INFLUENCE OF THE TARGET SHAPE ON THE TRANSITION RADIATION FROM RELATIVISTIC ELECTRONS

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The problem of transition radiation from relativistic electrons in a translucent disc and a sphere is considered. It is shown that the shape and transverse dimensions of the target may significantly affect the spectral and angular characteristics of transition radiation in the frequency range $\omega \le c\gamma a^{-1}$ (γ is the Lorentz factor of the electron and a is the characteristic transverse size of the target).

The process of transition radiation of a relativistic electron in a thin layer of a substance develops in a spatial region extended along the particle propagation direction, whose length $l_c = 2\gamma^2\lambda$ considerably exceeds the radiated wavelength λ (γ is the Lorentz factor of the electron). In [1] it was demonstrated that not only longitudinal l_c but also the transverse distance ρ_{eff}, responsible for the formation of transition radiation, may have a macroscopic size considerably exceeding the radiated wavelength. The order of magnitude of these distances is determined by the relation $\rho_{\text{eff}} \sim \gamma\lambda$. Moreover, when the characteristic transverse size of the target a is comparable to ρ_{eff}, the influence of the transverse dimensions of the target on the transition radiation must be taken into account.

In the present work, using problems of transition radiation from relativistic electrons in a translucent disc and a sphere as the simplest examples, it is demonstrated that the angular characteristics of transition radiation depend quite noticeably on the geometry of the target. It is also demonstrated that the angular distributions of transition radiation depend significantly on the frequency of the emitted photon for wavelengths $\lambda \sim a/\gamma$. This means that the diagnostics of targets can be based on a study of the characteristics of transition radiation. This problem becomes especially important for investigations of nanostructures.

We now consider the transition radiation of a relativistic electron passing with constant velocity $\boldsymbol{v}$ through a thin layer of a substance with permittivity $\varepsilon(\boldsymbol{r}, \omega)$ slightly different from unity:

$$\varepsilon(\boldsymbol{r},\omega) = 1 + \varepsilon_1(\boldsymbol{r},\omega), \tag{1}$$

where $\varepsilon_1 \ll 1$. In this case, the equation for the Fourier component of the electric field $\boldsymbol{E}(\boldsymbol{r}, \omega)$ can be written in the form (for example, see [2, 3])

$$\left(\Delta + \frac{\omega^2}{c^2} - \nabla\,\text{div}\right)\boldsymbol{E}(\boldsymbol{r},\omega) = \frac{4\pi i\omega}{c}\boldsymbol{j}(\boldsymbol{r},\omega) + \frac{\varepsilon_1\omega^2}{c^2}\boldsymbol{E}(\boldsymbol{r},\omega)\,, \tag{2}$$

where $\boldsymbol{j}(\boldsymbol{r}, \omega)$ is the Fourier component of the particle current. Using the Green's function method (see [4]), we can easily derive the following expression for the spectral and angular radiant density:

$$\frac{d^2S}{d\omega\, do} = \frac{1}{4\pi^2 c}\left|\boldsymbol{k}\times\boldsymbol{I}\right|^2, \tag{3}$$

where $\boldsymbol{k}$ is the wave vector in the direction of radiation propagation, $|\boldsymbol{k}| = \omega/c$, and

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$$\boldsymbol{I} = \frac{i\omega}{4\pi}\int d^3 r\, \varepsilon_1(\boldsymbol{r},\omega)\boldsymbol{E}(\boldsymbol{r},\omega)e^{-i\boldsymbol{kr}} . \tag{4}$$

Formula (4) demonstrates that to find the spectral and angular radiant density, we must know the field $\boldsymbol{E}(\boldsymbol{r}, \omega)$ within the limits of the target. The spectral and angular density of radiation emitted by the electron when it passes through the target of finite dimensions is determined by the sum of the particle field and the field of the response of the medium within the limits of the target.

In the simplest case in which the target perturbs the particle field only slightly (that is, $|\varepsilon_1| \ll 1$), the solution of Eq. (2) may be expanded in a series in powers of ε_1 [2–5]. The first term of this expansion represents the field of the particle moving with constant velocity $\boldsymbol{v}$ in vacuum:

$$\boldsymbol{E}^e(\boldsymbol{r},\omega) = \left[\frac{\boldsymbol{\rho}}{\rho}\frac{e\omega}{v^2\gamma}2K_1(\omega\rho/v\gamma) - i\frac{\boldsymbol{v}}{v}\frac{e\omega}{v\gamma^2}2K_0(\omega\rho/v\gamma)\right]e^{i\omega z/v} . \tag{5}$$

Here e is the electron charge, z and $\boldsymbol{r}$ are coordinates parallel and orthogonal to $\boldsymbol{v}$, and K_0 and K_1 are the MacDonald functions of the zero and first orders, respectively [6].

Substituting Eq. (5) into Eq. (4), we obtain

$$\boldsymbol{I} = -\frac{i\omega}{4\pi}\int d^3 r\, \varepsilon_1(\boldsymbol{r},\omega)\boldsymbol{E}^e(\boldsymbol{r},\omega)e^{-i\boldsymbol{kr}} . \tag{6}$$

Formulas (3) and (6) determine the spectral and angular radiant density in the translucent target. They are valid for a thin layer of the substance if the condition

$$|\varepsilon_1 a_z \omega/c| \ll 1 \tag{7}$$

is satisfied within the limits of the target, where a_z is the thickness of the target. Moreover, if the thickness of the target is small compared to the radiated wavelength λ, integration over the thickness of the target in Eq. (6) can be performed analytically, and integration over the transverse coordinates will be determined by the geometry of the target surface. Let us present the results of integration for the simplest cases in which the electron passes through the center of a thin disc of radius b and through the center of a sphere of radius R.

For the thin disc of radius b, we have

$$\frac{d^2 S}{d\omega\, do} = \frac{e^2}{4\pi^2 c}\left(\frac{\varepsilon_1 a_z \omega}{c}\right)^2 \frac{\sin^2\theta\cos^2\theta}{(\sin^2\theta + \gamma^{-2})^2} F^2\left(\gamma\sin\theta, \frac{\omega}{\omega_\perp}\right), \tag{8}$$

where θ is the emission angle and $\omega_\perp = v\gamma/b$, $F(y, x)$ is the function defining the influence of the transverse dimensions of the disc on the transition radiation [5]:

$$F(x, y) = \frac{y^2+1}{y}\int_0^x u\, du\, K_1(u)J_1(yu)` . \tag{9}$$

For high frequencies ($\omega \gg \omega_\perp$) and $\gamma\theta \sim 1$, the function F is close to unity. This means that for frequencies $\omega \gg \omega_\perp$, the transition radiation in the finite disc is analogous to that in the disc of infinite transverse dimensions.

For low frequencies ($\omega \le \omega_\perp$) and $\gamma\theta \sim 1$, we have

$$F(x, y) \approx \frac{1}{4}(y^2+1)x^2 . \tag{10}$$

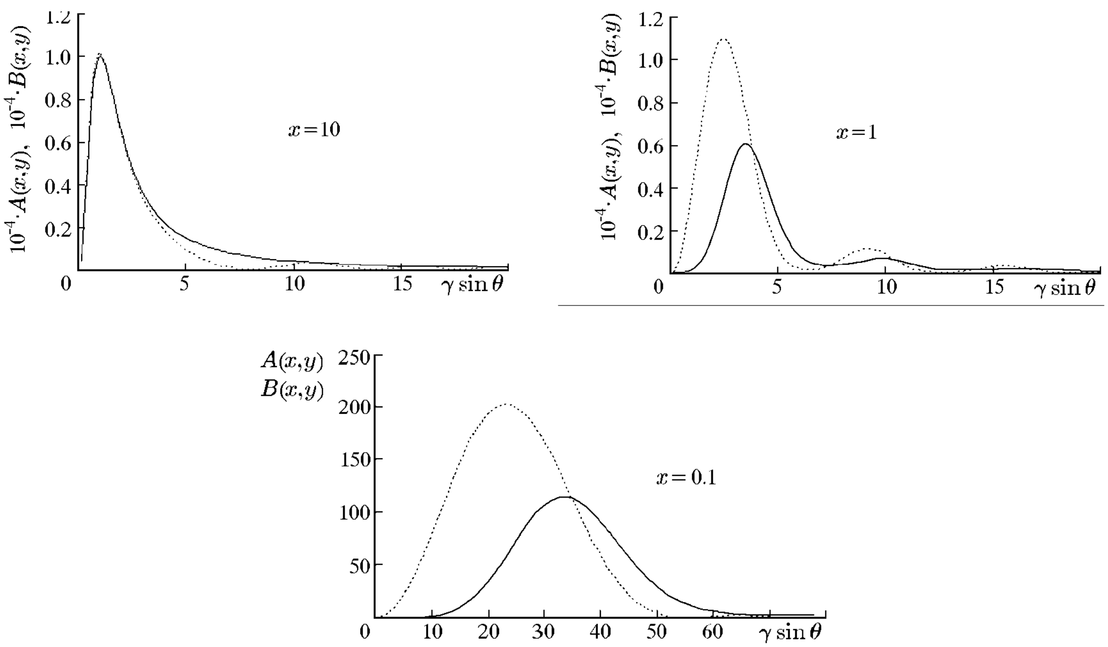

Fig. 1. Angular distributions of transition radiation from electrons with energy 100 MeV passing through the center of the thin disc (solid curves) and sphere (dashed curves) as functions of the frequency of emitted photons.

The spectral and angular radiant density in this frequency range decreases rapidly as the frequency increases. Thus, the characteristics of transition radiation from the electron in the disc change at $\omega \sim \omega_\perp$. An analysis of Eq. (8) also demonstrates that for $\omega > \omega_\perp$ the maximum in the spectral and angular radiant density is at the angles $\theta \sim \gamma^{-1}$, whereas for $\omega \leq \omega_\perp$ this spectral and angular radiation distribution broadens considerably (Fig. 1).

For a thin sphere of radius R, we obtain

$$\frac{d^2S}{d\omega\, do} = \frac{e^2}{4\pi^2 c}\left(\frac{\varepsilon_1 a\omega}{c}\right)^2 A\left(\gamma\sin\theta, \frac{\omega}{\omega'_\perp}\right), \tag{11}$$

where a is the thickness of the layer of the sphere, $\omega'_\perp = v\gamma/R$, and

$$A(x,y) = x^2\gamma^2\cos^2\theta\left\{\int_0^x u\,du\,K_1(u)J_1(yu)\frac{\cos([1+y^2\sqrt{x^2-u^2}]/[2\gamma])}{\sqrt{x^2-u^2}}\right\}^2. \tag{12}$$

Figure 1 shows plots of the function $A(x, y)$ and the function

$$B(x,y) = \frac{\sin^2\theta\cos^2\theta}{(\sin^2\theta+\gamma^{-2})^2}F^2(x,y)$$

for the indicated values of the parameter $x = \omega/\omega_\perp$. The calculations were done for electrons with energy 100 MeV. The results obtained demonstrate that for frequencies $\omega \geq 10\omega_\perp$, the maximum of the angular distribution of transition radiation is at $\theta \sim \gamma^{-1}$, and for these angles the radiation distribution virtually does not depend on the shape of the target surface. For frequencies $\omega \leq \omega_\perp$, the angular radiation distribution depends significantly on this parameter. In this case, the maximum of the angular radiation distribution is shifted toward angles θ larger than $\theta \sim \gamma^{-1}$. Thus, the transverse dimensions and the shape of the target surface affect strongly the spectral and angular characteristics of transition radiation from the relativistic electron for frequencies $\omega \leq \omega_{\perp}$.

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