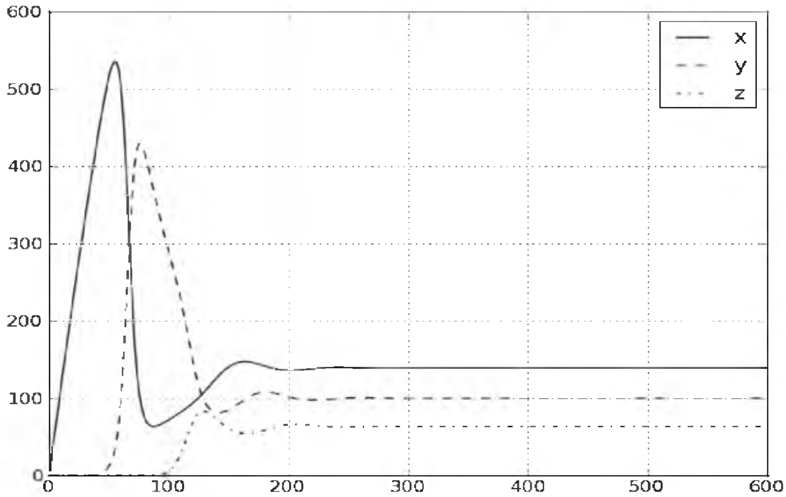
At the second singular point the Jacobian matrix (12) will be simplified a little provided that the explicit expression for y^* will be inserted into it:

$$A = \begin{pmatrix} -\alpha_1 - \frac{\alpha_3\beta_1}{\beta_2} - \gamma_1 z^* & -\beta_1 x^* & -\gamma_1 x^* \\ \frac{\alpha_3\beta_1}{\beta_2} + \gamma_1 z^* & -\alpha_2 + \beta_1 x^* - \beta_2 z^* & \gamma_1 x^* - \alpha_3 \\ 0 & \beta_2 z^* & 0 \end{pmatrix} \tag{15}$$

But the relevant characteristic cubic equation for λ will be rather lengthy. The numeric experiments based on this model that conjugate to the previous ones are represented in the Figure 2.



Model parameters and calculated coordinates of the nontrivial singular point:
$a = 10$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\alpha_3 = 0.1$, $\beta_1 = 0.0001$, $\beta_2 = 0.001$, $\gamma_1 = 0.001$, $x^* = 138.68$, $y^* = 100$, $z^* = 63.07$.



Model parameters and calculated coordinates of the nontrivial singular point:
$a = 12$, $\alpha_1 = 0.005$, $\alpha_2 = 0.05$, $\alpha_3 = 0.1$, $\beta_1 = 0.0005$, $\beta_2 = 0.001$, $\gamma_1 = 0.005$, $x^* = 60.3$, $y^* = 10.07$, $z^* = 3.39$.

Figure 2. Results of the numeric experiments on the basis of the model (10)

In both numeric experiments (Fig. 1, 2) we have obtained the faint stable focuses upon satisfaction of the condition $x^* > y^* > z^*$ with the wave of amount of the candidates of sciences surpassing the wave of number of the non-degree scientific personnel and the wave of number of doctors of sciences surpassing the wave of number of candidates of sciences.

Please note that a great number of parameters in the models (1, 10) make it difficult to conduct the comprehensive analysis close to the actual area of the scientific personnel training. The specific empiric studies concerning evaluation of the model parameters and relations between the stationary levels of the different scientific personnel categories are required. The numerical calculations were made on the basis of the special program created with the use of SciPy (Scientific Python).

Conclusion

So, there was conducted the qualitative and numeric analysis of the previously constructed mathematic model of the scientific personnel training and dynamics. By the qualitative study of this model the linear stability analysis and Routh-Hurwitz criteria were used, by the numeric study – the special program created with the use of SciPy (Scientific Python). The numeric experiments conducted with the model showed that the Routh-Hurwitz criteria obtained by means of the analytic method were satisfied.

Further approbation of this model requires performance of the specific empiric studies concerning evaluation of the model parameters and relations between the stationary levels of the different scientific personnel categories.

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