THE PROBABILISTIC NATURE OF SOIL FORMATION AND EROSION - ON MATHEMATICALLY MODELED DEFLATION PROCESSES

Burjak D. V., Lisetskiy, F. N.› I. I. Mechnikov[1]

When designing soil protection plans for agricultural landscapes and proposing anti-erosion measures on the basis of calculations, energy parameters for rainfall and runoff during snow thawing, wind velocity, and in some cases, soil properties are subject to probability characteristics. Also, prediction of erosion by water and/or by wind hazards is made to a specific probability level. Values of tolerable soil loss rate, correlated with the given estimations, are of a statical nature, which are connected with the method of their determination and lack of dynamic models of soil formation processes.

Thus, the problem of construction of such a dynamic model based on the data obtained from a finite time interval (T - time interval or total age, from 0 to 6000 years) has been developed to form the most authentic representation of the process development to which these data are related, i.e., of its basic characteristics.

New approach employing modern methods of spectral estimation has been used to solve stated problem. The essence of these methods is connected with wide application of model representations of analyzed processes, taking account of peculiar to them inherent relationships which are usually neglected in classic spectral analysis.

[1]Polytechnical University, Odessa, Ukraine
State University, Odessa, Ukraine

We have examined a process resulting a synchronous λn H(t) representing humus horizon thickness:

$$H(t) = V(t) - W(t) \qquad (1)$$

where V(t) - function of increase in H(t) owing to soil formation process; W(t) - function of decrease in H(t) owing to exhibiting soil degrading processes (rainfall erosion, deflation). It should be noted that EQ. (1) is also valid when investigating soil formation process under automorphous conditions.

Speed of soil formation process may be reasonably represented in a form of differential equation:

$$\frac{dH}{dt} = \lambda (H_{\infty} - H) + \Omega(t, H(\cdot)) \qquad (2)$$

where λ - bioclimatic coefficient; H_{∞} - height (quasi-climax) thickness of humus horizon over a zonal autonomous landscape; Ω(t, H(·)) - operator-function (disturbance function) representing fluctuations of soil profile formation and degradation processes occurring through a system of hierarchically arranged cycles (such as solar activity, hydrothermal regime, etc.).

Consistent approach has enabled us to consider all methods of spectral estimation as methods of data approximation with the aid of any model accepted ([5]). Characteristic feature of the offered method is that the model describing variation of humus horizon thickness with time synthesized three models:

1st model - $H_{\tau}(t)$ indicating deterministic tendency of temporal row (trend);

2nd model - $H_{\iota f}(t)$ describing over century cyclic recurrence (cycles represented dimensionally as $n \cdot 10^2$, $n \in [6;14]$) of functions V(t) and

W(t) (low-frequency spectrum);

3rd model - $H_{hf}(t)$ reflecting cycles represented

dimensionally as $n \cdot 10^2$, $n \in [1;6]$ (high-

frequency spectrum)

FIRST model, $H_\tau(t)$, is analytical one obtained from the solution of differential equation representing soil formation age from 0 to 10,000 years (holocene):

$$\frac{dH}{dt} = \lambda(H_\infty - H) \quad (3)$$

where under the appropriate initial conditions H(-2000) = 0 is a prehistory of soil formation, and may be represented as ([4]):

$$H_\tau(t) = \alpha g(\frac{F_f}{F_z})^\beta \exp[\gamma \operatorname{Re} xp(-\delta \frac{R^\nu}{p})](1 - k\exp(-\lambda t)) \quad (4)$$

where α, β, γ, δ, ν, k - empirically determined constants; g - coefficient representing granulometric composition of soils; F_f - actual productivity of vegetation; F_z - zonal productivity of vegetation; R - radiation balance of the Earth's surface; p - annual amount of precipitation.

SECOND model, $H_{\iota f}(t)$, is a classical one (i.e., model obtained by using classical methods of spectral estimation; we used Fourier transformation) which may be represented as:

$$H_{\iota f}(t) = \frac{T}{N}[\sum_{\eta=0}^{N-1} h_\eta \exp(-j2\pi t n T)]^2 \quad (5)$$

where h_o, ..., h_{n-1} - finite variety of initial data obtained by instrumental (C^{14}, etc.) and soil-archaeological methods; $\frac{1}{2T} \le t \le \frac{1}{2T}$ (T, as mentioned above, is time interval from 0-6000 years).

THIRD model, $H_{hf}(t)$, is a parametrical model of autoregression of moving average ([5]):

$$H_{hf}(t) = Tp_{\nu} \cdot \left| \frac{1+\sum_{l=1}^{q} b_l \exp(-j2\pi t l T)}{1+\sum_{m=1}^{s} a_m \exp(-j2\pi t m T)} \right|^2 \tag{6}$$

where p_{ω} - variance of initiating sequence (of white noise); s and q - orders of corresponding models of autoregression and moving average (specially selected and varied to achieve required compromise between values of variance, resolution and bias); a_m ($M=\overline{1,s}$) and $b_l(l=\overline{1,q})$ - parameters of chosen models.

Thus, FINAL model is appeared as:

$$H(t) = \bar{H}(t) + sgnH_{hf}(t), \tag{7}$$

where

1) $\bar{H}(t) = H_\tau(t) + sgnH_{lf}(t)$

$$\text{at } sgnh_{lf}(t) = \begin{cases} +H_{lf}(t) & if \quad H_{lf}(t) \geq H_T(t), \\ -H_{lf}(t) & if \quad H_{lf}(t) < H_T(t), \\ \forall t \in & [-\frac{1}{2T}, \quad \frac{1}{2T}] \end{cases}$$

$$2)\ sgnH_{hf}(t) = \begin{cases} +h_{hf}(t) & if \quad H_{hf}(t) \geq \bar{(t)}, \\ -H_{hf}(t), & if \quad H_{hf}(t) < \bar{H}(t), \\ \forall t \in & [-\frac{1}{2T} \quad \frac{1}{2T}] \end{cases}$$

(It is understood that $H_{lf}(t)$ and $H_{hf}(t)$ describe the disturbance function).

Note the following. If we consider that t -> + ∞, singular quasi linear differential-operator equation (2) is derived. Then disturbance function Ω(t, H(·)) may be considered as operator function defined in some class of functions asymptotically equal at t -> + ∞ to the solution of Eq. (3), i.e., to the solution of the form (4). For this case we established sufficient conditions (on the basis of [1] and [2] where method developed by R. G. Grabovskaya has been used) at which for the arbitrary solution of the Eq. (3) particularly, for the solution of (4), exists asymptotically equal to said at t-> + ∞ solution of the initial Eq. (2), satisfying specific estimations.

Reverting to final model (7), note that the presence of classical side (5) is based on the fact that classical methods are most structurally stable or, as usually spoken now, most robust in a sense of spectral estimations in such cases when most substantial values of function are attempted to be estimated at correlation shifts varying from 0 to some peak value M. In our case, this is a description of low frequency signals. Model (5) constructed by pedochronological data has been examined by us against low frequency component of solar activity which is a principal generator of cyclic recurrence in the biosphere ([3]). The fact that astroclimatic cycles give rise to soil formation process allows to make use of constructed model (5), as follows, over-century cycles of solar activity established with its aid, for long-term forecasts of soil formation rates variations in a natural trend.

As is presently known, highly important in estimation of soil resource formation speed are long-term cycles of solar activity in terms of $n \cdot 10^2$ years (n∈ [16]) which spectra contain sharply expressed peaks and deep troughs, that is what defines high-frequency component. Such pattern is of particularizing importance attaching probability nature to the model. Classical model is not applicable to such a case as so-called "window" effect arises in the given situation, i.e., faint spectral lines are masked by side lobes of stronger spectral lines. Attempts to describe high-frequency signals led to the application of parametric models, specifically, of model (6) suitable for modeling both sharp peaks and deep troughs. This model is parameterized by some finite and (sic!) small set of coefficients. As a result, less biased spectral estimations are obtained to have higher resolution. Furthermore, model (6) has more degrees of freedom than other parameteric models, the - before it should be expected has spectral estimations obtained with its aid will display wider possibilities for reproducing forms of different spectra. It should be also noted that practically no studies of statistical characteristics of spectral estimations have been made against the model (6) and most of information on their properties is based on the results of a few model experiments. Unfortunately, available annual instrumental observations of solar activity, firstly, cover short periods of time (as from 1700) and secondly are not applicable for modeling without having been previously examined.

Thus, it may be easily seen that final model (7) integrates basic components of soil formation process in time and is of probability nature. It is also obvious that only generalities in statistical estimations of modulations of different signs may be determined at a polyperiodical nature of variation of humus horizon thickness in holocene. For the last 6000 years of soil formation the substantial excess in thickness trend values (up to 90mm) from the total number of cycles within low frequency component has been observed in 50% of cases. Within high frequency modulations the probability of excess of thickness trend values (up to 50mm) has reached 64%, while the probability of substantial reduction of H(t) (up to 50mm) at the expense of soil degrading processes has reached 50% of the total number of cycles for the last 6000 years.

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